

# Modeling the Reliability and the Performance of a Wind Farm Using Cyclic Non-Homogenous Markov Chains

Theodoros V. Tzioutzias<sup>a</sup>, Agapios N. Platis<sup>a\*</sup>, Vasilis P. Koutras<sup>a</sup>

<sup>a</sup>University of the Aegean Department of Financial and Management Engineering, Chios, Greece

---

**Abstract:** Reliability issues concerning wind power installations are of prior interest due to the increasing production of electricity through wind energy around the world. Attaining the highest performance of a wind farm lies on two factors: wind intensity and mechanical failures. Wind intensity is related to the geographical characteristics of the farm, while mechanical failures are related to the wind turbines reliability. The latter issue is the one studied in this paper. In order to achieve higher performance levels when assuming the wind capacity as constant for a certain installation and a certain period, we can increase reliability by reducing mechanical failures and increasing repair rates. The objective of this paper is to model reliability's impact on the overall performance of the wind farm. To this direction, a Continuous Time Markov Chain (CTMC) and a Cyclic Non-Homogenous Markov Chain (CNHMC) are used to model the system. CNHMCs are adopted in order to capture the periodicity of the wind intensity. The results of the above models are compared in order to identify which one fits better the characteristics of the system.

**Keywords:** Wind Power, Reliability, Cyclic Non-Homogenous Markov Chains (CNHMC), Asymptotic Probability Distribution.

---

## 1. INTRODUCTION

The production of electricity using renewable energy resources is an urgent matter. The continuous CO<sub>2</sub> emissions from industry aggravate the global warming phenomenon. That is the main reason of EU's promise that by 2020 the 20% approximately of the total demand of electricity will come from renewable energy resources, with wind energy being the leader with a percentage between 14-18%.

The technology that uses the power of wind in order to produce electricity is very growing. Wind generator companies come up with new ideas that can achieve better performance year after year. But the performance of a wind farm is a compromise between the wind that blows in the area where the generators are located and the reliability of that system. Modeling such a system is of prior importance in evaluating properly the reliability indices. A simple and easy to understand method for modelling is the deterministic criteria, but for the evaluation of the reliability, probabilistic methods seem to be a more accurate and reliable method. Due to the nature of the wind and its changes over time, modelling becomes more interesting and difficult.

In this paper, we evaluate the reliability of a wind park considering not only weather data from a certain area, but also the mechanical failures. In order to model our problem we use classical probabilistic methods, with which we will be able derive our results for different intensities of the wind taking also into account the mechanical failures. Our model is based on the categorization of wind intensity in four categories and on the transitions between them using Markov models [1]. Beyond the classical CTMC approach, we expand our study by using CNHMC in order to capture the periodicity revealed due to seasonal wind changes.

In our study, we consider wind data provided by the Hellenic National Meteorological Service (HNMR), from the meteorological station which is located in the island of Chios in the area of airport. The data refer to the last five years (from January 2008 to July 2013). We had a measurement about the speed of the wind once every three hours, so we had 8 measurements per day. Because of the volatile nature of wind and the different intensities over the time we categorize the wind speed into four categories, including also a no-wind state. Every transition from a state to another is possible,

even more from a state with a strong wind to no-wind state. Combining wind intensity categories with the state of a system regarding machine failures, we develop a model with eight states in total; the first four refer to the states without any machine failure, and the rest to the states where a failure occurs

The application of probabilistic methods for modeling of wind power was addressed by different authors in the past. Gouveia and Matos in [1], inspired from the previous researches, develop a new methodology by which the wind speed was categorized in four categories, no-wind state (0% production), light wind (30% production), strong wind (70% production), and very strong wind (100% production). In their model, the transitions need not to be only in adjacent wind levels, but it is possible to have a transition between the states excessive wind to no wind state.

This paper is organized as follows: The methodologies used are described in Section 2; we model the same problem using Homogenous Markov Chains and Cyclic Non-homogenous Markov Chains. In section 3 we apply the above methodologies using real data that HNMR provide us, in section 4 we present an example and finally in section 5 we come up with our concluding remarks.

## 2. HOMOGENEOUS AND NON-HOMOGENEOUS MARKOV MODELS

A continuous-time Markov chain is a probability model which takes values in countable set. The time spent in each state takes only real and non-negative values according to an exponential distribution. The evolution of the system in time, according to the Markov property, does not depend on the historical behavior and the previous system states, but only on the current system state [1-2, 5].

Non-homogenous Markov chains are more complex, and the reason is why every transition rate between the states is a function of time and more accurate of a global clock [2, 3, 5].

In order to have a better understanding of the two cases lets assumes a system with two possible states, operation and failure. The rate of transition from the operational state to the failure state is called failure rate ( $l$ ), while the transition from failure to operation repair rate ( $m$ ). Using a CTMC to model the system indicates constant rates  $l, m$ , whereas for the NHMC the transition rates  $l(t)$  and  $m(t)$  are time dependent. In addition, NHMC modeling is preferable when the systems to model have hazard rates that evolve with time; it is generally the case of systems that hazard rates depend on environmental parameters such as wind farms where the performance and the failures depend mainly on the wind intensity, and weather phenomena like thunders.

### 2.1. Transition probability and transition function

Let us assume that  $S = \{0,1,2,3,\dots,s\}$  is the state space of the system, and  $X = \{X_t; t \in \mathbb{IN}\}$  is a stochastic process on a probability space  $X_t$  which is a Markov chain if, for all  $k \in E$  and all  $t > 0$ , the following equation holds true [5]:

$$\Pr\{X_{t+1} = k \mid X_t = i_t, \dots, X_0 = i_0\} = \Pr\{X_{t+1} = k \mid X_t = i_t\}$$

As a transition probability of a Markov chain, we define the probability that a transition from state  $i$  to state  $j$  occurs in the time interval  $[t-1, t]$ . The probability  $p_{ij}(t) = \Pr\{X_t = j \mid X_{t-1} = i\}$  for all  $i, j \in S, t \in \mathbb{IN}$  is called transition function of the chain  $X$  and it is a one-step transition probability. We can also define multiple steps transition probabilities by  $p_{s,t}(i, j) = \Pr\{X_t = j \mid X_s = i\}$  and the transition probability function can be then derived [2]:

$$p_t(i, j) = p_{t,t+1}(i, j) \quad \text{and} \quad p_{tt}(i, j) = \mathbf{I}$$

The transition function follows the property known as *Chapman-Kolmogorov* equation, for all  $i, j \in S$

$$p_{t,s}(i, j) = \sum_{l=0}^k p_{t,l}(i) \cdot p_{l,s}(j) \quad \text{for all } t < l < s$$

A transition function  $p_t(i, j)$  with  $t \in \mathbb{IN}$  and  $i, j \in S$  is called cyclic or periodic with period  $h, (h > 1)$  if  $h$  is the smallest integer verifying the equation  $p_{th+r} = \Pr(i, j)$  with  $t, r, \in \mathbb{IN}$ , and  $i, j \in E$  [2, 4].

## 2.2. State Probability Vector

Let us now consider the initial distribution probability  $\mathbf{a} = (\alpha_i)_{i \in E}$  with,  $\alpha_i = \Pr(X_0 = i)$  and transition probability matrix  $\mathbf{p}_t$ . The Markov chain  $X$  is completely defined by the initial distribution  $\mathbf{a}$  and the transition probability matrix  $p_t$  with  $t \in \mathbb{IN}$ , and the transition probability from state  $i$  to  $j$  at time  $t$  is given by

$$\begin{aligned} p_j(t) &= \Pr(X_t = j) = \sum_{i \in E} \Pr(X_t = j | X_0 = i) \Pr(X_0 = i) \\ &= \sum_{i \in E} \alpha_i p_{i,j}(t) = (\mathbf{a} \mathbf{p}_{0,t})(j) = \left[ \mathbf{a} \left( \prod_{k=0}^{t-1} \mathbf{p}_k \right) \right](j) \end{aligned}$$

$p_{ij}(t)$  is called state probability at time  $t$ . The vector  $\mathbf{P}(t) = (P_1(t), \dots, P_s(t))$  is called state probability vector at time  $t$ . We can also write  $\mathbf{P}(t) = \mathbf{a} \mathbf{p}_{0,t}$ .

## 2.3. Asymptotic Behavior

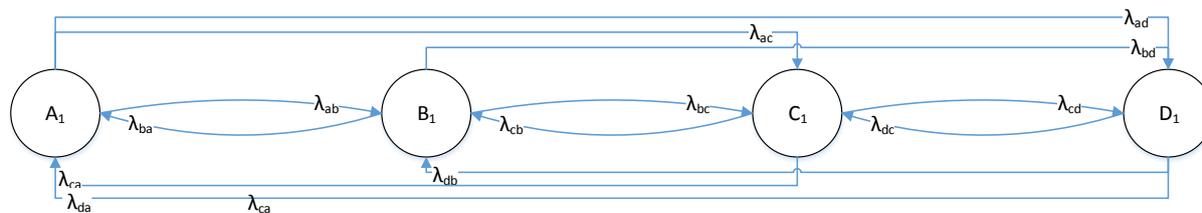
Let's assume that  $S_t$  is an embedded homogenous chain from  $X_t$ . As a result we have that  $S_t = X_{td}$  for every  $t > 0$ . If  $\mathbf{p}_t$  is the transition matrix of the NHMC  $X_t$ , then  $\mathbf{p}_{0,d}$  is the constant transition matrix of the HMC  $S_t$ . The asymptotic behavior of the transition matrixes is closely related with the known as ergodic properties of the Markov chain. The term ergodicity is related to the convergence of probability distributions  $p_0$  in time,  $\mathbf{p}^n \rightarrow \mathbf{\Pi}$  as  $n \rightarrow \infty$ , with  $\mathbf{\Pi}$  ergodic and  $\mathbf{P}$  the transition matrix probability, and assumes aperiodicity as a necessary condition. The above result can be used for the embedded HMC  $Y_t$ , in order to obtain the asymptotic results concerning the NHMC  $X_t$ .

## 3. MODELING THE SYSTEM OF WIND GENERATORS

### 3.1. Model description

The general model adopted and studied is a system consisting of  $n$  wind generators that are able to operate in different wind levels. Because of wind nature, we assume that its intensity can vary between 4 categories: the first category is the no-wind state and the last one is the excessive wind state. In our approach, the possible transitions are the transition from a wind category to another, without being necessary for the new category to be adjacent to the previous, and the transition from a state without failure to a state with a failure.

**Fig. 1 Markov chain model for four categories of wind**



Two different probability models are described below, a CTMC and a CNHMC, and we will use both models for our study. Firstly, we use a CNHMC to model a wind turbine and then a wind farm, where the hazard rates evolve periodically. Moreover, we will model the same case using a CTMC, and after deriving the results we will compare them with the results of the CNHMC model.

As previously above, the performance of a wind park depends on the intensity of the wind and the reliability of the generators. In order for the wind farm to achieve the best performance, the blowing wind in the area of interest should be able to give full production without any risk. Moreover the system is designed to operate without a mechanical failure that can cause loss of performance. The high repair rate and the very low failure rate are the keys to achieve the better performance. In addition, the failures of a system are highly dependent on weather conditions, while on the other hand the repairs depend on both the weather conditions and the working hours of the units. It is clear enough that seasonality plays also an important role for the repair and failure rates, and through our study we find out that it is not needed to keep a constant repair rate through all the seasons of the year, but if we reduce the repair rate in some periods of time the overall performance of the system will be the same as keeping the repair rate constant. For example, we may need fewer repairs in winter than in summer time without having lower production.

In our approach we are interested only the variation of the transition rate among the wind intensity states and we assume that not only the failure but also the repair rates are constant. This is because, initially we are interesting in studying the effects of the time dependent transition rates on system's asymptotic probability distribution. In the future we intend to extend our study by taking into account time dependent failure and repair rates.

### 3.2. Modelling of the wind transitions using Markov chains

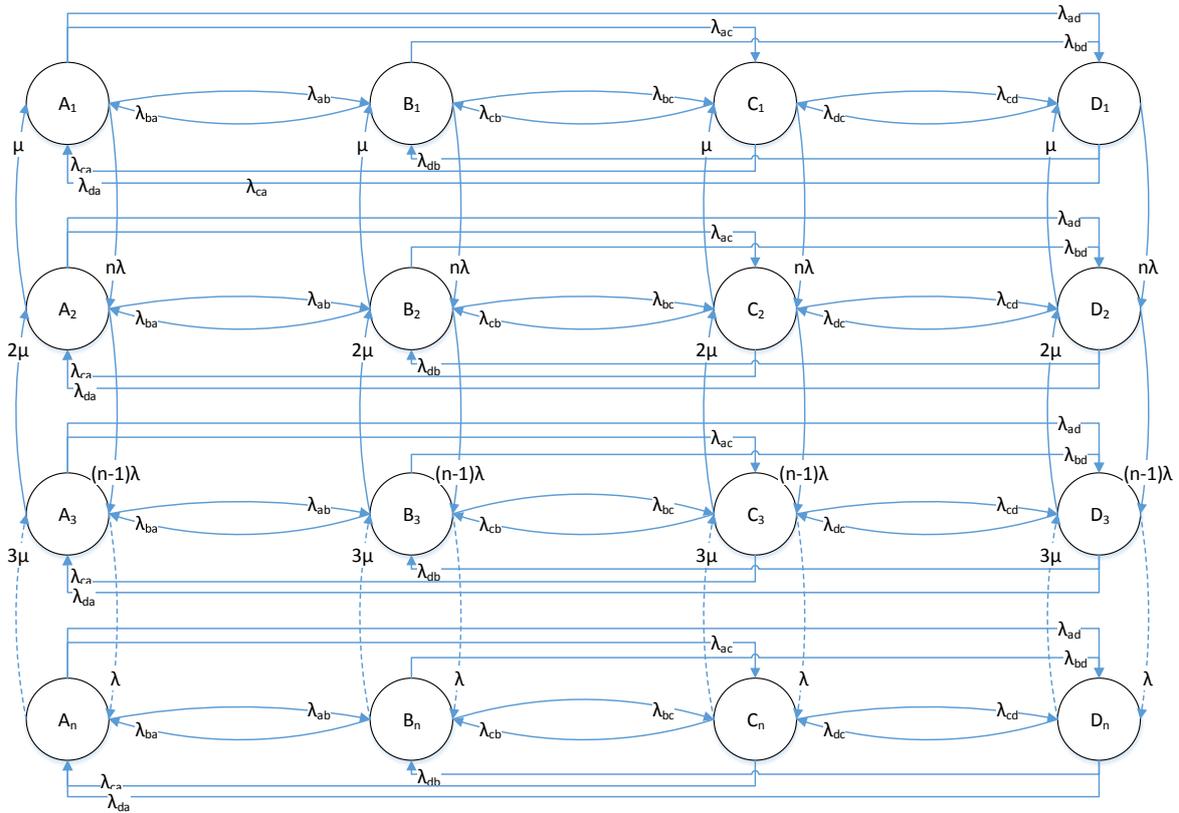
In our case, we will consider the wind variation through a season (spring, summer, autumn or winter). We rely on the data we got from the HNMS from January 2008 to June 2013, for every day, every three hours, from the meteorological station which is located in the area of Chios Island airport. Through appropriate data processing, we categorized the data according to the season and the year. As a result we have four seasons for every year from 2008 to 2012 and for the year 2013 we have only the spring. The next step was to categorize the wind according to the intensity into four categories. State A refers to the scenario where there is no wind or very weak wind unable to give any electricity production. States B and C are intermediate states and in state D we have winds able to provide full electricity production. In Fig.1 the Markov model for all the possible transitions between the four states of wind is represented. We consider that all the transitions are possible, even from state A without wind, to state D in which excessive winds, and the opposite [1].

### 3.3. Modeling a wind park using Markov chains

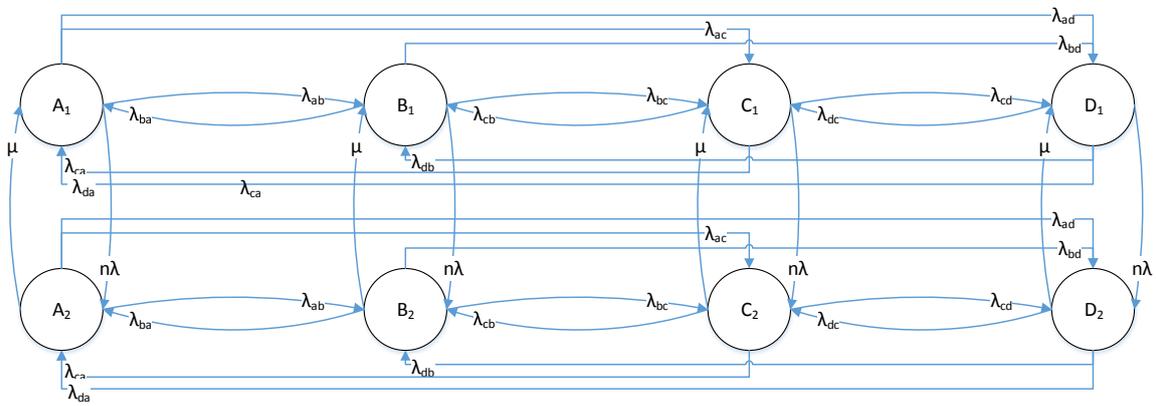
In order to completely model the operational scenario of a wind farm, apart from the wind states that are presented in Fig. 1, we should consider the number of the wind generators  $n$  in addition with the repair rates  $\mu$  and failure rate  $\lambda$ .

In Fig. 2 the final Markov model for a wind farm consisting of  $n$  turbines is shown, considering four different categories of wind intensity. A transitions occurs in three possible cases. Firstly when there is

**Fig. 2 Markov model of a wind farm consisting of n turbines**



**Fig. 3 Simplified Markov model considering aggregation**



a wind variation, secondly, when we have a mechanical failure of a unit and thirdly when we have a repair of a unit. As it is mentioned in [1], if we want to receive more accurate results we should consider more than four wind states.

As it is stated in [1], the aggregation of wind farms is very common, in order to have a simpler model without great number of states, and the main reason for that is the high loads of wind power that is being produced. A simplified diagram consisting of the eight states that are needed in order to combine the state of a unit with the state of other turbines' state is presented in Fig.3

In order to have a better understanding of the system and the time dependency, we study its behavior for every season from spring 2008 to summer 2013, and we set as a period a full year, or the four season of a year, spring, summer, autumn and winter.

The main aim of our work is to calculate the steady state probability of a wind park using CNHMC since we are interested in the long run behavior of the system. Initially the system starts at an operational state, and hence it can convert all the wind power to electricity.

A transition between two states is possible when we have variation in the intensity of the wind, for example from state B (low wind) to state D (extensive wind), in case of a failure, and in case of a repair. Considering the above, the generator matrix is:

$$\mathbf{Q} = \begin{bmatrix} y_{1,k} & \lambda_{ab,k} & \lambda_{ac,k} & \lambda_{ad,k} & n\lambda & 0 & 0 & 0 \\ \lambda_{ba,k} & y_{2,k} & \lambda_{bc,k} & \lambda_{bd,k} & 0 & n\lambda & 0 & 0 \\ \lambda_{ca,k} & \lambda_{cb,k} & y_{3,k} & \lambda_{cd,k} & 0 & 0 & n\lambda & 0 \\ \lambda_{da,k} & \lambda_{db,k} & \lambda_{dc,k} & y_{4,k} & 0 & 0 & 0 & n\lambda \\ \mu & 0 & 0 & 0 & y_{5,k} & \lambda_{ab,k} & \lambda_{ac,k} & \lambda_{ad,k} \\ 0 & \mu & 0 & 0 & \lambda_{ba,k} & y_{6,k} & \lambda_{bc,k} & \lambda_{bd,k} \\ 0 & 0 & \mu & 0 & \lambda_{ca,k} & \lambda_{cb,k} & y_{7,k} & \lambda_{cd,k} \\ 0 & 0 & 0 & \mu & \lambda_{da,k} & \lambda_{db,k} & \lambda_{dc,k} & y_{8,k} \end{bmatrix}$$

where

$$\begin{aligned} y_{1,k} &= -(\lambda_{ab} + \lambda_{ac} + \lambda_{ad} + n\lambda) \\ y_{2,k} &= -(\lambda_{ba} + \lambda_{bc} + \lambda_{bd} + n\lambda) \\ y_{3,k} &= -(\lambda_{ca} + \lambda_{cb} + \lambda_{cd} + n\lambda) \\ y_{4,k} &= -(\lambda_{da} + \lambda_{db} + \lambda_{dc} + n\lambda) \\ y_{5,k} &= -(\mu + \lambda_{ab} + \lambda_{ac} + \lambda_{ad}) \\ y_{6,k} &= -(\mu + \lambda_{ba} + \lambda_{bc} + \lambda_{bd}) \\ y_{7,k} &= -(\mu + \lambda_{ca} + \lambda_{cb} + \lambda_{cd}) \\ y_{8,k} &= -(\mu + \lambda_{da} + \lambda_{db} + \lambda_{dc}) \end{aligned}$$

$k = 1, 2, 3, 4$  stands for season  $k$ ,  $\lambda_{ab,k}$  is the transition from state A to the state B in season  $k$ ,  $\lambda$  is the failure rate,  $n$  the number of the wind turbines and  $\mu$  is the repair rate [1,3-4].

One of the advantages of modelling using CNHMC is that we can determine the steady state probabilities  $\boldsymbol{\pi}$  of the  $k$ -th season from the equation ([3]):

$$\boldsymbol{\pi}_k = \boldsymbol{\pi}_{k-1} \mathbf{P}_{k-1} \quad (1)$$

where  $\mathbf{P}_{k-1}$  is the probability matrix for the  $(k-1)$ -th season which is derived from the equation

$$\mathbf{p}_k = \mathbf{Q}_k \mathbf{h} + \mathbf{I} \quad (2)$$

with  $h$  a constant number near 1 and  $\mathbf{I}$  the identity matrix.

In order to derive the steady state probability of the first period we should solve the linear system

$$\pi_1 = \pi_0 \mathbf{p}_0 \text{ with } \pi_0 = \boldsymbol{\pi} \quad (3)$$

with  $\boldsymbol{\pi}$  the steady state probability vector and  $p_0$  is the probability matrix for the first season.

The stationary probability distribution can be computed by solving the linear system

$$\pi_0 = \pi_0 \mathbf{p}_0 \mathbf{p}_1 \mathbf{p}_2 \mathbf{p}_3$$

with the constrains that  $\sum_{1 \leq i \leq s} \pi_0(i) = 1$  which results to  $\pi_0(\mathbf{p}_0 \mathbf{p}_1 \mathbf{p}_2 \mathbf{p}_3 - \mathbf{I}) = \mathbf{0}$  with the additional condition  $\boldsymbol{\pi}_0 \mathbf{I}_s = \mathbf{1}$ , with  $\mathbf{I}_s$  is being a  $(s + 1)th$ -dimension vector with ones.

Finally we define the following matrix

$$\mathbf{A} = \mathbf{p}_0 \mathbf{p}_1 \mathbf{p}_2 \mathbf{p}_3 - \mathbf{I}, \quad (4)$$

and the following system of linear equation

$$\mathbf{u} \mathbf{A}' = \mathbf{b}$$

where  $\mathbf{A}'$  is the matrix  $\mathbf{A}$  where the last column has been replaced by ones and  $\mathbf{b}$  is an 8-dimension zero vector with one in the last column,

$$\mathbf{b} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]$$

and the stationary probability  $\mathbf{u}^{CNHMC}$  can be calculated solving

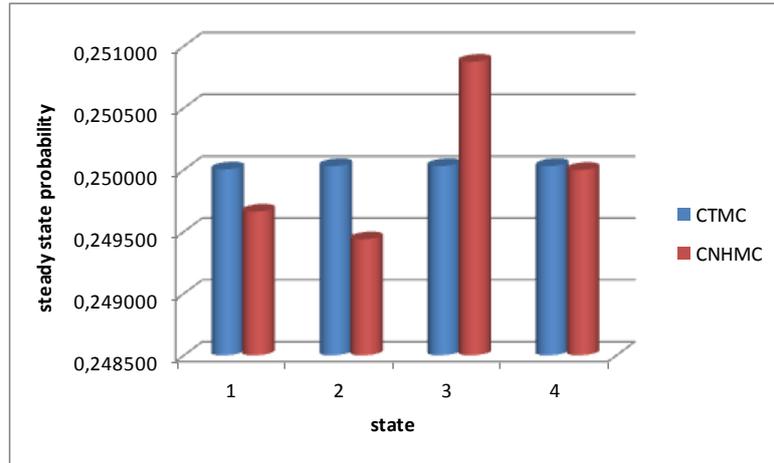
$$\mathbf{u} \mathbf{A}' = \mathbf{b} \quad (5)$$

#### 4. NUMERICAL ILLUSTRATION

The first season for which the HNMS provide us data is the spring of 2008. For this season the generation matrix is the following

$$\mathbf{Q}_{spg2008} = \begin{bmatrix} 0.001444 & 0.000926 & 0.000463 & 0.000000 & 0.000055 & 0.000000 & 0.000000 & 0.000000 \\ 0.001389 & -0.015333 & 0.012963 & 0.000926 & 0.000000 & 0.000055 & 0.000000 & 0.000000 \\ 0.000000 & 0.012500 & -0.059777 & 0.047222 & 0.000000 & 0.000000 & 0.000055 & 0.000000 \\ 0.000000 & 0.001852 & 0.046296 & 0.048203 & 0.000000 & 0.000000 & 0.000000 & 0.000055 \\ 2.000000 & 0.000000 & 0.000000 & 0.000000 & -2.001389 & 0.000926 & 0.000463 & 0.000000 \\ 0.000000 & 2.000000 & 0.000000 & 0.000000 & 0.001389 & -2.015278 & 0.012963 & 0.000926 \\ 0.000000 & 0.000000 & 2.000000 & 0.000000 & 0.000000 & 0.012500 & -2.059722 & 0.047222 \\ 0.000000 & 0.000000 & 0.000000 & 2.000000 & 0.000000 & 0.001852 & 0.046296 & -2.048148 \end{bmatrix}$$

**Fig. 4 Asymptotic probability distribution comparison**



and the transition probability according to (2) is

$$\mathbf{P}_{spg2008} = \begin{bmatrix} 0.999299 & 0.000450 & 0.000225 & 0.000000 & 0.000027 & 0.000000 & 0.000000 & 0.000000 \\ 0.000674 & 0.992556 & 0.006294 & 0.000450 & 0.000000 & 0.000027 & 0.000000 & 0.000000 \\ 0.000000 & 0.012500 & -0.059777 & 0.047222 & 0.000000 & 0.000000 & 0.000055 & 0.000000 \\ 0.000000 & 0.001852 & 0.046296 & 0.048203 & 0.000000 & 0.000000 & 0.000000 & 0.000055 \\ 2.000000 & 0.000000 & 0.000000 & 0.000000 & -2.001389 & 0.000926 & 0.000463 & 0.000000 \\ 0.000000 & 2.000000 & 0.000000 & 0.000000 & 0.001389 & -2.015278 & 0.012963 & 0.000926 \\ 0.000000 & 0.000000 & 2.000000 & 0.000000 & 0.000000 & 0.012500 & -2.059722 & 0.047222 \\ 0.000000 & 0.000000 & 0.000000 & 2.000000 & 0.000000 & 0.001852 & 0.046296 & -2.048148 \end{bmatrix}$$

By the time we have the transition probability of the four season of the year,  $\mathbf{P}_{spg2008}$ ,  $\mathbf{P}_{sum2008}$ ,  $\mathbf{P}_{aut2008}$ ,  $\mathbf{P}_{win2008}$  we can calculate matrix  $\mathbf{A}$  from (3):

$$\mathbf{A} = \begin{bmatrix} 0.005874 & 0.005144 & 0.000686 & 0.000017 & 0.000027 & 0.000000 & 0.000000 & 0.000000 \\ 0.004862 & -0.044692 & 0.038090 & 0.001713 & 0.000000 & 0.000026 & 0.000001 & 0.000000 \\ 0.000938 & 0.035896 & -0.099709 & 0.062848 & 0.000000 & 0.000001 & 0.000025 & 0.000002 \\ 0.000046 & 0.003407 & 0.061340 & -0.064821 & 0.000000 & 0.000000 & 0.000002 & 0.000026 \\ 0.994126 & 0.005143 & 0.000686 & 0.000016 & 0.999972 & 0.000000 & 0.000000 & 0.000000 \\ 0.004862 & 0.955308 & 0.038089 & 0.001713 & 0.000000 & -0.999974 & 0.000001 & 0.000000 \\ 0.000938 & 0.035895 & 0.900291 & 0.062848 & 0.000000 & 0.000001 & 0.999975 & 0.000000 \\ 0.000046 & 0.003406 & 0.061340 & 0.935179 & 0.000000 & 0.000000 & 0.000002 & -0.999974 \end{bmatrix}$$

Now, we can calculate the stationary probability for the year 2008 using (5)

$$\mathbf{u}_{2008}^{CNHMC} = [0.249662 \quad 0.249438 \quad 0.250873 \quad 0.249999 \quad 0.000007 \quad 0.000007 \quad 0.000007 \quad 0.000007]$$

We use the same methodology also for the years from 2009 to 2012, and we obtain the stationary probability for each year. Our next step is to calculate the average stationary probability for that five years, which is the following

$$\mathbf{u}^{CNHMC} = [0.249890 \quad 0.249908 \quad 0.250188 \quad 0.249987 \quad 0.000007 \quad 0.000007 \quad 0.000007 \quad 0.000007]$$

In contrast, when using a CTMC instead of CNHMC, considering the same data set, firstly we do not assume any periodicity for the transition rates. Instead, the transition rates are calculated as the number

of the transitions between the four states of wind through the period of time from January 2008 to May 2013. Hence the stationary probability distribution for the wind park system can be derive by solving the system of linear equations:

$$\mathbf{u}^{CTMC} \mathbf{Q} = \mathbf{0}, \sum_{i=0}^{i=n} u^{CTMC}(i) = 1$$

where  $\mathbf{u}^{CTMC}$  is the steady state probability vector for the continuous time the Markov chain and the  $(8 \times 8)$  generation matrix is ([4]):

$$\mathbf{Q} = \begin{bmatrix} y_1 & \lambda_{ab} & \lambda_{ac} & \lambda_{ad} & n\lambda & 0 & 0 & 0 \\ \lambda_{ba} & y_2 & \lambda_{bc} & \lambda_{bd} & 0 & n\lambda & 0 & 0 \\ \lambda_{ca} & \lambda_{cb} & y_3 & \lambda_{cd} & 0 & 0 & n\lambda & 0 \\ \lambda_{da} & \lambda_{db} & \lambda_{dc} & y_4 & 0 & 0 & 0 & n\lambda \\ \mu & 0 & 0 & 0 & y_5 & \lambda_{ab} & \lambda_{ac} & \lambda_{ad} \\ 0 & \mu & 0 & 0 & \lambda_{ba} & y_6 & \lambda_{bc} & \lambda_{bd} \\ 0 & 0 & \mu & 0 & \lambda_{ca} & \lambda_{cb} & y_7 & \lambda_{cd} \\ 0 & 0 & 0 & \mu & \lambda_{da} & \lambda_{db} & \lambda_{dc} & y_8 \end{bmatrix}$$

The steady state probability distribution can be then computed:

$$\mathbf{u}^{CTMC} = [0.250003 \quad 0.250003 \quad 0.250003 \quad 0.250003 \quad 0.000007 \quad 0.000007 \quad 0.000007 \quad 0.000007]$$

Apart from the limiting probabilities of the last for states, it is clearly shown in Fig. 4, that there is a difference between the state probabilities of the first four states in the case of the CNHM model. This is due to the fact that CNHMC despite the complexity can give us more accurate results because the model is not generalized but it is analyzed in periods of time and finally we have a total result for the time of interest, while in CTMC the model is not analyzed and we have only a general point of view for the overall time of interest.

## 5. CONCLUSION

In this paper, we calculated the steady state probabilities of a wind park using two different types of probabilistic methods, CTMC and CNHMC. Our main purpose was to compare these two approaches and find out which of the two is the more appropriate and accurate to model our problem.

We received weather data from the HNMS for a particular place in the island of Chios, we created four categories of wind according to the intensity, A was the no wind state, B was the intensity of wind able to give us 30% electricity production, C was the state which was able to give 70% production and finally, D was the state in which we have full production of electricity. We studied the reliability of the system using Markov Chains, pointing out and analyzing the use of CNHMC, which is proven to be a more accurate approach for our case in comparison with the classical CTMC model. The accuracy of the results when considering multiple wind states lies on the fact that the CNHMC approach can capture the dependencies of the transition rates on time. Despite the fact that modeling with CNHMCs is more complicated it is worthwhile to apply the corresponding analysis in order to achieve the limiting distribution for all the seasons for each year and additionally the overall limiting distribution for the period of interests.

At last, assessments of reliability can also be studied using much more simple probabilistic methods, but the use of CNHMC is more accurate method able to model systems like the system we study or even more complicated.

Although we studied only the variation of the transition rates among the wind intensity states by assuming constant failure and repair rates, we intend to extend our study by taking into account time dependent failure and repair rates. In authors' opinion, by this we will manage to provide a more detailed model for the operational scenario of a wind park.

## References

- [1]. E. M. Gouveia, M. A. Matos. "*Evaluating operational risk in a power system with a large amount of wind power*", Electric Power Systems Research, 79, pp. 734-739, (2009)
- [2]. A.N. Platis, N.E. Limnios, M.L. Du. "*Asymptotic Availability of Systems Modelled by Cyclic Non-Homogenous Markov Chains*", Annual Reliability and MAINTAINABILITY Symposium (1997)
- [3]. V.P. Koutras, A.N. Platis, G.A. Gravvanis. "Optimal server resource reservation policies for priority classes of users under cyclic non-homogenous markov modelling", European Journal of Operational Research, 198, pp. 545-556, (2009).
- [4]. V.P. Koutras, A.N. Platis, G.A. Gravvanis. "On the optimization of free resources using non-homogenous Markov chain software rejuvenation model", Reliability Engineering and system safety, 198, pp. 545-556, (2009).
- [5]. A. Papoulis, S. U. Pillai, "Probability, Random variables and Stochastic Processes" McGraw-Hill, 2002