

Emergency Resource Allocation for Disaster Response: An Evolutionary Approach

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Abstract: The efficient response to a disaster plays an important role in decreasing its impact on affected victims. In some cases, the high volume of potential casualties as well as the urgency of a fast response increase the complexity of the disaster response mission. Such cases have created a need for developing an effective and efficient disaster response strategy. This paper focuses on developing a multi-objective optimization model and an evolutionary algorithm as a first step to generate optimal emergency medical response strategies characterized by the selection of: (1) locations of temporary emergency units, (2) dispatching strategies of emergency vehicles to evacuate injured victims to the temporary emergency units, and (3) number of victims to evacuate to each unit. The objectives of the optimization model are to minimize response time and cost of the response strategy. The evolutionary algorithm is used to solve the model and find a set of Pareto optimal solutions where each solution represents a different emergency medical response strategy. This approach can help decision-makers to evaluate the trade-offs among different strategies. Three experiments are provided to discuss the model.

Keywords: Resource Allocation, Disaster Response, Emergency Logistics, Multi-Objective Optimization, Evolutionary Algorithm.

1. INTRODUCTION

During the past two decades, the volume of casualties and the size of population affected by natural disasters have surged. For example, during the first decade of the 21st century, over 3.65 million people were affected by disasters. This represents a sharp increase over the 1990s, when roughly 3.1 million people were affected by similar disasters [1]. This increase has raised international concerns about the emergency preparedness [2]. The ability of government agencies to quickly respond with medical needs is crucial for limiting the impact of disasters on the population. Failure to allocate needed medical resources in a timely manner may result in worsening the disaster situation and increasing the number of casualties. For example, a US Congressional investigation found that when Hurricane Katrina hit the southern US coast in 2005, federal, state and City Bgencies did not succeed on implementing decisive response actions. Many of the disaster management procedures were implemented improperly or were inapplicable, including the evacuation plan, leading to preventable deaths and further delays in disaster relief [3].

It has become common to define four phases of emergency operation management: mitigation, preparedness, response, and recovery [4]. Even though all phases are interrelated, this section only focus on the response phase as it is related to the approach presented in this paper. The response phase is post-disaster plans conducted immediately after a disaster strikes. It includes activities such as implementing relief plans and providing all necessary emergency services. Whenever disasters strike, the critical need for effective and efficient emergency response and rapid deployment of resources (e.g. medical resources, food, clothing, shelter, etc.) creates an economically costly and complex planning paradigm. In this respect, many researchers have focused on: (1) improving emergency management performance [5] and/or (2) developing approaches dealing with allocation and deployment of emergency resources [6]. French [7] discussed methods to structure decision support systems for emergency response specifically for nuclear accidents. He pointed out the case of multi-attribute aspects of such decision where there are

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conflicting objectives. With respect to disaster relief plans, Barbarosoglu & Arda [8] proposed a two-stage stochastic programming model to plan the transportation of first-aid supplies in a disaster. Liu & Zhao [9] discussed a logistic relief network and proposed a multi-objective model for quick response to relief demands. Balcik & Beamon [10] proposed a mixed integer programming model for facility location and stock pre-processing for disaster relief. Ozdamar & Yi [11] proposed a model for vehicle dispatch in disaster relief planning. Other research projects have been conducted on the dispatch of emergency resources. Fiedrich et al. [12] presented an optimization model for allocating emergency resources after an earthquake. The objective of the model is to minimize the total number of fatalities during the Search-and-Rescue period. The approach implemented a computer-based decision support system in order to improve the efficiency of the model. Kondaveti & Ganz [13] introduced a decision support framework to find the optimal deployment and dispatching of emergency resources. However, the activities in the response phase (dispatching of relief provisions and emergency resources) cause a major vehicle routing and scheduling problem, very well known in the field of transportation and logistics. The problem was first introduced by Dantzig & Ramser [14]. Since then, it has been widely analyzed in many research studies. Beck et al. [15] studied the differences between the vehicle routing and shop scheduling problems, and developed an understanding of the problem characteristics. The study showed that the routing technology is superior to the scheduling technology, which makes routing technology able to perform well on open shop scheduling problems.

Although several studies have contributed to improving disaster response plans, specifically the emergency resources scheduling problem, there is still a lack of techniques for providing optimal strategies for emergency resources to respond effectively and efficiently to disasters. Also, the previously proposed approaches can be improved using state-of-the-art optimization methods to help decision-makers assign medical resources optimally. It is important to develop a model that allows decision makers to become aware of response time and cost when they select their preferred disaster response strategy.

As such, this paper proposes a multi-objective (MO) optimization model (along with an evolutionary algorithm for its solution) to facilitate the design of emergency medical response strategies characterized by the selection of: (1) TEU locations, (2) routes of emergency vehicles, and (3) number of victims to transport to each TEU. The multi-objective optimization model optimizes two objectives: (1) response time, defined as the total time it takes to evacuate all victims from the affected areas to the TEUs, and (2) cost, which is a function of the ambulances and TEUs procurement cost, and the operational cost of the strategy based on distance travelled by the emergency vehicles. To solve the multi-objective optimization model, an evolutionary algorithm (EA) called Probabilistic Solution Discovery Algorithm (PSDA), which was first used by Ramirez-Marquez and Rocco [16], is used to generate an approximate Pareto set of non-dominated solutions. The algorithm is used to generate multiple emergency medical response strategies – where each strategy represents a solution – and find an approximate set of Pareto optimal solutions. This approach can help decision-makers to evaluate the trade-offs among strategies with different response time and cost values.

The remaining sections of this paper continue as follows: Section 2 discusses the structure of the proposed response plan, introduces the equations used to assess the values of response time and cost, and explains the proposed multi-objective optimization model and the evolutionary algorithm. Section 3 includes experiments with corresponding results. Finally, Section 4 summarizes and concludes the paper with future research directions

2. PROPOSED METHEDOLOGY

2.1. Assumptions and notation:

1. Each emergency vehicle can only transport one victim at a time

2. TEUs have unlimited capacity
3. The model considers only transporting victims with non-life threatening injuries

Notation	
i, j, k	indices representing a city section with coordinates $(x_i, y_i), (x_j, y_j), (x_k, y_k)$
N	total number of sections
t_{ij}	time to travel from section i to section j
d_{ij}	road distance between section i and section j (miles)
MC_i	binary variable defining if there is a medical center at section i
TEU_j	binary decision variable defining if there is a TEU in section j
E_k	binary variable defining if an event occurred in section k
K_i	number of emergency vehicles available at medical center in section i
Z_j	number of emergency vehicles assigned to TEU in section j
Q_k	number of victims in section k
V_{ij}	decision variable defining number of vehicles dispatched from MC_i to TEU_j
S_{jk}	decision variable defining the number of victims to be evacuated from section k to TEU_j
a_{ij}	fraction of emergency vehicles sent from MC_i to $TEU_j, a_{ij} \in [0,1]$
b_{jk}	fraction of victims be evacuated to $TEU_j, b_{jk} \in [0,1]$
TC	total cost of response strategy
RT	response strategy time
C^T	ambulance operating cost per mile (\$/mile)
C_j^F	fixed cost associated with creating a TEU at section j (\$)
C^A	procurement cost of one ambulance (\$)
PSDA	
U	number of generations, $u=1, \dots, U$
W	number of solutions per generation, $w=1, \dots, W$
γ_j^u	appearance probability of allocating a TEU in section j at generation u
P^u	Pareto set of optimal solutions at generation u
Acronyms	
MC	Medical Centers
TEU	Temporary Emergency Unit
EMS	Emergency Medical Services

2.2. Mathematical formulation

In scenes of mass-casualty incidents, health professionals need to sort and prioritize victims for emergency transportation and treatment. In general, victims are classified into four groups: (1) those with unsalvageable life-threatening injuries, (2) those with salvageable life-threatening injuries, (3) those with non-life threatening injuries, and (4) those not seriously injured. This research considers the third group whose victims need to be transported to Temporary Emergency Units (TEUs). For such group, the emergency response includes selecting the area where TEUs must be allocated, and dispatching emergency vehicles to the corresponding TEUs. Emergency vehicles then begin transporting victims from the scene to the TEUs until evacuating all victims.

This paper considers two types of medical facilities: medical centers (MC) and temporary emergency units (TEU). MCs are established medical centers that operate 24 hours and include hospitals and Emergency Medical Services (EMS) from where emergency vehicles are dispatched. TEUs are temporary facilities created at stable areas to provide medical treatment to disaster victims. This paper only addresses victims with non-life threatening injuries who do not need major medical intervention, the role of MC is only to dispatch emergency vehicles and victims are not transported to the medical centers (MCs). When a mass-casualty incident occurs, an emergency medical response strategy must include: (1) allocation of TEUs to areas unaffected by the event, (2) dispatching of emergency vehicles from MCs to TEUs, (3) dispatching of emergency vehicles from TEUs to the incident sites, from which victims are transported to the TEUs, and (4) the number of emergency vehicles to be used as well as the number of victims to be evacuated.

The mathematical model presented in this paper proposes that the region, where the event can potentially occur, is divided into N sections. The following binary variables are defined:

$$E_k = \begin{cases} 1 & \text{if a mass - casualty incident affects section } k \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$MC_i = \begin{cases} 1 & \text{if there is a medical center at section } i \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$TEU_j = \begin{cases} 1 & \text{if there is a TEU at section } j \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The number of emergency vehicles available at each MC is represented by K_i , and is assumed known a-priori by decision-makers. However, the actual number of emergency vehicles sent from MC_i to TEU_j is denoted by V_{ij} , and is calculated as a fraction of K_i mapped to the largest previous integer value. The fraction a_{ij} is a random number, between 0 and 1, generated by the algorithm.

$$V_{ij} = \lfloor (a_{ij} * K_i * TEU_j) \rfloor \quad \text{where } a_{ij} \in [0,1] \quad (4)$$

Note that an MC can dispatch emergency vehicles to different TEUs, and a TEU can receive emergency vehicles from different MCs. Thus, the total number of emergency vehicles dispatched to a TEU is represented as Z_j , and is obtained following Equation (5).

$$Z_j = \sum_{i=1}^N V_{ij} \quad (5)$$

The total number of victims (Q_k) in such conditions is assumed to be a function of the population density impacted by the disaster. The fraction b_{jk} is a random number, between 0 and 1, generated by the algorithm. Thus, S_{jk} , the number of victims to be evacuated from section k to TEU_j , is equal to the largest previous integer of a fraction of the total number of victims at section k as shown in Equation (6).

$$S_{jk} = \lfloor (b_{jk} * Q_k * TEU_j) \rfloor \quad \text{where } b_{jk} \in [0,1] \quad (6)$$

The objectives of the MO optimization model are to minimize: response time and cost. To solve the model, an EA is proposed. It produces an approximate set of Pareto optimal solutions where each solution represents a response strategy with different values of response time and cost. Each strategy is characterized by the selection of: (1) TEU locations, (2) routes of emergency vehicles, and (3) number of victims to transport to each TEU. The optimization model is comprised of three decision variables, two objectives, and two constraints.

The decision variables were discussed in the previous section, and include: (1) TEU location (TEU_j), (2) number of emergency vehicles to be sent from MCs to TEUs (V_{ij}), and (3) number of victims to be evacuated from the section where the event happened to each TEU (S_{jk}).

The objective functions are as follows: the first objective is to minimize the total cost (TC), and the second objective is to minimize the response time (RT). Apparently, those two objectives seem to be conflicting. Usually, for this type of problems a solution with less response time incurs higher cost and vice versa. In the proposed model, the total cost (TC) accounts for the operation cost of each ambulance (C^T) including transportation from MCs to TEUs and evacuation of victims from events to TEUs, the procurement cost of each ambulance (C^A), and the initial cost of allocating a TEU in a given section (C_j^F). The response time (RT) represents the average time it takes to send emergency vehicles from MCs

to TEUs plus the maximum of the average time needed to evacuate victims from the affected sections to each TEU.

Each emergency vehicle can only evacuate one victim at a time. The constraint number one in the model is represented by Equation (7) and is used to ensure that all victims are evacuated. The constraint number two, shown in Equation (8), is used to guarantee that the total number of emergency vehicles utilized from a MC does not exceed the total number of emergency vehicles available at that unit.

$$\min TC = \sum_i \sum_j V_{ij} * d_{ij} * C^T + \sum_j \sum_k S_{jk} * d_{jk} * C^T + \sum_j C_j^F * TEU_j + \sum_i \sum_j V_{ij} * C^A$$

$$\min RT = \frac{\sum_i \sum_j V_{ij} * t_{ij}}{\sum_i \sum_j V_{ij}} + \max_{1 \leq j \leq N} \left(\frac{\sum_k 2 * S_{jk} * t_{jk}}{Z_j} \right)$$

$$\text{s.t.} \quad \sum_j S_{jk} = Q_k \quad \forall k = 1, \dots, N \quad (7)$$

$$\sum_j V_{ij} \leq K_i \quad \forall i = 1, \dots, N \quad (8)$$

where d_{ij} and d_{jk} represent the distance between section i and j , and section j and k , respectively.

3. ILLUSTRATIVE EXAMPLE AND RESULTS

This study considers City B, which is divided into 25 sections, enumerated as shown in Figure 1. The city has four Medical Centers (MCs): one is a hospital, located at section 23, and the other three are EMSs, located at sections 7, 10, and 24. These medical centers have ambulances used for transporting injured people. Public records show that the number of ambulances operated by MCs around the world tends to vary between 3 and 20. Based on these figures, an estimated total of 50 ambulances would be available across City B medical centers, distributed as illustrated in Table 1.

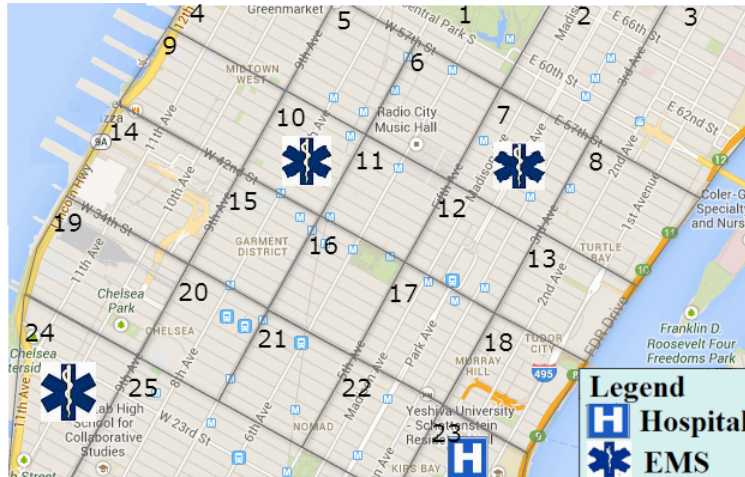


Figure 1 – City B sections and medical centers (MCs).

Table 1 – Number of emergency vehicles at Medical Centers (MCs)

Medical Center (MC)	Number of available ambulances
EMS at section 7	10
EMS at section 10	10
Hospital at section 23	20
EMS at section 24	10
Total	50

The proposed emergency response plans to disasters affecting City B are designed using an MO optimization model and include (1) allocating temporary emergency units (TEUs) at city sections, (2) dispatching emergency vehicles, and (3) evacuating disaster victims to the TEUs. It is important to mention that the cost of a TEU is not the same in all sections because of differences in the cost and availability of facilities, and because of potential area issues at each section. The cost of a TEU in each section, shown in Figure 2, is estimated based on several factors, including population density as many studies have associated land prices with population [17]. Furthermore, the operating cost associated with travelling from one section to another is assumed to be \$5 per mile. Each ambulance incurs a fixed procurement cost of \$400, regardless of its travel distance. In addition, travelling speed is assumed to be 5 minutes per mile.

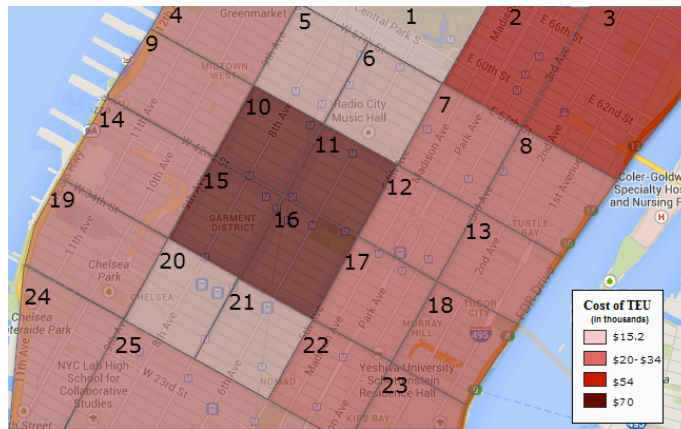


Figure 2 – Initial cost of a TEU at each section in thousands of dollars

Two experiments are discussed in this section in order to illustrate how the model is applied. In these experiments, it is assumed that City B could be affected by disasters in different locations. Experiment 1 assumes a disaster has affected three sections. The MO optimization model is designed for this case and solved with PSDA to generate nearly optimal response strategies characterized by different values of response time and cost. The second experiment discusses 7 scenarios. Each scenario assumes a disaster affecting three different sections in order to investigate possible behavior in the response strategies.

3.1. Experiment 1

In this experiment, it is assumed that a disastrous event affected sections 18, 19, and 20 of the city, causing damage and leaving 900 people with injuries that need immediate medical attention, as illustrated in Table 2. It is important to mention that these numbers are initial estimates and can never be guaranteed to be accurate. But, designing response strategy needs these estimates as inputs into the MO optimization model.

Table 2 – Events data

Event	Location	Injured victims
Event 1	18	510
Event 2	19	290
Event 3	20	100
Total	-	900

The emergency response strategy for this experiment was generated through the implementation of the PSDA. The algorithm was coded in *Mathematica* and run on a laptop computer with Intel Core I5-550M, 2.66 GHz processor, 4 GB RAM, and a Windows 7 operating system. In the implementation process, five generations were run with 1000 solutions in each generation. A total of 41 Pareto solutions were obtained, as shown in Figure 3. Each solution corresponds to an emergency response strategy with a unique combination of response time and cost values. Figure 3 also shows that the total cost increases as response time decreases.

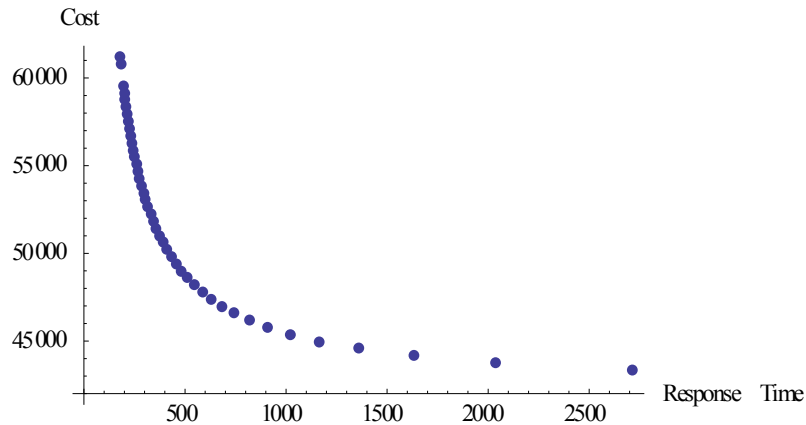


Figure 3 – Pareto Solutions for Experiment 1

As solutions evolve, they converge towards the same TEU allocation. Interestingly, some solutions with more than one TEU appeared in the Pareto set in the earlier generations. However, these solutions were too expensive and the response times were not short enough. Consequently, in the latter Pareto set, new solutions, with one TEU, have replaced the old solutions, with more than one TEU, as they incur lower costs and deliver the same or even lower response times.

According to the final results, only one TEU should be allocated at section 7, and all victims from the three events should be transported to this TEU. Table 3 contains the solution characteristics of a sample of the Pareto solutions. For instance, solution number 25 has a response time of 432.6 minutes, and costs \$49,821, and utilizes a total of 19 emergency vehicles. Out of those 19 emergency vehicles, 9 are from the MC at section 7, 1 from the MC at section 10, 8 from the MC at section 23, and 1 from the MC at section 24. Figure 4 illustrates graphically the locations of the incidents and the TEU location.

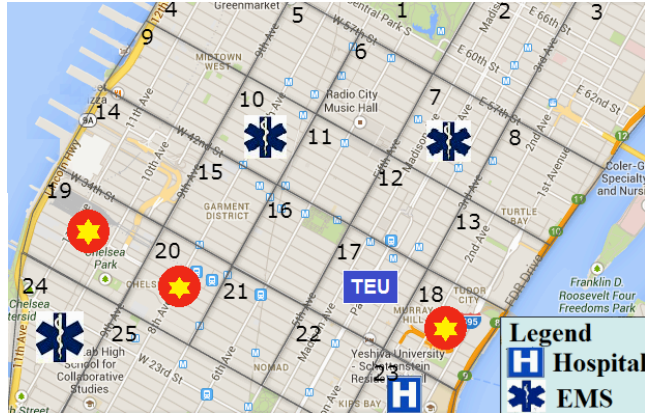


Figure 4– Incident and TEU locations

Table 3 – Samples of Pareto solutions

Solution Number	Response Time (minutes)	Total Cost (\$)	Number of ambulances from MC at section j to TEU at section 17				
			MC ₇	MC ₁₀	MC ₂₃	MC ₂₄	Total
1	178.84	61205.5	10	9	18	10	47
5	203.75	58753.8	9	7	19	6	41
9	224.534	57107.8	10	7	19	1	37
13	251.258	55491.5	5	10	18	0	33
20	317.442	52653.5	10	6	10	0	26
25	432.684	49821	9	1	8	1	19
28	512.672	48602.8	8	1	7	0	16
34	817.5	46175	10	0	0	0	10
35	908.611	45777.5	4	3	2	0	9
39	1632.6	44163	1	4	0	0	5
40	2037.75	43751	4	0	0	0	4
41	2716.33	43349	2	1	0	0	3

3.2. Experiment 2

In Experiment 2, seven incident scenarios, in which disastrous events affected City B in various locations, are examined and analyzed in order to discuss and uncover possible trends in the nearly optimal response strategies. Each scenario represents an incident affecting three sections and causing damage and injuries in these sections as illustrated in Table 4. For example, scenario 2 indicates that the incident affected sections 1, 2 and 3. Each incident causes a total of 900 injured people. Scenario 1 is the one discussed in the previous experiment. It is assumed that when an incident occurs at a section having an MC, such as sections 7, 10, 23, and 24, emergency vehicles operated at the MC are still functioning and can be dispatched.

Table 4 – Locations of the incident scenarios

Scenario no.	2	3	4	5	6	7	8
Affected Sections	1,2,3	4,5,6	7,8,9	10,11,12	13,14,15	16,17,18	21,22,25

Ten generations were run for each scenario with 1000 solutions in each generation. An average of 35 Pareto solutions per scenario was obtained for the 8 scenarios. The characteristics (min, max, and

average) of the solutions for each scenario are illustrated in Table 5. The table shows that scenarios 1 and 6 shared the same TEU location (at section 17). However, the response time and cost vary considerably in all of the scenarios, and that apparently because of the various aspects (i.e., location, population, cost of the land, etc.) associated with each scenario. Scenario 5 is the cheapest one as it costs an average of \$44064.32. Whereas, scenario 3 is the most expensive one (average cost is \$148,444.39). Scenario 1 has the shortest response time (average RT is 505.82 minutes) while Scenario 2 has the longest response time (average RT is 505.82 minutes).

Table 5 - Solution characteristics of scenarios 1-8

Scenario No.	TEU location	Cost			Response Time		
		Avg	Max	Min	Avg	Max	Min
1	17	52164.31	61205.5	43751	505.82	2037.75	178.84
2	8	96869.82	109331	83828	2042.2	4813.40	1124.45
3	1	148444.39	157446	136784	1126.6	2344.63	589.39
4	4	98035.18	107366	89699	1211.33	4127.67	509.32
5	6	44064.32	52362	36825.5	722.20	1975.07	314.71
6	17	88802.76	97641	81740.7	1134.52	2403.81	540.50
7	21	47581.49	55314.50	39899.70	769.43	2062.47	363.36
8	23	68317.06	75165.30	62230	972.95	1952.50	535.57

Figure 5 illustrates the TEU locations for the 8 scenarios. In each scenario, the response strategy calls for the allocation of only one TEU. Two scenarios (1 and 7) share one location as a TEU location. In general, the TEUs, MCs, and incident locations show that the best location of a TEU is close to incident locations, rather than to MC locations. A total of 7 sections have been chosen as best TEU locations for the 8 scenarios

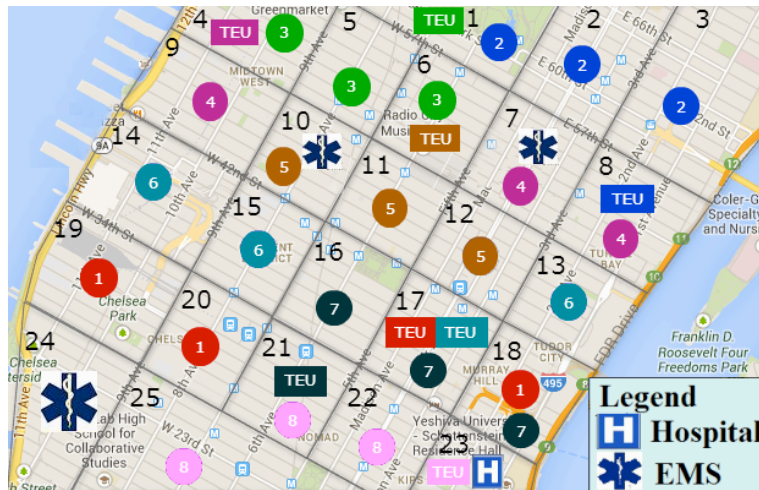


Figure 5 - TEU and incident locations for each scenario

4. CONCLUSIONS AND FUTURE RESEARCH

This study presented a MO optimization approach that helps with design of response strategies to a disastrous event with the intention to optimize the response time and cost. The model temporarily locates emergency units in stable areas and dispatches emergency vehicles to evacuate affected victims, who have non-life threatening injuries but need medical attention. An evolutionary algorithm called PSDA was used to obtain the approximate Pareto set of optimal solutions where each solution represents an emergency response strategy. This approach enables decision-makers to tradeoff response strategies based on values of response time and cost. The future extension of this study is to consider some criteria:

finding shortest path from medical/emergency centers to events, taking into account path failures, victim classification based on the level of injuries, and considering stochastic events.

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