Uncertainty Propagation in Dynamic Event Trees - Initial Results for a Modified Tank Problem

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Abstract: The coupling of plant simulation models and stochastic models representing failure events in Dynamic Event Trees (DET) is a framework to model the dynamic interactions among physical processes, equipment failures, and operator responses. The benefits of the framework, as a number of applications show, include, for instance, the capability to account for the aleatory timing of equipment failures or operator actions on sequence outcomes and to consider the impact of the number of available trains (rather than having to identify the bounding cases). The integration of physical and stochastic models may additionally enhance the treatment of uncertainties. Probabilistic Safety Assessments as currently implemented, e.g. for Level 1, propagate the (epistemic) uncertainties in the probability distributions for the failure probabilities or frequencies; this approach does not consider propagate uncertainties in the physical model (parameters). The coupling of deterministic (physical) and probabilistic models in integrated simulations such as the DET allows both types of uncertainties to be considered. The starting point in this work is to consider wrapping an epistemic loop, in which the epistemic distributions are sampled, around the DET simulation. To examine the adequacy of this approach, and to allow different approaches and approximations (for uncertainty propagation) to be compared, a simple problem is proposed as a basis for comparisons. This paper presents initial results on uncertainty propagation in DETs, obtained for a tank problem that is derived from a similar one defined for control system failures and dynamic reliability. An operator response has been added to consider stochastic timing.

Keywords: Epistemic and aleatory uncertainties, Dynamic PSA, Monte Carlo simulation, Dynamic Event Tree Analysis

1. INTRODUCTION

Typical accident scenario in a Nuclear Power Plant (NPP) involves complex interactions between physical process and safety systems (safety equipment and operator response). The response of a safety system is inherently random in nature, which is often referred as aleatory uncertainty [1]. The response of physical process can also have aleatory elements; for example, initial level, break size, break location, etc. Dynamic event tree (DET) analysis provides a framework to simulate the accident scenario considering the dynamic interactions [2], where mathematical models of physical process and safety systems are used. The limitations in assessing the parameters of these models introduce another type of uncertainty, which is often referred as epistemic uncertainty [1]; for example, demand failure probability of safety equipment, human error probabilities, and thermal hydraulic parameters. These epistemic variables can significantly impact the simulated accident dynamics and ultimately the risk estimate; for example, uncertainty in TH parameter or operator response can change the outcome of an accident sequence affecting the final risk estimate. Hence risk quantification must consider both epistemic and aleatory uncertainties in both physical and safety system models along with their dynamic interactions.

In the current PSA practice [3], accident sequence models are first developed and then solved for a cut set equation. A point estimate of risk (e.g. Core Damage Frequency for level-1 PSA) can then be obtained using mean values for the PSA parameters. A Monte Carlo simulation is run to propagate epistemic uncertainty in PSA parameters. The obtained CDF distribution thus accounts for the aleatory and the epistemic uncertainties of safety system responses, e.g. demand failures. However, the current approach does not propagate uncertainties in TH parameters through to the risk model outcomes. The

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success criteria definitions are the interface between the physical system simulations and the PSA models; they are normally calculated with point estimates of Thermal-Hydraulic (TH) parameters, using bounding parameter values in some cases.

For similar problems in the literature [4, 5], a two-loop Monte Carlo simulation has been used. epistemic variables are sampled in the outer loop while aleatory variables are sampled in the inner loop. In this work, the inner loop dealing with the aleatory response is a DET simulation. There are two DET approaches in the literature to consider dynamic interactions in NPP, discrete DET [6-8] and MCDET [9] approaches; these studies demonstrated the potential of DET approaches in addressing the complex interactions providing insights for risk assessment. Further, DET approaches have been found to be useful to assess the impact of dynamics [10] and the detrimental effects of bounding [11] in the quantification of risk. The DET approach can also provide a framework to consider epistemic and aleatory uncertainties.

In this work, the discrete DET framework along with epistemic uncertainty analysis is applied to quantify risk and identify important contributors in the light of uncertainties and dynamics. The analysis determines the impacts of physical uncertainties and safety system uncertainties on the accident evolution and final risk estimate. Monte Carlo simulation with convergence criteria for epistemic uncertainty analysis and appropriate discretization strategies for DDET are considered. To examine the adequacy of this approach, and to allow different approaches and approximations (for uncertainty propagation) to be compared, a simple problem is proposed as a basis for comparisons. This paper presents initial results on uncertainty propagation in DETs, obtained for a tank problem that is derived from a similar one defined for control system failures and dynamic reliability [12]. An operator response has been added to consider stochastic timing. The results from DDET approach are compared with analytical solution including, important risk contributors and uncertainty importance measures.

The paper is organized as follows. Section 2 explains the methodology and its elements. Section 3 presents application to the tank problem, its analytical solution, and its comparison with DDET approach. The detailed analysis of the results and discussion is presented in Section 4. Finally, conclusions are given in Section 5.

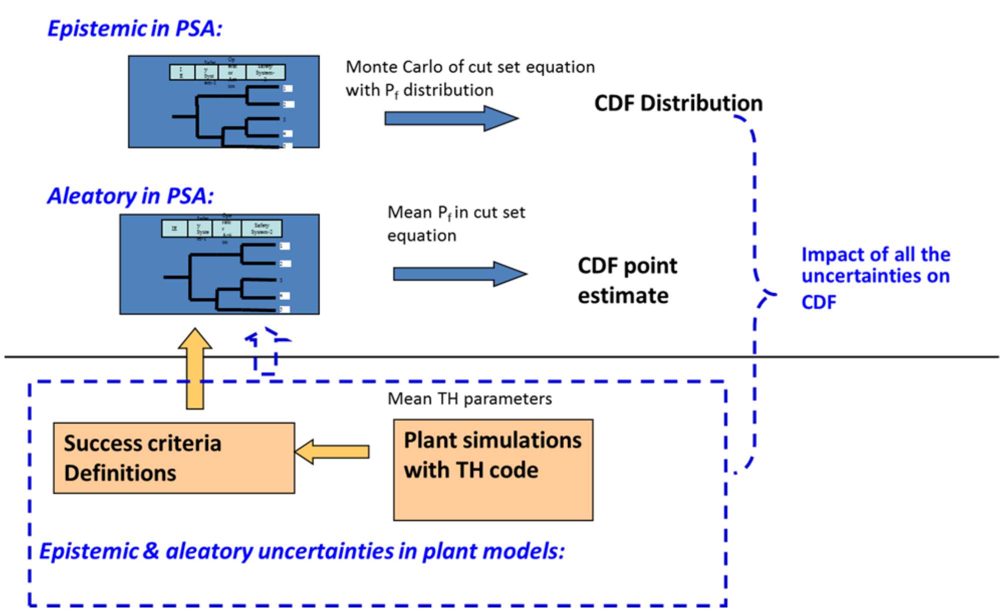

Fig. 1: Propagation of uncertainties in the current PSA practice

2. DYNAMICS AND UNCERTAINTIES IN RISK QUANTIFICATION

The classical combination of fault tree and event tree analyses is used to develop risk models in Probabilistic Safety Assessment of NPPs. Success criteria requirements (for fault tree development) and sequence outcomes (for event tree development) are derived based on plant simulations with thermal-hydraulic codes. PSA model is then solved for a cut set equation, which is subsequently quantified to estimate core damage frequency (CDF) using plant specific or/and generic reliability data. In order to account for epistemic uncertainty in PSA parameters such as demand probabilities of safety systems, HEPs, etc., Monte Carlo simulation is used to quantify the uncertainty in CDF. Fig. 1 shows the current practice of uncertainty propagation in PSA; epistemic and aleatory uncertainties in stochastic elements (safety systems and operator responses) are properly accounted, but these uncertainties are also present in the physical process. For example, epistemic uncertainties in physical process (TH model) parameters or the natural variability of break size or location could change the structure of the event trees or success criteria definitions, subsequently the risk estimate. Any uncertainties related to the success criteria are treated through enveloping / bounding [11, 13]. Thus the uncertainties in PSA models and TH models are separately treated and they are not propagated across the interface. All these uncertainties (both epistemic and aleatory) in the dynamic interactions of physical process and stochastic systems (safety systems and operator responses) must properly be accounted in the risk assessment.

Table 1: Epistemic and aleatory uncertainties in safety and physical models

	Aleatory	**Epistemic**
Safety equip. & OAs (PSA)	Demand failure probability, failure times, recovery time of safety equipment	Parameter uncertainty of discrete & continuous aleatory variables
	Response time of OA	
Physical process (TH)		TH parameters
	Aleatory variables in TH model (discrete & continuous)	Parameter uncertainty of TH aleatory variables

Table 1 gives a summary of the uncertainties involved in risk calculations. In safety system models, equipment failures on demand, time to failure (during operation), recovery time, and operator response times are aleatory variables, usually characterized with binomial distribution, exponential distribution, and lognormal distribution respectively. The parameters of these aleatory distributions are epistemic variables. Both these variables in PSA studies were well explored in the literature and used in practice [4, 5]. In plant physics models, the uncertainty in TH parameters is epistemic in nature; the examples of aleatory variables are initial levels and break size. Thus there are four types of variables in a full scope risk model and their propagation to final risk is the problem under consideration. Although some studies considered TH epistemic variables by wrapping an epistemic loop, these uncertainties were not propagated up to final risk quantification [9]. Some of these limitations are due to issues with DET quantification of risk [11]. Besides propagation of the uncertainties, ranking of uncertainty parameters and important risk contributors considering uncertainties is also necessary [14, 15], which help to see their individual impact on risk, and further in uncertainty and risk management. In addition to considering epistemic variables of both TH and PSA models in risk quantification, the current study also considers aleatory variables of physical process. A solution is proposed here to this problem of integrated treatment of uncertainties (both epistemic and aleatory in both TH and PSA models) in quantifying risk and its uncertainties.

$$CDF = f(TH_p, L, P_f, OA); TH_p : THparameter, L : level, P_f : demand\ prob, OA : Operator\ time$$

$$TH_p = Uniform(a,b)\ \&\ L = Normal(\mu_L, \sigma_L); \mu_L = Lognormal(median_L, ef_L)$$

$$P_f = Binomial(P); P = Lognormal(median_P, ef_P)\ \&\ t_{OA} = Weibull(\alpha_{OA}, \beta_{OA}); \alpha_{OA} = Lognormal(median_{\alpha_{OA}}, ef_{\alpha_{OA}})$$

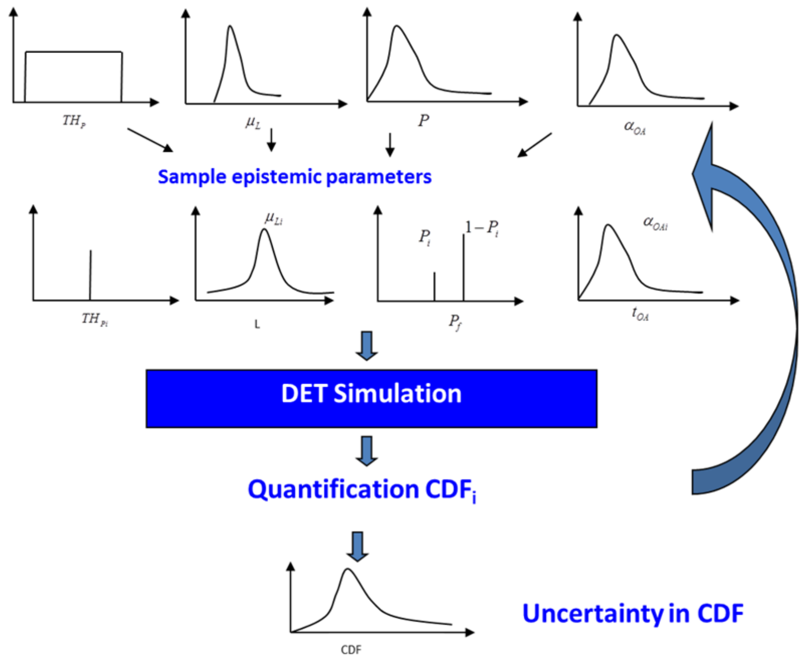

Fig. 2: Computational framework – DDET along with epistemic uncertainty analysis

Computational Framework – DDET along with epistemic uncertainty analysis

The objective is quantifying the risk, identifying accident sequences, and ranking of important parameters in the light of uncertainties and dynamics. The premise of the solution approach, while considering uncertainties and dynamics in accident scenario simulation, is based on the primary distinction of parameters on the nature of uncertainty, and not based on the physical vs. safety systems. The two-loop Monte Carlo simulation strategy, which was used in the literature for similar problems, is adapted for the current problem. The inner loop modeling the aleatory response is replaced by a DDET simulation. In this way, the aleatory response is addressed by the DDET rather than by means of Monte Carlo sampling. Mathematically, risk is a function of epistemic and aleatory parameters of the model, as depicted in equation (1): it has two variables relating physical process and two variables relating to safety systems. 'TH_p' is an epistemic variable (physical) and physical level 'L' is an aleatory variable; P_f and OA are aleatory variables (safety system) and their distribution parameters are epistemic variables. Let us assume that the probability distributions for all these epistemic and aleatory variables are available. The following steps are involved in the computational methodology as shown in Fig. 2.

i. **Epistemic sampling:** The epistemic variables are sampled based on their distribution. In this case, epistemic parameter TH_p and epistemic parameters of aleatory variables, viz. μ_L, P, and α_{OA}, are sampled and they are treated as constants in the next step. A convergence criterion for epistemic Monte Carlo sampling is required to check the accuracy and keep the number of computations to manageable size. A criterion that uses the acceptable percentage of error and confidence levels as inputs is used for the implementation. Details of convergence criterion are discussed in section 3.4.

ii. **Aleatory physical variables:** DDET simulation considers both physical and safety system aleatory variables. The aleatory variables of physical process are different from safety systems. While the initial conditions or boundary conditions of accident initiator depend upon the aleatory physical variables, the aleatory variables of safety systems influence post-accident initiation and drive accident evolution.

Branches are generated for each of the aleatory physical variables. As they are not time dependent, these branches correspond to accident initiating event. If any of these variables is continuous (e.g. level in this case), they are discretized on a logarithmic scale as discussed in section 3.3.

iii. **Aleatory safety system variables:** DDET tool simulates accident scenario with the boundary conditions from steps 1 and 2, branches are generated as and when the safety system are demanded. The response of aleatory safety system variables can be discrete (for example success or failure on demand P_i or $1\text{-}P_i$) or continuous such as operator response time. Appropriate discretization is used if the variables are continuous. The logarithmic discretization strategy ensures optimal number of branches with less conservatism at reduced computations. The branches of aleatory physical variables are simulated subsequently.

iv. **Quantification of risk:** Each DDET generated from step iii is evaluated to quantify risk and important contributors. These results correspond to an epistemic sample i. The computation is switched over to next epistemic sample, i.e. step i. The computations continue until the convergence criterion is satisfied.

v. **Quantification of uncertainty in Risk Estimates:** The following measures of risk are obtained from the simulations: Epistemic uncertainty distribution of final risk, important sequences, uncertainty importance measures (ranking of uncertainty parameters), ranking of risk contributors along with their uncertainties.

The next section presents an application of this computational framework to simple tank problem and also its comparison with analytical results.

3. APPLICATION TO A SIMPLE TANK PROBLEM

3.1 Depleting Tank Problem

A tank problem has been derived from a similar one defined for control system failures and dynamic reliability [12]. An operator response has been introduced to consider stochastic timing. There is a cylindrical tank of diameter 'D' with an initial water level of H_i. Tank starts depleting due to a spurious signal that opens a valve, which has a diameter of the leak as 'd'. Alarm is the cue for operator action. Operator has to close the valve before the tank level reaches a critical level H_f. The objective is to estimate the likelihood of the tank reaching a critical level considering all epistemic and aleatory uncertainties in the scenario. Time taken for a depleting tank to reach a level H_f based on Bernoulli's equation is [16, 17]:

$$TW = \frac{A}{aC}\left(\sqrt{H_i} - \sqrt{H_f}\right)\sqrt{\frac{2}{g}} \qquad (1)$$

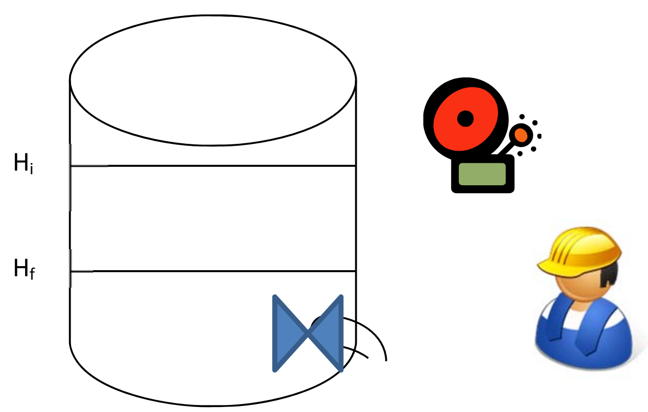

Fig. 3: A depleting tank with an initial level H_i and a critical level H_f

Nomenclature:

TW – Time
a – area of the hole
H_f – Critical tank level
g – gravitational force
P(V) – Valve failure prob
A – area of tank
H_i – Initial tank level
C – discharge coefficient
T_{OA} – Response time of operator

The tank depletes to critical level when the operator does not act before a time, which is the time taken for the tank to reach the critical level. Operator has a cue from an alarm, which is due to fall of level, and in response operator needs to close the valve. The valve needs to function on demand to stop the leak. The tank failure depends on the failures on demand of alarm and valve, and human response. The time dependent (dynamic) element in the problem is human response time competing with the time taken by the tank to reach the critical level, which depends on initial level and other constants.

Table 2: Epistemic and Aleatory uncertainties in the tank model

	Aleatory Variables	**Epistemic Variables**
Safety System Models	Demand failure probability of Valve (2e-4)	P-Lognormal(1.24e-4, 5)
	Response time of OA –g(t_{OA}) Lognormal(360s, 2)	Error factor-Uniform(1.8, 2.2)
Physical Process Models		Discharge coefficient C Uniform(0.72, 0.98)
	Initial tank level Hi Normal(10, 0.3)m	
Other data used in calculations	Diameter of the tank -2m Critical tank level – 2m Diameter of the hole -0.05m g – 9.8m/sec^2	

Table 2 gives the summary of aleatory and epistemic uncertainties assumed in the analysis. In physical process model, tank level is an aleatory variable and discharge coefficient is an epistemic variable. In safety system models, demand failure probabilities of valve and alarm, and operator response time are aleatory variables, where as their distribution parameters are epistemic variables.

3.2 Analytical Solution

This section presents the analytical solution for the system failure probability, as a baseline result with which to compare the DDET solution, in both cases, with uncertainties.

Tank failure probability FP can be expressed as a function of likelihoods of alarm, valve, and human error probability, which is shown in equation (2).

$$FP = f\big(P(A), HEP, P(V)\big) \tag{2}$$

The failure probability of alarm and valve are independent of physical parameters or time dependent elements. But the HEP is the probability of the aleatory variable response time (R) exceeding another aleatory variable time window (W) or time taken for the tank level to reach the critical level (see Fig. 4). The time window is an aleatory variable as it is a function of initial level, which is another aleatory variable. HEP is shown in equation (3), which can be simplified using reliability theory on load-resistance or stress-strength concept [18] as shown below:

$$HEP = P(R > W) \tag{3}$$

Differential HEP is the probability of response time falling in the interval 'dr' around r and the time window being smaller than the value 'r' simultaneously is

$$d(HEP) = f_R(r)dr \int_0^r f_W(w)dw$$

The HEP is given as the probability of time window 'W' being smaller than the response time 'R' for all possible values of R.

$$HEP = \int_0^\infty f_R\,(r)dr \int_0^r f_W(w)dw = \int_0^\infty f_R(r)F_W(r)dr \tag{4}$$

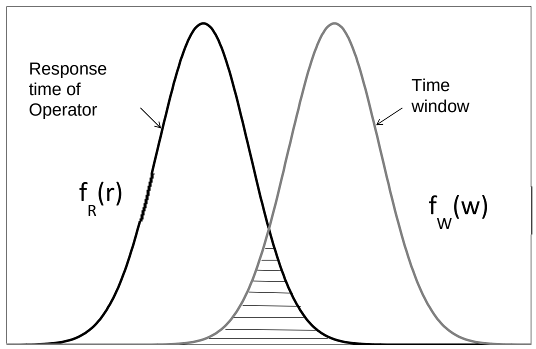

Fig. 4 Operator response time and time window

PDF of response time is known, but PDF of time window is not known; as time window is a function of core level whose pdf is known, we can derive its PDF using transformation of random variables [19] as shown below: Equation (1) can be simplified to

$$W = \frac{A}{aC}\left(\sqrt{H_i} - \sqrt{H_f}\right)\sqrt{\frac{2}{g}} = k_1\sqrt{H} - k_2\;;\;\; where\; k_1 = \frac{A}{aC}\sqrt{\frac{2}{g}}\; and\; k_2 = \sqrt{H_f} \times k_1 \tag{5}$$

As mentioned in Table 2, H_i is a normal distribution, we have to find probability density or cumulative distribution function (CDF) of 'W'. The CDF of W can be expressed as

$$F_W(w) = P(W \le w) \tag{6}$$

Equation (5) can be rearranged to derive H as a function of w:

$$H = \left(\frac{w + k_2}{k_1}\right)^2$$

Substituting equation (5) in equation (6) and expanding further:

$$F_W(w) = P\left(k_1\sqrt{H} - k_2 \le w\right) = P\left(H \le \left(\frac{w + k_2}{k_1}\right)^2\right) = \int_0^{\left(\frac{w+k_2}{k_1}\right)^2} f_H(h)dh$$

$$F_W(r) = \int_0^{\left(\frac{r+k_2}{k_1}\right)^2} f_H(h)dh \tag{7}$$

Substituting equation (7) in equation (4) gives the HEP for final calculations:

$$HEP = \int_0^\infty f_R(r) \int_0^{(\frac{r+k_2}{k_1})^2} f_H(h)dh.dr \tag{8}$$

Numerical integration method has been used to solve equation (8) for HEP and with the data mentioned in Table 2.

3.3 Discrete DET Solution

The discrete DET approach has been applied on the tank problem. DDET is shown in Fig. 5. Continuous aleatory variables, viz., tank level and operator response times are discretized. The alarm and valve have two branches either success or failure.

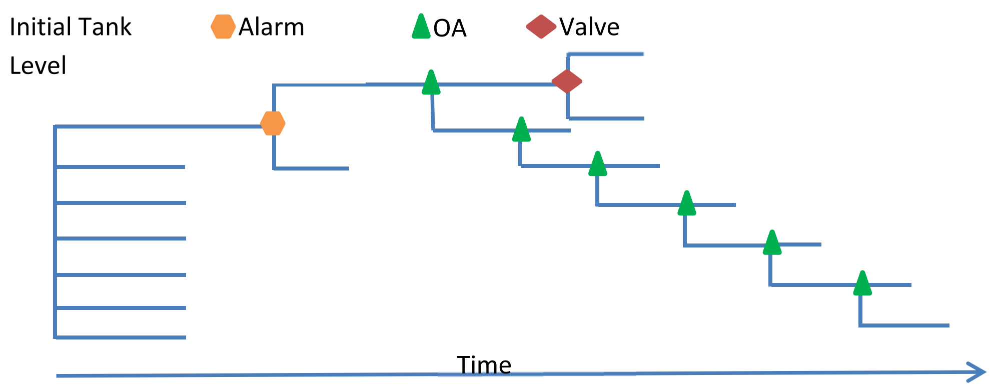

Fig. 5: Discrete DET of the tank problem considering aleatory uncertainties

The initial tank level and operator response time are discretized as they are continuous random variables. The discretization strategies used in the literature [7, 20] are 3 percentiles which normally represent low, median, and high values. This strategy is reasonable for a qualitative understanding of the sequences, but, in quantification, can results in overestimation. It is also important to know if the variable to be discretized is sensitive as a whole or in certain parts (e.g. upper or lower tails) of the distribution. Since the tank level is sensitive for all the values, it was discretized linearly on the whole distribution. The logarithmic discretization strategy ("log strategy") is used in case of operator response distribution on the upper tail (between 0.9 and 1.0 in cum. prob.). The premise for selecting this range is that human error probability (HEP) is assumed to be in the range of 0.0001 and 0.1; the lower values than this range would not contribute significantly compared with other risk contributors and the higher values would make only a marginal error. 5 different discretization strategies (4, 5, 7, 10, and 20 Branches; the last 3 with log strategy, see Table 3) are considered and their results are compared with analytical result. Fig. 6 shows the 7-branch log strategy, where the tail is divided into 3 branches (intervals) in log scale; the remaining 4 branches correspond to 5%, 50%, 90%, and skip, which are necessary to see quick, normal, late, and never actions. Like the 7-branch log strategy, the 10- and 20-branch strategies discretize cumulative probabilities between 0.9 and 1.0 in log scale into 6 and 16 branches respectively. Table 3 shows the discretization strategies for 4, 5, and 7 branches used in the calculations. The percentiles of response time and the branch probabilities are also shown.

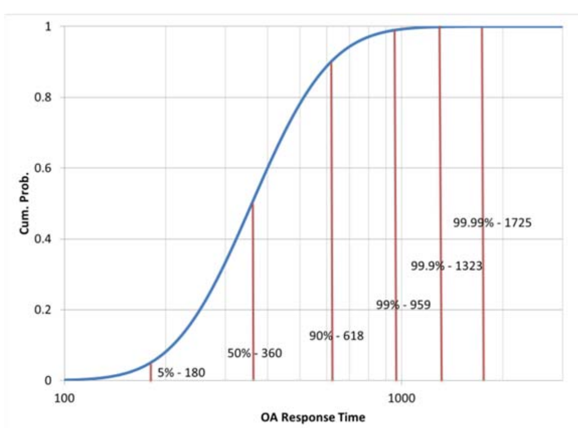

Fig. 6: Log discretization strategy for operator response time

Table 3: Discretization strategies for OA in DET simulations

	4-branch	5-branch	7-branch
Operator response time	(5, 50, 95)tiles, skip	(5, 50, 95, 99.9)tiles, skip	(5, 50, 90, 99, 99.9, 99.99)tiles, skip
Branch probability	0.05, 0.45, 0.45, 0.05	0.05, 0.45, 0.45, 4.9e-2, 1e-3	0.05, 0.45, 0.4, 9e-2, 9e-3, 9e-4, 1e-4

3.4 Uncertainty Propagation by Monte Carlo Simulation with Convergence Criteria

The uncertainty (epistemic) in distribution parameters of aleatory variables are propagated using Monte Carlo simulation which is widely used in current PSA practice [3]. The convergence criterion is based on the specified confidence level and percentage error. The method proposed by Driels and Shin [21] in Monte Carlo simulations of weapon effectiveness is adapted for this problem. Let risk y is a function of epistemic variables whose uncertainties to be propagated. Let 'n' is the initial number of Monte Carlo simulations run (sample size). Sample mean and standard deviation are calculated. The current percentage error and estimate of number of runs required to achieve a specified percentage of error are determined using the equations 9 and 10 [21]. Assuming 'y' as a normally distributed random variable, the percentage error of the mean risk is

$$\boldsymbol{E} = \frac{\mathbf{100} * \boldsymbol{Z_c} * \boldsymbol{S_y}}{\bar{\boldsymbol{y}} * \sqrt{\boldsymbol{n}}} \quad (9)$$

Where Z_c confidence coefficient, S_y standard deviation, and mean of sample is $\bar{y}$.

A relationship between the number of trial runs necessary, confidence interval, and acceptable error is shown in Equation 10.

$$\mathbf{n} = \left[\frac{\mathbf{100} * \boldsymbol{Z_c} * \boldsymbol{S_y}}{\boldsymbol{E} * \bar{\boldsymbol{y}}}\right]^{\mathbf{2}} \quad (10)$$

It was reported that the estimate of number runs convergence quickly after a few initial runs. This convergence method has been applied in the current calculations.

4. RESULTS AND DISCUSSION

The methods discussed in the previous section, i.e. analytical and DDET method, have been applied on the tank problem to determine the failure probability. In the first set of calculations, aleatory uncertainties are only considered and epistemic parameters are kept at their mean values; the second set of calculations considers both epistemic and aleatory uncertainties. The comparison between analytical and DDET aleatory results are shown in Table 4 The analytical method solved with numerical integration technique is the reference result. Several discretization strategies are compared with the reference result. DDET with 3%tile and 4%tile methods that were used in the literature are found to be conservative in estimation. The former overestimates by 83.9 times and the latter depend on the percentile assigned to skip action giving different results. The sensitive to skip percentile indicates it may change from case to case. Although it is obvious that larger the number of discretization levels the better accuracy in DDET calculations, log discretization strategy is found to give satisfactory results with few number of branches; for example, the percentage errors are 98% and 31% for 7 and 10 log branches respectively and the 20 branch (log) case converged with the reference result.

Monte Carlo sampling for epistemic calculations uses the convergence criteria discussed in the previous section. The criterion uses 95% confidence level and 5% error with respect to estimated mean. Comparing the epistemic mean with aleatory results (Table 4) from the analytical method, the former is higher than the latter indicating ignoring epistemic uncertainties could underestimate the risk.

Table 4: Comparison of failure probability without considering epistemic uncertainties

	Analytical	**DDET-discretization**						
	Numerical Integration	4 Br.	5 Br.			7 Br.	10 Br.	20 Br.
			99%tile*	99.9%tile	99.99%tile*			
Failure Probability	5.98e-4	5.02e-2	1.02e-2	1.19e-3	5.02e-2	1.19e-3	7.83e-4	6.43e-4
Overestimation		83.9	17	1.98	83.9	1.98	1.31	1.07

*Sensitive cases for 5 branch discretization

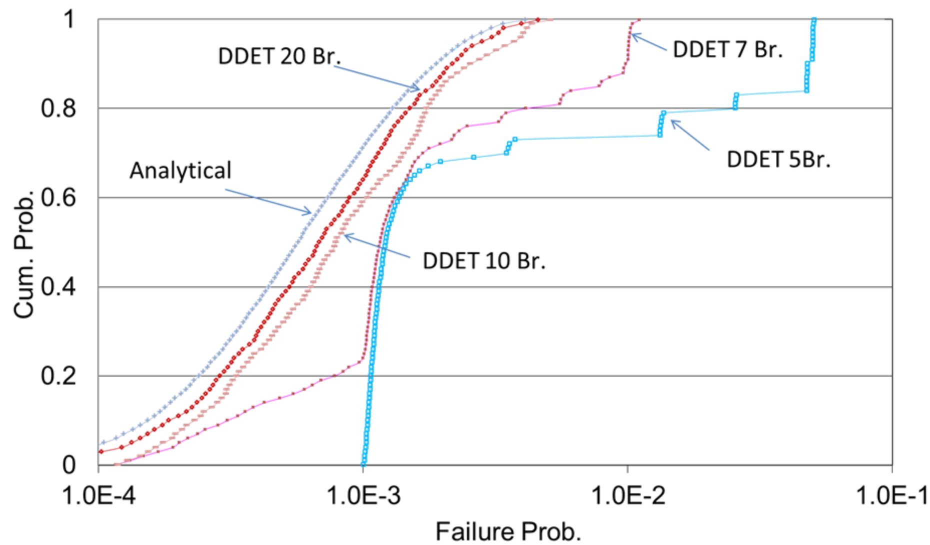

Fig. 7: Cumulative probability functions for epistemic uncertainty in failure probability

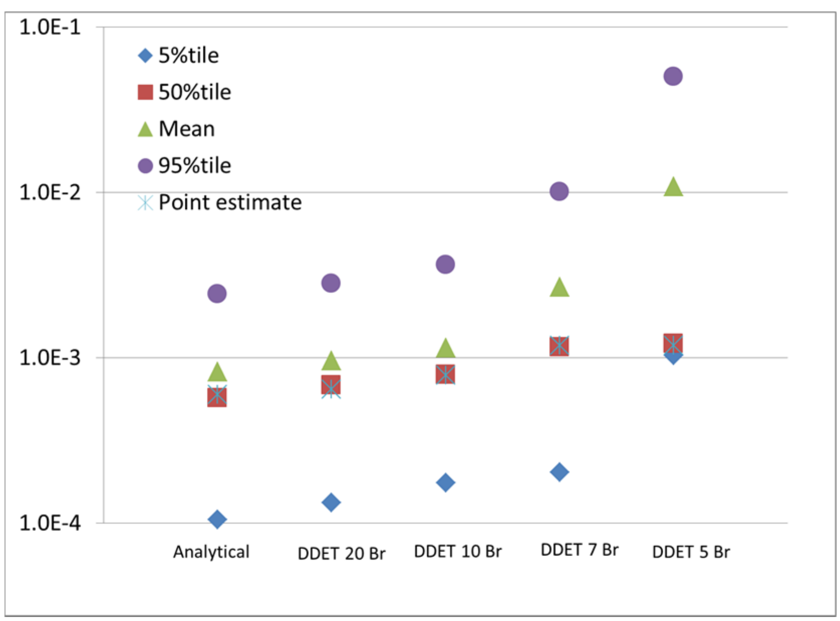

Fig. 8: Comparison of methods - Percentiles and mean of failure probability

In epistemic calculations, the discretization strategies are more thoroughly tested and compared with analytical results. Fig. 7 shows the cumulative probability functions for epistemic uncertainty in failure probability of the methods under consideration; further, the comparison of percentiles and mean among the methods is shown in Fig. 8. The median of all methods are close, but the upper tails of 5-branch and 7-branch are longer than other CDFs. The 20-branch and 10-branch approaches are in fairly good agreement with analytical CDF including the mean and tails. As expected 5-branch CDF is conservative, the log discretization strategies with a few more branches (7br., 10-branch) shifts CDFs close to the analytical result. Provided the percentiles focus on tails of operator response time distribution as in log strategy (e.g. 90, 99, 99.9, 99.99, skip), the results of 5-branch can be close to results of 7-branch.

Importance measures (risk contributors) and their uncertainties, and uncertainty importance measure (Pearson correlation coefficient method) of epistemic parameters have been calculated. The comparison of results among the methods reveals that operator error and its distribution parameter are top contributors to risk and its uncertainty. The ranking order is same among all methods, but the risk and uncertainty contribution of valve and its distribution parameters in 5-branch case are underestimated because of overestimation of operator error in aleatory calculations.

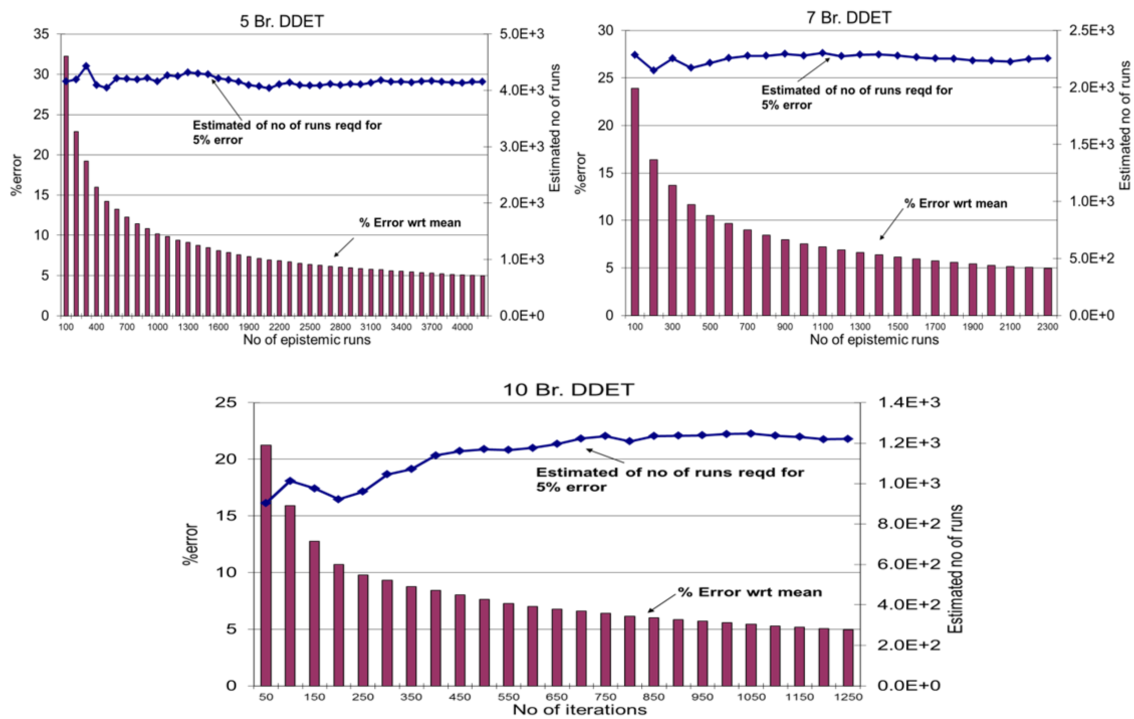

Fig. 9: Convergence criteria and no of runs required – comparison

The DDET approach gives an overestimate of risk in general, but there is no chance of underestimation. Log discretization strategy in DDET reduces conservatism in risk estimate to a great extent and only with few branches involved. Nevertheless, it is important to note that the number of runs gets multiplied as the number of continuous variables to be discretized increases. On the other hand, the number of continuous variables to be discretized in the simulations of nuclear power plant (NPP) accident scenarios are limited, for instance in MLOCA scenario of PWR type NPP there are only two safety functions whose response is continuous.

Convergence criteria and the number of runs

Accuracy costs, but acceptable error in results with limited number of runs is quite important during NPP accident scenario simulations, which challenges even today's computational resources. Convergence criteria used in current calculations monitors the current percentage error and estimates the number of runs required to achieve a given percentage of error with respect to mean. Fig. 9 shows such monitoring plots for DDET with 5, 7, and 10 branch cases. In DDET with 5-branch case, after 500 runs it consistently estimates the required runs to be 4200 runs, and the percentage error after 4200 runs matches 5%. This trend of estimating runs required after a few initial runs is noticed in all other cases as well; only plots for 5, 7, 10-branch DDET cases are shown in Fig. 9. This online convergence criterion could be very useful when the complex nuclear scenarios are explored. Depending on the estimated number of runs and current percentage error after a few initial runs (with initially considered values of percentage error and confidence level), the criterion can be modified to reduce the number of runs or improve the accuracy.

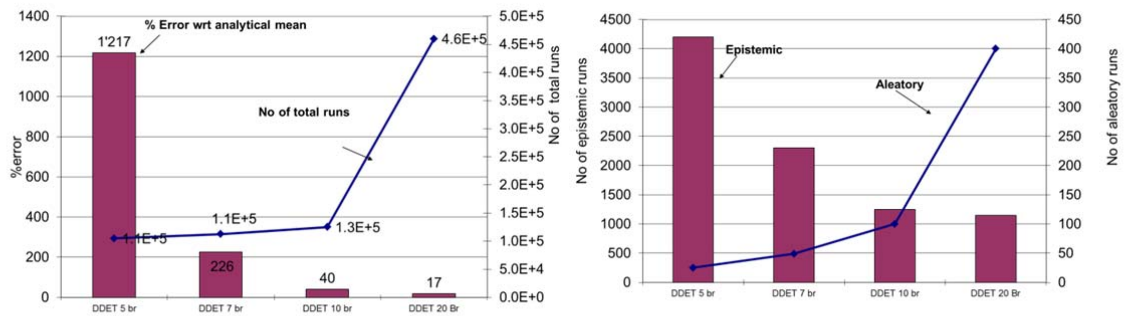

Fig. 10: Comparison of runs and %error among DDET methods

Epistemic Vs Aleatory Runs

An interesting relationship between the number of epistemic and aleatory runs is present. The number of epistemic runs is inversely proportional to the number of aleatory runs before it becomes stable. Their product which gives total number of runs and the percentage error with respect to analytical mean are also dependent. Fig. 10 shows such a relation among the DDET cases. The epistemic runs represent the total number of Monte Carlo simulations sampling epistemic parameters, where each simulation produces a DDET; whereas the aleatory runs represent the number of sequences in each DDET. The number of epistemic runs decreases as the number of aleatory runs increases, and both lines meet in DDET 10-branch case. Looking at the total number of runs versus percentage error with respect to analytical mean, larger number of runs required to reduce the error from 40% to 17%. The optimal point is DDET 10-branch case, which gives less error with a few runs.

Having an optimal number of discretization levels (branches) helps the analyst. However, determining the optimal number of discretization levels is quite challenging in real plant applications and it should not be the aim in such cases. The number of branches may be added in steps and the change in risk estimate among them shall be monitored. When the change is not substantial, it indicates the convergence.

5. CONCLUSIONS

In risk quantification, both epistemic and aleatory uncertainties are present due to inherent physical and safety system variability and their model parameters. Ignoring any one could lead to inappropriate estimation of risk and its uncertainties. DDET along with epistemic uncertainty analysis has been proposed as an approach for solving the problem of integrated treatment of uncertainties and dynamics in quantifying risk. The approach has been demonstrated with the application to modified tank problem. The analytical solution provided as a means of comparison with the obtained results from

DDET cases. The proposed log discretization strategy reduces the conservatism in risk estimate, which is present in discretization strategies used in the literature. The log strategy needs only a few branches to converge with analytical results. Nevertheless, in DDET approach the number of aleatory runs gets multiplied as the number of continuous variables to be discretized increases. The number of epistemic runs depends on the number of aleatory runs. But increasing the aleatory runs beyond a point does not increase the accuracy significantly, but only increases total computational time significantly. Optimal allocation of computational resources between epistemic and aleatory runs ensures accuracy in risk estimate. The convergence criterion in epistemic calculations helps to monitor the current percentage error and estimates number of runs required to achieve a specified accuracy. The computational resources can be used more efficiently to improve accuracy in risk estimate. In more complex problems like NPP accident scenarios, the online convergence criteria will be particularly useful.

Acknowledgements

This work has been performed in the frame of a joint project with PSI's Laboratory for Reactor Physics and Systems Behaviour (LRS). It was partly supported by swissnuclear, the nuclear energy section of the organisation of Swiss electricity grid operators.

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