Stochastic programming decision for inland container liner route stowage planning with uncertain container weight

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Abstract: For the internal trade export containers, the uncertainty of weight information which is provided by the cargo owners when booking the shipping space has made the route stowage planning decision more complex. Existing research lacks the consideration of uncertain factors for route stowage planning decision in container shipping. Based on the stochastic programming theory, the stochastic parameter is utilized to describe the uncertain container weight to propose the stochastic programming model (SPM) for inland container liner route stowage planning decision with the objective of minimizing the ship stack occupancy number on the route. Due to the stochastic constraints in SPM, the chance constraints are utilized to describe them to construct the stochastic chance-constrained programming model (SCPM). Since SPM and SCPM cannot be solved with the standard solvers, two different methods are proposed to convert and solve them. First the deterministic equivalent model (DEM) of SCPM is obtained through the deterministic equivalent transformation. Then the robust optimization model (ROM) is obtained based on SPM under the idea of robust optimization. Experimental and simulation results show that both DEM and ROM can be used to solve the proposed problem efficiently.

Keywords: Inland container liner shipping, route stowage planning, uncertain container weight, stochastic programming, robust optimization.

1. INTRODUCTION

Owing to the relatively limited capacity of the inland container ship, unlike in maritime container shipping, the carriers are more emphasis on the capacity utilization in the inland container liner shipping. The limited capacity also results in a general sensitivity of stability to the stowage plan. A change in the stowing positions or weights of a few containers can alter a ship’s stability. This makes an efficient stowage plan crucial to the operation of an inland container ship. For the transportation cost reduction, the cargo owners sometimes conceal the accurate weight information of domestic trade export containers causing the uncertain container weight in stowage planning. The uncertain container weight makes the route stowage planning decision in inland container liner shipping more complex.

Current research on the stowage planning typically focuses on maritime container shipping and often assumes the container information of each port known to make the route stowage plans. A considerable amount of work utilizes the heuristic algorithms [1-3], genetic algorithms [4-5], multi-phase approaches [6-7] and integer programming [8-9]. The research lacks the consideration of uncertainty and cannot satisfy the requirement of inland container liner route stowage planning decision with uncertain container weight.

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The weight of domestic trade export containers in the inland container liner shipping has its uncertainty. The stochastic parameter is utilized to describe it, and the route stowage planning decision problem with stochastic parameters in the constraints belongs to a stochastic programming problem. To date, the stochastic programming research in the container shipping usually concerns the liner ship route schedules [10-11], ship schedules [12-13], cargo mix problem [14], empty container reposition [15], berth allocations [16-17] and so on. These studies have shown that the application of stochastic programming in container shipping can achieve the effective absorption of the stochastic disturbance to ensure the robustness of the scheme.

In conclusion, most research on the stowage planning problem generally focuses on the maritime container shipping and lacks the consideration of stochastic disturbance. The stochastic programming can be utilized to deal with stochastic disturbance in decision making. In this study, the inland container liner route stowage planning decision with uncertain container weight is researched based on the stochastic programming theory. The stochastic programming model (SPM) is constructed with the objective of minimizing the ship stack occupancy number on the route. Due to the stochastic constraints in SPM, the stochastic chance-constrained programming model (SCPM) is constructed by describing them with chance constraints. Since there are stochastic parameters in SPM and SCPM, they cannot be solved with the standard solvers. Thus, two different methods are proposed for solving the proposed models. For SCPM, its deterministic equivalent model (DEM) is obtained by using the deterministic equivalent transformation in the chance-constrained programming and the inequation of absolute value. For SPM, it is converted into the robust optimization model (ROM) considering the optimization of the worst case based on the idea of robust optimization.

The rest of the paper is organized as follows. The problem is described in Section 2. Section 3 presents the mathematical formulations for the problem. The solving methods are introduced in Section 4. Experiments are shown in Section 5, and in Section 6, the conclusion is presented and future work is discussed.

2. PROBLEM DESCRIPTION

In the inland container liner route stowage planning decision, the stowage plan of current port will be the input for stowage planning of next port on the route as shown in Figure 1. The ship visits the ports sequentially to transport the containers between different ports on the route. The ship planners use the information provided by the cargo owners when booking the shipping space to make the pre-stowage plan of current port and reserve the space for the subsequent ports. The stowage planners at current port combine the collection process information of domestic trade export containers and the pre-stowage plan to complete the stowage plan for the ship.

For the domestic trade export containers, the cargo owners sometimes conceal the accurate weight information. It will cause the uncertain container weight in stowage planning. The ship planner makes the stowage plan based on this information causing the weak robustness of the plan. This may make the pre-stowage plan provided by the ship planner becoming infeasible and the stowage planner at port needs to take a lot of extra time to modify the pre-stowage plan.

Therefore, the inland container liner route stowage planning needs to consider the disturbance of uncertain container weight for domestic trade export containers. The stowage plans for each port on the route are made by considering the transportation demands between different ports comprehensively while satisfying the ship's constraints of stability, strength, and capacity. The ship stack occupancy number on the route is minimized considering the peculiarity of inland container liner shipping to ensure the efficient utilization of the occupied stacks and ship's capacity.

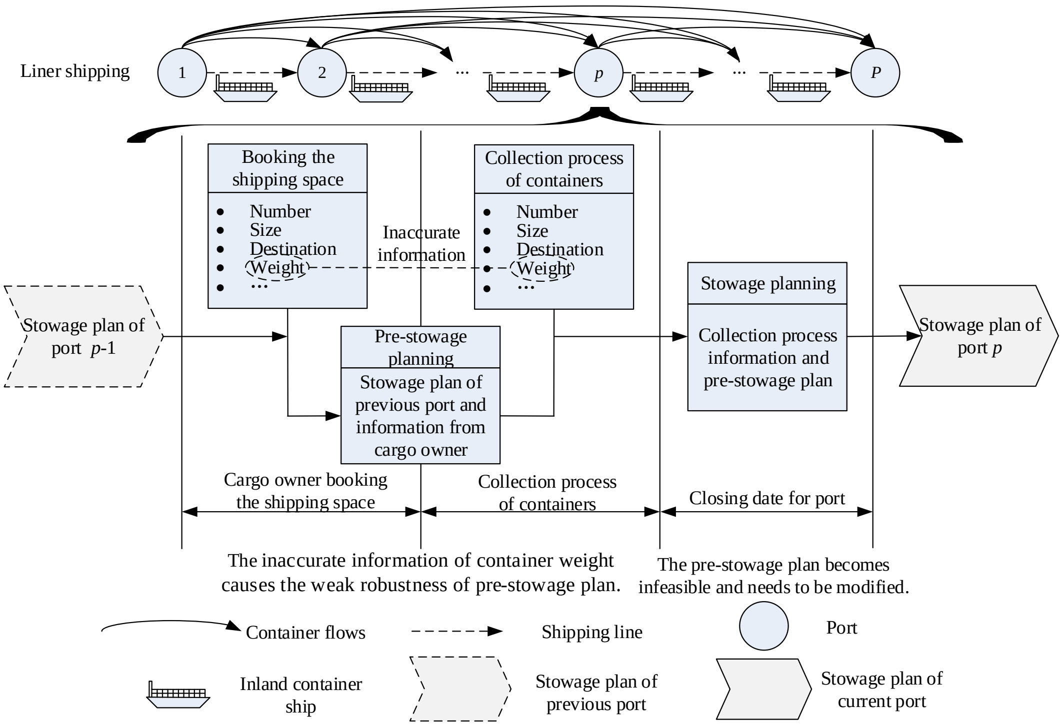

Figure 1 Inland container liner route stowage planning decision

3. MATHEMATICAL FORMULATIONS

The structure of a typical inland container ship on the Yangtze River is shown in Figure 2. For the mathematical formulations, the ship stacks are divided into different stack sets according to the front half, back half, left half, and right half of the ship.

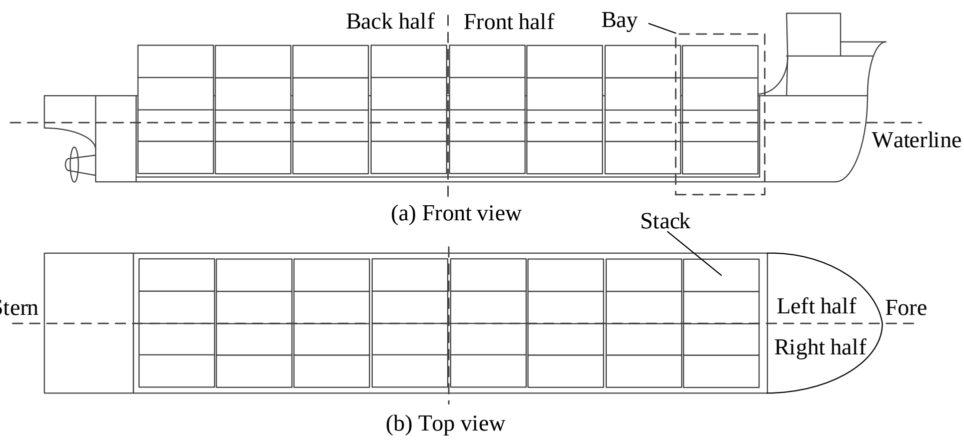

Figure 2 The structure of an inland container ship

(1) Sets

P : Set of ports on the route

$Q(p)$: Set of $o-d$ shipping at port p , $Q(p) = Q_t(p) \bigcup Q_s(p), \forall p \in P$

$Q_t(p)$: Subset of $o-d$ shipping passing through port p , $Q_t(p) = \{a \mid o, p, d \in P, o \prec p \prec d, a = (o,d)\}$

$Q_s(p)$: Subset of $o-d$ shipping starting at port p , $Q_s(p) = \{a \mid o, p, d \in P, o = p \prec d, a = (o,d)\}$

G : Set of container weight classes (light, medium, heavy), $G = \{1,2,3\}$

J : Set of ship stacks, $J = J_F \bigcup J_A = J_L \bigcup J_R$

J_F : Subset of ship stacks in the front half of ship

J_A : Subset of ship stacks in the back half of ship

J_L : Subset of ship stacks in the left half of ship

J_R : Subset of ship stacks in the right half of ship

(2) Parameters

$N_g(a)$: Number of containers with weight class g to be loaded at port p and destined for port d (Unit: TEU), $\forall a = (o,d) \in Q_s(p), p \in P,\ g \in G$

w_g : Average weight of containers with weight class g (Unit: ton), $\forall g \in G$

$\tilde{w}_g$: Stochastic weight of containers with weight class g (Unit: ton), $\forall g \in G$

ΔLG : Maximum longitudinal weight tolerance of ship (Unit: ton)

ΔCG : Maximum horizontal weight tolerance of ship (Unit: ton)

SW_j : Maximum load weight of stack j (Unit: ton), $\forall j \in J$

ST_j : Maximum stacking tiers (or capacity limit) of stack j (Unit: TEU), $\forall j \in J$

L : A large number, $L = 1000$

(3) Variables

$x_{jg}(a)$: Container number of weight class g in stack j with $o-d$ shipping a , $\forall\ g \in G,\ j \in J, a \in Q(p),\ p \in P$

$y_j(a)$: $y_j(a)$=1 , if stack j stows containers with $o-d$ shipping a ; otherwise $y_j(a)$=0 . $\forall j \in J, a \in Q(p),\ p \in P$

The SPM for inland container liner route stowage planning decision with uncertain container weight can be formulated as follows:

$$f = \min \sum_{p \in P} \sum_{a \in Q(p)} \sum_{j \in J} y_j(a) \tag{1}$$

$$\sum_{j \in J} x_{jg}(a) = N_g(a), \forall p \in P, a \in Q_s(p), g \in G \tag{2}$$

$$\sum_{j \in J} y_j(a) \geq 1, \quad \forall p \in P,\ a \in Q(p) \tag{3}$$

$$\sum_{a \in Q(p)} y_j(a) \leq 1, \forall p \in P,\ j \in J \tag{4}$$

$$y_j(a) \leq \sum_{g \in G} x_{jg}(a) \leq L \cdot y_j(a), \forall j \in J, a \in Q(p), p \in P \tag{5}$$

$$\sum_{a \in Q(p)} \sum_{g \in G} x_{jg}(a) \leq ST_j, \forall j \in J,\ p \in P \tag{6}$$

$$\sum_{a \in Q(p)} \sum_{g \in G} x_{jg}(a) \cdot \tilde{w}_g \leq SW_j, \forall j \in J, p \in P \tag{7}$$

$$\left| \sum_{a \in Q(p)} \sum_{g \in G} \left(\sum_{j \in J_F} x_{jg}(a) \cdot \tilde{w}_g - \sum_{j \in J_A} x_{jg}(a) \cdot \tilde{w}_g \right) \right| \leq \Delta LG, \forall p \in P \tag{8}$$

$$\left|\sum_{a\in Q(p)}\sum_{g\in G}\left(\sum_{j\in J_L} x_{jg}(a)\cdot\tilde{w}_g-\sum_{j\in J_R} x_{jg}(a)\cdot\tilde{w}_g\right)\right|\le \Delta CG,\ \forall p\in P \tag{9}$$

$$x_{jg}(a)\ge 0,\ x_{jg}(a)\in Z,\forall j\in J, g\in G, a\in Q(p), p\in P \tag{10}$$

$$y_j(a)=\{0,1\},\ \ \forall j\in J,\ p\in P, a\in Q(p) \tag{11}$$

The objective function (1) minimizes the ship stack occupancy number on the route to ensure the efficient utilization of the occupied stacks and the optimized ship capacity utilization. Constraint (2) ensures that all the containers can be loaded at each port on the route. Constraint (3) ensures that all the containers within same $o-d$ shipping should occupy at least one stack. Constraint (4) ensures that each stack can at most be occupied by containers within one $o-d$ shipping to avoid overstowage. Constraint (5) defines the relationships between different variables. $x_{jg}(a)>0$ means that there are containers with $o-d$ shipping a in stack j, in which case $y_j(a)$ should be equal to 1. Similarly, $x_{jg}(a)=0$ means that there are no containers with $o-d$ shipping a in stack j, in which case $y_j(a)$ should be equal to 0. Constraints (6) and (7) ensure that the load weight and capacity constraints of each stack are within limits at each port. Constraints (8) and (9) ensure that the longitudinal and horizontal weight differences of the ship are within its required limit at each port. Constraints (10) and (11) constrain the respective values of several variables used in this formulation.

The SPM cannot be solved by using the standard solvers as constraints (7)-(9) are all stochastic constraints. The stochastic constraints are described by using chance-constrained programming based on the stochastic programming theory. Then the SCPM for the proposed problem can be formulated as follows：

$$\Pr\left\{\sum_{a\in Q(p)}\sum_{g\in G} x_{jg}(a)\cdot\tilde{w}_g\le SW_j\right\}\ge\alpha,\ \forall j\in J, p\in P \tag{12}$$

$$\Pr\left\{\left|\sum_{a\in Q(p)}\sum_{g\in G}\left(\sum_{j\in J_F} x_{jg}(a)\cdot\tilde{w}_g-\sum_{j\in J_A} x_{jg}(a)\cdot\tilde{w}_g\right)\right|\le \Delta LG\right\}\ge\alpha,\ \forall p\in P \tag{13}$$

$$\Pr\left\{\left|\sum_{a\in Q(p)}\sum_{g\in G}\left(\sum_{j\in J_L} x_{jg}(a)\cdot\tilde{w}_g-\sum_{j\in J_R} x_{jg}(a)\cdot\tilde{w}_g\right)\right|\le \Delta CG\right\}\ge\alpha,\ \forall p\in P \tag{14}$$

α is the confidence requirement ($\alpha=0.95$).

$$(\text{SCPM})\ \left\{f=\min\sum_{p\in P}\sum_{a\in Q(p)}\sum_{j\in J} y_j(a):(2)\sim(6),(12)\sim(14),(10),(11)\right\}.$$

The lower bound model (LBM) of the proposed problem can be obtained by deleting the constraints (7)-(9) in SPM. The result of LBM can be used as the theory lower bound because it ignores the constraints which consider the load weight and stability limits of the ship.

$$(\text{LBM})\ \left\{f=\min\sum_{p\in P}\sum_{a\in Q(p)}\sum_{j\in J} y_j(a):(2)\sim(6),(10),(11)\right\}.$$

4. SOLVING METHODS

Due to the stochastic parameters in the proposed models SPM and SCPM, they cannot be solved by using the standard solvers. So we proposed two different solving methods to convert and solve them with a standard solver. For SCPM, considering the stochastic parameters subject to normal distributions, we use the deterministic equivalent transformation in the chance-constrained programming and the inequation of absolute value to generate the DEM. For SPM, we generate the ROM considering the optimization of the worst case.

4.1. Deterministic Equivalent Transformation

In equation (15), E_{pg} represents the number difference of containers with same weight class between front half and back half of the ship at each port. In equation (15), F_{pg} represents the number difference of containers with same weight class between left half and right half of the ship at each port.

$$E_{pg} = \sum_{a \in Q(p)} (\sum_{j \in J_F} x_{jg}(a) - \sum_{j \in J_A} x_{jg}(a)), \forall g \in G, p \in P \tag{15}$$

$$F_{pg} = \sum_{a \in Q(p)} (\sum_{j \in J_L} x_{jg}(a) - \sum_{j \in J_R} x_{jg}(a)), \forall g \in G, p \in P \tag{16}$$

Constraints (13) and (14) can be converted based on equations (15) and (16) as follows:

$$\Pr\left\{\left|\sum_{g \in G} E_{pg} \cdot \tilde{w}_g\right| \le \Delta LG\right\} \ge \alpha, \forall p \in P \tag{17}$$

$$\Pr\left\{\left|\sum_{g \in G} F_{pg} \cdot \tilde{w}_g\right| \le \Delta CG\right\} \ge \alpha, \forall p \in P \tag{18}$$

Considering the inequation of absolute value (19), we can obtain the inequations (20) and (21) as follows:

$$\left|\sum_{i \in I} A_i\right| \le \sum_{i \in I} |A_i|, A_i \in R \tag{19}$$

$$\left|\sum_{g \in G} E_{pg} \cdot \tilde{w}_g\right| \le \sum_{g \in G} \left|E_{pg} \cdot \tilde{w}_g\right| = \sum_{g \in G} \left|E_{pg}\right| \cdot \tilde{w}_g, \forall p \in P \tag{20}$$

$$\left|\sum_{g \in G} F_{pg} \cdot \tilde{w}_g\right| \le \sum_{g \in G} \left|F_{pg} \cdot \tilde{w}_g\right| = \sum_{g \in G} \left|F_{pg}\right| \cdot \tilde{w}_g, \forall p \in P \tag{21}$$

Then, constraints (17) and (18) can be converted based on inequations (20) and (21) as follows. When constraints (22) and (23) are satisfied, the constraints (17) and (18) must be satisfied.

$$\Pr\left\{\sum_{g \in G} \left|E_{pg}\right| \cdot \tilde{w}_g \le \Delta LG\right\} \ge \alpha, \forall p \in P \tag{22}$$

$$\Pr\left\{\sum_{g \in G} \left|F_{pg}\right| \cdot \tilde{w}_g \le \Delta CG\right\} \ge \alpha, \forall p \in P \tag{23}$$

We consider all the containers have a maximum weight deviation of 1 ton from the weight provided by the cargo owners based on the regulations from the Ministry of Transport of China. The stochastic weight of containers with weight class g should satisfy the inequation below.

$$w_g - 1 \le \tilde{w}_g \le w_g + 1, \forall g \in G \tag{24}$$

Then,

$$\beta = \Pr\left(w_g - 1 \le \tilde{w}_g \le w_g + 1\right) \approx 1, \forall g \in G \tag{25}$$

In expression (25), β represents the probability of stochastic weight $\tilde{w}_g \in [w_g - 1, w_g + 1]$. We assume the stochastic weights of containers with different weight classes subject to normal distributions of their average weights. We can get the expression (26) below based on the probability calculation formula of normal distribution by setting $\beta = 0.995$.

$$\Pr\left(w_g - 1 \le \tilde{w}_g \le w_g + 1\right) = \phi(\frac{w_g + 1 - \mu}{\sigma}) - \phi(\frac{w_g - 1 - \mu}{\sigma}) = 0.995, \forall g \in G \quad (26)$$

In expression (26), μ and σ represent respectively the expectation and standard deviation of normal distribution. $\phi(\cdot)$ represents the standard normal distribution function. We take $\mu = w_g$ into the expression (26):

$$\phi(\frac{1}{\sigma}) - \phi(\frac{-1}{\sigma}) = 2\phi(\frac{1}{\sigma}) - 1 = 0.995, \forall\ g \in G \quad (27)$$

The σ in expression (27) is equal to 0.356 from the standard normal distribution table, then the stochastic weight $\tilde{w}_g$ subject to the normal distribution below:

$$\tilde{w}_g \sim N(w_g, 0.356^2), \forall g \in G \quad (28)$$

According to the deterministic equivalents [18], the constraints (12), (22) and (23) can be converted based on expression (28) as follows:

$$\sum_{a \in Q(p)} \sum_{g \in G} x_{jg}(a) \cdot w_g + \phi^{-1}(\alpha) \sqrt{\sum_{a \in Q(p)} \sum_{g \in G} [x_{jg}(a)]^2 \cdot 0.356^2} \le SW_j, \forall j \in J, p \in P \quad (29)$$

$$\sum_{g \in G} \left|E_{pg}\right| \cdot w_g + \phi^{-1}(\alpha) \sqrt{\sum_{g \in G} \left|E_{pg}\right|^2 \cdot 0.356^2} \le \Delta LG, \forall p \in P \quad (30)$$

$$\sum_{g \in G} \left|F_{pg}\right| \cdot w_g + \phi^{-1}(\alpha) \sqrt{\sum_{g \in G} \left|F_{pg}\right|^2 \cdot 0.356^2} \le \Delta CG, \forall p \in P \quad (31)$$

Due to the constraints (29)-(31) all contain the nonlinear terms, we consider to convert them based on the inequation of absolute value below:

$$\sqrt{\sum_{i \in I} A_i^2} \le \sum_{i \in I} A_i, A_i \in R \text{ and } A_i \ge 0 \quad (32)$$

According to the inequation (32), when constraints (33)-(35) are satisfied, the constraints (29)-(31) must be satisfied.

$$\sum_{a \in Q(p)} \sum_{g \in G} x_{jg}(a) \cdot w_g + 0.356 \cdot \phi^{-1}(\alpha) \sum_{a \in Q(p)} \sum_{g \in G} x_{jg}(a) \le SW_j, \forall j \in J, p \in P \quad (33)$$

$$\sum_{g \in G} \left|E_{pg}\right| \cdot w_g + 0.356 \cdot \phi^{-1}(\alpha) \sum_{g \in G} \left|E_{pg}\right| \le \Delta LG, \forall p \in P \quad (34)$$

$$\sum_{g \in G} \left|F_{pg}\right| \cdot w_g + 0.356 \cdot \phi^{-1}(\alpha) \sum_{g \in G} \left|F_{pg}\right| \le \Delta CG, \forall p \in P \quad (35)$$

Then the deterministic equivalent model for SCPM can be formulated as follows：

$$(\text{DEM}) \left\{ f = \min \sum_{p \in P} \sum_{a \in Q(p)} \sum_{j \in J} y_j(a) : (2) \sim (6), (15), (16), (33) \sim (35), (10), (11) \right\}.$$

4.2. Robust Optimization Model

Robust optimization is a common method to solve the problem with uncertainty. The optimization of the worst case is considered based on the idea of robust optimization. Considering satisfying the limits of the worst case, the constraints (7)-(9) which contains the stochastic container weight in SPM are converted based on the equation (15), equation (16), and inequation (24) as follows:

$$\sum_{a\in Q(p)}\sum_{g\in G} x_{jg}(a)\cdot\left(w_g+1\right)\le SW_j,\ \forall j\in J, p\in P \tag{36}$$

$$\left|\sum_{g\in G} E_{pg}\cdot(w_g+1)\right|\le \Delta LG, \forall p\in P \tag{37}$$

$$\left|\sum_{g\in G} F_{pg}\cdot(w_g+1)\right|\le \Delta CG, \forall p\in P \tag{38}$$

When constraints (36)-(38) are satisfied, the constraints (7)-(9) in SPM must be satisfied. Then the robust optimization model for the proposed problem can be formulated as follows:

$$(\text{ROM})\ \left\{f=\min\sum_{p\in P}\sum_{a\in Q(p)}\sum_{j\in J} y_j(a):(2)\sim(6),(15),(16),(36)\sim(38),(10),(11)\right\}.$$

5. EXPERIMENTS

5.1 Test Instances

According to the Chinese national standard GB/T 19283-2010, two typical inland container ship types on the Yangtze River are listed in Table 1.

Table 1 Inland container ship on the Yangtze River

NO.	Bay	Column	Tier	Load weight of stack/t	Stack	Capacity/TEU	ΔLG /t	ΔCG /t
S1	8	4	3	45	32	96	30	30
S2	12	4	4	60	48	192	30	30

Here, we assess a series of test instances based on real-world scenarios of inland container liner shipping on the Yangtze River. Table 2 lists the number of operational ports for four shipping lines; for each line, three vessel loading rates are listed in Table 3. All the test instances are coded in a way like S1-L1-C45. The first part S1 represents the ship type. The second part L1 represents the shipping line. The third part C45 represents the vessel loading rate.

Table 2 Inland container shipping lines on the Yangtze River

Shipping line	L1	L2	L3	L4
Number of ports	4	5	6	7

Table 3 Different vessel loading rates for the ship

NO.	C45	C65	C85
loading rate	45%	65%	85%

5.2 Computational Results

5.2.1 Results of container liner route stowage planning

All the mathematical model including SPM, DEM, LBM, and ROM are solved using a standard solver Gurobi 7.5.1. The stochastic weight parameters in SPM are calculated with their average weights. All tests are run on an Intel Core I7-5500U 2.40 GHz processor with 4 GB of RAM.

The average weight of containers with different weight classes are set as 7 ton, 14 ton, and 21 ton respectively. First, the CPU time limit is set as 60 s. When the result is not ideal or the model cannot be solved, the time limit is altered to 600 s. The results of container liner route stowage planning for different ships are shown in Table 4 and Table 5. In Tables 4-5, f represents the ship stack occupancy number on the route. T represents the CPU time for solving each instance (Unit: s). gap represents the gap in units of f between each model (SPM, DEM, ROM) and LBM (Unit: %).

Table 4 Results of different models for ship S1

Instance	LBM		SPM			DEM			ROM		
	f	T	f	gap	T	f	gap	T	f	gap	T
S1-L1-C45	48	0.94	48	0	60	48	0	60	48	0	60
S1-L1-C65	67	4.87	67	0	60	67	0	60	67	0	60
S1-L1-C85	85	0.77	85	0	60	86	1.18	60	86	1.18	60
S1-L2-C45	60	1.5	60	0	60	60	0	60	60	0	60
S1-L2-C65	84	0.98	84	0	457.15	84	0	600	84	0	600
S1-L2-C85	113	1.84	115	1.77	60	116	2.65	60	114	0.88	60
S1-L3-C45	84	14.49	84	0	60	84	0	60	84	0	60
S1-L3-C65	101	6.1	103	1.98	60	102	0.99	60	102	0.99	60
S1-L3-C85	154	60	154	0	600	156	1.3	600	155	0.65	600
S1-L4-C45	99	60	99	0	60	99	0	60	99	0	60
S1-L4-C65	130	60	132	1.54	60	130	0	60	131	0.77	60
S1-L4-C85	173	60	175	1.16	600	177	2.31	600	174	0.58	600
Average	**99.83**	**22.62**	**100.50**	**0.54**	**183.10**	**100.75**	**0.70**	**195**	**100.33**	**0.42**	**195**

Table 5 Results of different models for ship S2

Instance	LBM		SPM			DEM			ROM		
	f	T	f	gap	T	f	gap	T	f	gap	T
S2-L1-C45	63	0.74	63	0	2.22	65	3.17	60	65	3.17	60
S2-L1-C65	93	0.75	96	3.23	60	96	3.23	60	96	3.23	60
S2-L1-C85	121	0.94	122	0.83	60	122	0.83	60	122	0.83	60
S2-L2-C45	86	1.49	86	0	60	86	0	60	86	0	60
S2-L2-C65	125	1.49	125	0	5.44	126	0.8	60	126	0.8	60
S2-L2-C85	164	1.7	164	0	60	164	0	60	164	0	60
S2-L3-C45	111	2.75	111	0	12.83	111	0	8.26	111	0	6.5
S2-L3-C65	146	1.79	150	2.74	60	150	2.74	60	150	2.74	60
S2-L3-C85	205	3.25	205	0	60	205	0	60	207	0.98	60
S2-L4-C45	145	60	146	0.69	60	146	0.69	60	146	0.69	60

S2-L4-C65	190	60	191	0.53	60	191	0.53	60	190	0	60
S2-L4-C85	249	19.04	249	0	60	249	0	60	249	0	60
Average	**141.50**	**12.83**	**142.33**	**0.67**	**46.71**	**142.58**	**1**	**55.69**	**142.67**	**1.04**	**55.54**

Tables 4-5 show the ship stack occupancy number on the route and CPU time for each instance. It is seen from the results that it found 15 lower bound values by solving SPM with an average CPU time of 114.9 s. The average gap between SPM and LBM is 0.61 %. It found 12 lower bound values by solving DEM with an average CPU time of 125.34 s. The average gap between DEM and LBM is 0.85 %. It found 11 lower bound values by solving ROM with an average CPU time of 125.27 s. The average gap between ROM and LBM is 0.73 %. The results show that SPM is better than DEM and ROM when considering the quality of solution and CPU time, while the differences between DEM and ROM are quite small.

Due to the consideration of uncertain container weight, DEM and ROM are more difficult to be solved compared to the SPM. However, the average gap between these two models and LBM are both within 1 %. And the SPM cannot ensure the feasibility of stowage plan when the stochastic disturbance occurs.

5.2.2 Monte Carlo stochastic simulation

The Monte Carlo stochastic simulation is adapted to verify the robustness of stowage plans from different models. The stochastic weights of different weight classes are subject to the uniform distributions [6, 8], [13, 15], and [20, 22] respectively. Each instance is simulated 1000 times continuously and the results are listed in Table 6. The number in the table represents the pass rate of the stowage plan corresponding to each instance in 1000 times' simulation.

Table 6 Results of Monte Carlo stochastic simulation

Instance	SPM	DEM	ROM
S1-L1-C45	0.761	1	1
S1-L1-C65	0.750	1	1
S1-L1-C85	0.762	1	1
S1-L2-C45	0.748	1	1
S1-L2-C65	0.746	1	1
S1-L2-C85	0.753	1	1
S1-L3-C45	1	1	1
S1-L3-C65	0.736	1	1
S1-L3-C85	0.754	1	1
S1-L4-C45	1	1	1
S1-L4-C65	0.769	1	1
S1-L4-C85	0.747	1	1
S2-L1-C45	0.746	1	1
S2-L1-C65	1	1	1
S2-L1-C85	0.763	1	1

S2-L2-C45	0.746	1	1
S2-L2-C65	0.758	1	1
S2-L2-C85	1	1	1
S2-L3-C45	0.758	1	1
S2-L3-C65	0.757	1	1
S2-L3-C85	0.755	1	1
S2-L4-C45	0.769	1	1
S2-L4-C65	0.768	1	1
S2-L4-C85	0.759	1	1
Average	**0.796**	**1**	**1**

The simulation results show that SPM can only ensure a few examples passing the test which means it has a weak robustness. Both DEM and ROM have a good robustness as all the examples can pass the test while satisfying the confidence requirement.

6. CONCLUSION

In this study, we analyzed the inland container liner route stowage planning with uncertain container weight based on the stochastic programming theory. Due to its peculiarity, the existing decision making method for maritime shipping is difficult to meet the requirement of the inland container liner route stowage planning decision. The stochastic programming model and stochastic chance-constrained programming model are generated considering the capacity utilization on the route for the proposed problem. The stochastic chance-constrained programming model is converted into a deterministic equivalent model and solved by a standard solver. The stochastic programming model is converted into a robust optimization model and solved by the same solver. Experimental and simulation results show that the stochastic programming model has a weak robustness because it cannot ensure the feasibility of stowage plan when the stochastic disturbance occurs. The stochastic chance-constrained programming model and the robust optimization model can be used to solve the proposed problem. They can both achieve the effective absorption of the stochastic disturbance to ensure the robustness of liner route stowage plan. In future research, we will develop some robust optimization algorithms to obtain better solutions with less CPU time.

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References

[1] D. Ding and M.C. Chou. "*Stowage planning for container ship: A heuristic algorithm to reduce the number of shifts*". European Journal of Operational Research, 246, pp. 242-249, (2015).

[2] I.D. Wilson and P.A. Roach. "*Container stowage planning: a methodology for generating computerized solutions*". The Journal of the Operational Research Society, 51, pp.1248-1255, (2000).

[3] M.H.Y. Low, M. Zeng, W.J. Hsu, S.Y. Huang, F. Liu and C.A. Win. "*Improving Safety and Stability of Large Containerships in Automated Stowage Planning*". IEEE Systems Journal, 5(1), pp. 50-60, (2011).

[4] A.T. Azevedo, E.F. Arruda, L.L. Salles-Neto, A.A. Chaves and A.C. Moretti. "*Solution of the 3D Stochastic Stowage Planning for Container Ships through Representation by Rules*". Fourth International Workshop Proceedings, Eureka, pp. 120-129, (2013).

[5] A. Imai, K. Sasaki, E. Nishimura and S. Papadimitriou. "*Multi-objective simultaneous stowage and load planning for a container ship with container rehandle in yard stacks*". European Journal of Operational Research, 171, pp.373-389, (2006).

[6] I.D. Wilson, P.A. Roach and J.A. Ware. "*Container stowage pre-planning: using search to generate solutions, a case study*". Knowledge-Based System, 14, pp.137-145, (2001).

[7] J.G. Kang and Y.D. Kim. "*Stowage planning in maritime container transportation*". Journal of the Operational Research Society, 53, pp.415-426, (2002).

[8] D. Ambrosino, M. Paolucci and A. Sciomachena. "*Experimental evaluation of mixed integer programming models for the multi-port master bay plan problem*". Flexible Services and Manufacturing Journal, 27, pp.263-284, (2015).

[9] D. Ambrosino, M. Paolucci and A. Sciomachena. "*A MIP heuristic for multi-port stowage planning*". Transportation Research Procedia, 10, pp.725-734, (2015).

[10] Q. Meng and S. Wang. "*Short-term liner ship fleet planning with container transshipment and uncertain container shipment demand*". European Journal of Operational Research, 223(1), pp.96-105, (2012).

[11] J.X. Dong, C.H. Lee and D.P. Song. "*Joint service capacity planning and dynamic container routing in shipping network with uncertain demands*". Transportation Research Part B, 78, pp.404-421, (2015).

[12] Ng, M.W. Distribution-free vessel deployment for liner shipping [J]. European Journal of Operational Research. 2014, 238: 858–862.

[13] M.W. Ng. "*Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand*". Transportation Research Part B, 74, pp.79-87, (2015).

[14] J.S.K. Ang, C.X. Cao, M. Parlar and H.Q. Ye. "*Two-Stage Stochastic Integer Programming Model for Multiperiod Sea Cargo Mix Problem in Container Shipping Industry*". IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 39(2), pp.460-465, (2009).

[15] B. Zhang, C.T. Ng and T.C.E. Cheng. "*Multi-period empty container repositioning with stochastic demand and lost sales*". Journal of the Operational Research Society, 65, pp.302-319, (2014).

[16] E. Ursavas, X. Stuart and S.X. Zhu. "*Optimal policies for the berth allocation problem under stochastic nature*". European Journal of Operational Research, 255, pp.380-387, (2016).

[17] N.Umang, M. Bierlaire and A.L. Erera. "*Real-time management of berth allocation with stochastic arrival and handling times*". Journal of Scheduling, 20(1), pp.67-83, (2017).

[18] A. Charnes and W.W. Cooper. "*Deterministic equivalents for optimizing and satisficing under chance constraints*". Management Science, 11(1), pp.18-39, (1963).