

# USING SUBSET SIMULATION TO QUANTIFY STAKEHOLDER CONTRIBUTION TO RUNWAY OVERRUN

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**Abstract:** This paper studies the use of sensitivities to quantify the extent to which individual airline departments (stakeholders) contribute to the incident of runway overrun. For that purpose, we present a model of the incident runway overrun. The incident model is based on the dynamics of aircraft and describes the functional relationship between contributing factors leading to the incident. The incident model also takes operational dependencies into account. Model input are the probability distributions of the contributing factors, which are obtained by fitting distributions to data of a fictive airline. By propagating the probability distributions through our incident model, we are able to make statistical valid statements of the occurrence probability of the incident itself. Therefore, we use the subset simulation method. By estimating the design point using the samples of the subset simulation we obtain the sensitivities by applying the First-Order Reliability method. The sensitivities are used to quantify stakeholder contribution to the incident runway overrun by allocating the various stakeholders to the contributing factors.

**Keywords:** Runway Excursion, Incident Model, Subset Simulation, Sensitivity, First-Order Reliability Method

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## 1. INTRODUCTION

Certain incidents (e.g. runway excursion) occur rarely if at all for a single airline. Yet, the probability of such an incident is also not equal to zero, making it difficult to calculate the incident probability based purely on historical rates. If the airline increases its sample size with data from worldwide statistics (e.g. annual safety reports), a second problem arises, namely when data is collected across multiple airlines, it is currently impossible to correct for the effects of airline-specific safety cultures, flight procedures, types of aircraft, routes, training, etc.

Our hypothesis is as follows: even if the incident itself cannot be observed within the flight operation of the single airline, it is possible to measure the contributing factors leading to the incident. By modeling the contributing factors with probability distributions and knowing the functional relationship between them, we are able to calculate the probability that the actual incident will occur. We use the subset simulation method to compute the incident probability.

However, the individual contribution of each factor to the incident has not yet been quantified. In addition, it is desirable for safety management systems to know the contribution of stakeholders (e.g. departments of an airline such as training, or flight operation) to the safety level of an airline. To overcome this fact, this paper studies the use of sensitivities obtained from the subset simulation method to quantify such contributions. By tagging the contributing factors with the stakeholders, we are also able to assess the contribution of stakeholders to the risk of experiencing an incident that is analyzed.

The remainder of this paper is as follows: First, we describe in Sec. 2 the incident model including the physical and operational dependencies. Furthermore, we will describe the modeling process of the contributing factors that are the model input. In Sec. 3, we will present the method of subset simulation. Section 4 describes the estimation of the sensitivities and Sec. 5 concludes the paper.

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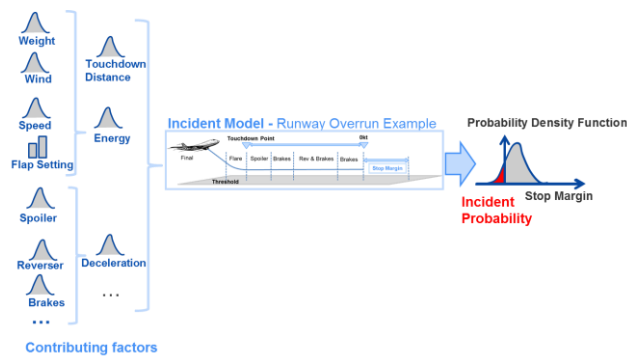
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## 2. RUNWAY OVERRUN EXAMPLE

### 2.1. General concept

Runway excursion is one of the most frequently occurring incidents worldwide [1]. Therefore, many studies have focused on determining the typical contributing factors leading to runway overruns and analyzing the dependencies of the contributing factors [2, 3]. Examples of typical contributing factors are: high-speed deviations from the target approach speed, high tailwind component, landings on a short runway, long landings (touching down late), or a wet runway. All of these contributing factors can be measured based on the operational flight data of an airline. The physical relationship between these factors is known and equal for each airline. The statistical variance of each contributing factor, however, heavily depends on the flight operation of an airline. Figure 1 shows some contributing factors of the incident type runway overrun.

Figure 1: General Concept



As indicated on the left of Fig. 1, many of the contributing factors vary during flight operation (e.g. wind) or even exceed limits imposed by the airline. We input the measured distributions into the incident model that contains the functional relationships between the contributing factors. This allows us to quantify an estimate for the incident probability, even if an airline has not observed such an incident in their flight operation.

First, we present an incident model of the runway overrun that is based on the dynamics of an aircraft (physical approach) and also includes operational dependencies. Then, we identify for each contributing factor a corresponding probability distribution that describes the contributing factor based on flight operational data. For that purpose, we present measures, which allow us to determine the goodness of fit. Third, we input the distributions of the contributing factors to the incident model in order to quantify the occurrence probability of an overrun. However, when using classical Monte-Carlo techniques, a large sample size is required for small probabilities since the sample size is inversely proportional to the failure probability that is to be obtained. To overcome this fact, we use the subset simulation method [4].

### 2.2. Runway Overrun Model

In this section, we present the runway-overrun model. The first step is to define a metric that describes the closeness of a single flight to ending in a specific incident. We call such metrics incident metrics. Put it another way, the incident metric describes a safety margin of a single flight with respect to a certain incident. If that incident metric is less than zero, an incident occurred. Examples for such metrics could be the tail clearance in case of the incident tailstrike, or the vertical speed prior to touchdown with respect to hard-landing. For the runway overrun, we use the stop margin  $SM$  that is defined as follows:

$$SM = LDA - ALD \quad (1)$$

Here,  $LDA$  is the Landing Distance Available (runway length) and  $ALD$  refers to the Actual Landing Distance. Our incident model will compute the ALD. An overrun occurs if the SM is negative (Eq. 2):

$$Flight\ Operations \begin{cases} No\ Incident & if\ SM > 0 \\ Incident & if\ SM \leq 0 \end{cases} \quad (2)$$

In order to calculate the safety margin, we use the aforementioned incident model  $f_{Incident}$

$$SM = f_{Incident}(CF) \quad (3)$$

The function  $f_{Incident}$  includes the relevant functional relationships between the contributing factors  $CF$  such as flight dynamics (physics), flight procedures, and aircraft systems. The output is the incident metric based on the input samples of the  $CF$ , here the stop margin.

The following Fig. 2 illustrates the concept. The red and dotted incident area equals the probability of the incident, which is unknown and has to be quantified. The aircraft symbols equal samples. If a sample is not within the incident area, no incident occurred. The distance between a sample and the incident area represents the incident metric (safety margin). The greater the distance between the incident area and the sample, the greater is the value of the incident metric. The aim of this paper is to quantify the size of the incident area that equals the incident occurrence probability by applying the subset simulation method and use its samples to estimate sensitivities of the each contributing factor.

**Figure 2: Estimating Incident Probabilities**



The deceleration of an aircraft can be expressed as follows by using Newton's second law of motion [5],

$$\dot{V} = \frac{1}{m} [T - D - mg \cdot \sin \gamma - \mu_F (mg \cdot \cos \gamma - L)] \quad (4)$$

where  $\dot{V}$  equals the deceleration of the aircraft,  $m$  expresses the aircraft mass,  $g$  is the gravitation constant,  $\gamma$  equals the flight path (here it is assumed to be equal to the runway slope). The forces within this Eq. (4) are the propulsion (thrust) forces  $T$ , the aerodynamic drag  $D$  and the aerodynamic lift  $L$ . The friction coefficient during braking is a function of the runway condition and brake application (Eq. 5).

$$\mu = f(\text{runway condition, brake pressure}) \quad (5)$$

We use the following expressions to model the lift and drag:

$$L = \bar{q} \cdot S \cdot C_L = \frac{\rho}{2} V_A^2 \cdot S \cdot C_L = \frac{\rho}{2} (V_{GS} - V_W)^2 \cdot S \cdot C_L \quad (6)$$

$$D = \bar{q} \cdot S \cdot C_D = \frac{\rho}{2} V_A^2 \cdot S \cdot C_D = \frac{\rho}{2} (V_{GS} - V_W)^2 \cdot S \cdot C_D \quad (7)$$

In Eq. (6-7),  $\bar{q}$  represents the dynamic pressure,  $S$  the reference area of the aircraft,  $C_L$  the aerodynamic lift coefficient and  $C_D$  the corresponding aerodynamic drag coefficient. The dynamic pressure can be expressed by using the air density  $\rho$  and the aerodynamic speed  $V_A$ .

$$\bar{q} = \frac{\rho}{2} V_A^2 \quad (8)$$

Here, we express the aerodynamic speed using the ground speed  $V_{GS}$  as well as the wind speed  $V_W$ .

$$V_A = V_{GS} - V_W \quad (9)$$

This overrun model also incorporates operational dependencies. The touchdown behaviour of pilots heavily depends on the runway length. Specifically, it depends on the difference between LDA and the Required Landing Distance (RLD). The RLD can be obtained from aircraft manuals. The expected value of the touchdown distance  $\mu_{Touchdown}$ , i.e. the distance from the runway threshold until the aircraft touches the runway, can be obtained in feet by the following relationship [6]:

$$\mu_{Touchdown} = \begin{cases} 12.5\delta + 1300 & \text{if } \delta < 55 \\ 2000 & \text{if } \delta \geq 55 \end{cases} \quad (10)$$

Here the buffer is computed as follows:

$$\delta = \frac{LDA - LDR}{LDA} \quad (11)$$

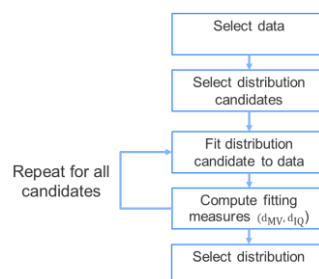
Eq. 10 shows that the smaller the buffer, i.e. the more critical a landing in terms of a runway overrun, the smaller the touchdown distance. However, as shown in [6], the standard deviation of the touchdown distance remains the same. If the buffer is greater than 55, the mean value for the touchdown distances does not change anymore.

### 2.3. Contributing Factors

In order to estimate the occurrence probabilities of incidents, we have to describe the statistics (distributions) of their contributing factors. For that purpose, we evaluate the operational data that can be obtained from an airline, e.g. by one's flight data monitoring (FDM) system. We distinguish between continuous and discrete contributing factors. Examples for discrete contributing factors are: the flap configuration for landing, or the runway condition (e.g. dry or wet). In contrast, examples for continuous contributing factors are: landing mass, or touchdown point. For the continuous ones, we fit continuous probability distributions to the data. Hence, we have a probabilistic model of each contributing factor.

As shown in the Fig. 1, many contributing factors vary during flight operation or even exceed airline-specific limits. Examples for such violations are tailwind components, late touchdowns, etc. As mentioned above, the incidents are usually the result of combined "extreme" contributing factors, such as landing out of the touchdown zone, late start of braking, or low runway friction. We account for this fact, by taking these "extreme" deviations from the nominal values into account. During the fitting process, we also consider that the probability distribution does not only have to represent values around the data's mean value but also describe the data in their boundary areas (left and right tail). This means, that the occurrence of such extreme values of contributing factors, such as the before mentioned touchdown distances or low friction coefficients are not underestimated. Due to the large amount of data, the following figure presents our algorithm approach to identify the distribution type that fits to the data best (Fig. 3).

**Figure 3: Fitting Algorithm**



Depending on the domain  $\Omega$  of each contributing factor, a set of distribution candidates is selected. Hence, we make sure that our probabilistic model for each contributing factor is valid. For example, values of the friction coefficients can only be positive and are in the range of zero and one, i.e.  $\Omega = [0; \infty)$ . Then we fit each of the distribution candidates to the data of the current contributing factor and calculate its goodness of fit. For that purpose, we use two measures, as used in Ref. [7], the first measure is the Integrated Quadratic Distance (IQD) which is defined as follows,

$$d_{IQ}(F,G) = \int_{-\infty}^{\infty} (F(t)-G(t))^2 w(t) dt \quad (12)$$

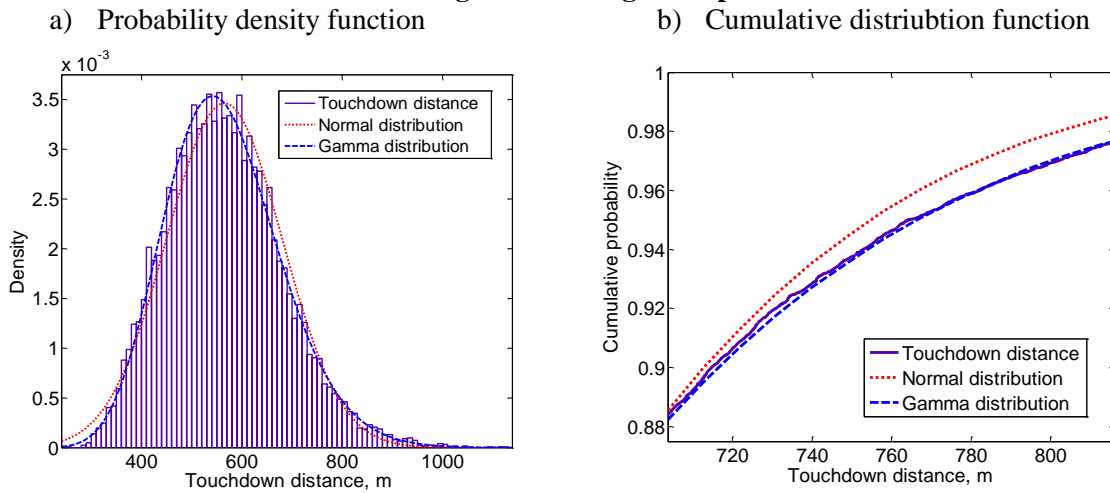
where  $G$  is the empirical distribution function (obtained from the data) and  $F$  the cumulative distribution function of the distribution candidate. In principle, a weighting function  $w(t)$  can be included that allows us to emphasize certain areas of interest, e.g. the tails of a distribution. The integration is adapted to the valid domain of the contributing factor. Recalling the friction coefficient example, the domain would be changed to  $\Omega = [0; \infty)$ . In order to counteract an over-fitting due to the weighting function, we also evaluate a second measure  $d_{MV}$  that is called the Mean Value Divergence.

$$d_{MV}(F,G) = \left[ \mu_F - \frac{1}{k} \sum_{i=1}^k y_i \right]^2 \quad (13)$$

Here,  $\mu_F$  is the mean value obtained by the distribution candidate  $F$ . This measure equals the difference between  $\mu_F$  and the empirical mean value based on the data. This ensures that also the first moment (i.e. mean value) of the distribution candidates fit closely to the empirical mean value.

The following figures (Fig. 4a, b) show an example of fitting various distribution types to the measured touchdown distances. The touchdown distance is defined as the distance between the runway threshold (begin of the runway) and the point at which the aircraft touches the ground. The fitting was performed without any weighting function. For this example, the Gamma distribution fits the data best.

**Figure 4: Fitting Example**



The following table shows the measures obtained for the five best fitting distributions of the touchdown distance that are shown in Fig. 4.

**Table 1: Fitting Measures**

| Rank | Distribution Type         | Integrated Quadratic Distance | Mean Value Divergence |
|------|---------------------------|-------------------------------|-----------------------|
| 1    | Gamma                     | 0.005                         | < 10e-4               |
| 2    | Generalized Extreme Value | 0.0176                        | 0.0202                |
| 3    | Nakagami                  | 0.0260                        | 0.0362                |
| 4    | Lognormal                 | 0.0392                        | 0.0142                |
| 5    | Birnbaum-Saunders         | 0.0397                        | < 10e-4               |

The fitting of the Normal (also known as Gaussian) distribution as shown in Fig. 4 was ranked as number ten, with an IQD of 0.1044. In addition, Figure 4b shows that large values of touchdown points (long-landings) are underestimated even if such long landings can be observed within the data.

### 3. SUBSET SIMULATION

#### 3.1. General Principle

As described in Sec. 2, we want to estimate the occurrence probabilities of incidents  $p_I$ , here of the runway overrun. As shown in Fig. 2, the probability  $p_I$  equals the size of the incident, and  $\pi(\theta)$  equals the probability density function of our incident metric SM. Hence, we can write

$$P(SM < 0) = E[p_I] = \iint_{\text{Flight Operation}} \mathbb{I}(\theta)\pi(\theta) d\theta = \iint_{\text{Incident}} \pi(\theta) d\theta \quad (14)$$

With the indicator function  $\mathbb{I}$

$$\mathbb{I}(\theta) = \begin{cases} 1 & \text{if incident} \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

In most of the cases, the probability density function of the incident metric is unknown and we have to estimate  $p_I$ . A possible and straightforward method to obtain an estimate  $\hat{p}_I$  for the probability is the Monte-Carlo approach, as follows.

$$p_I \approx \frac{1}{N} \sum_{i=1}^N \mathbb{I}(\theta_i) = \hat{p}_I \quad (16)$$

However, for rare events, with small occurrence probabilities, the number of required samples increases. In order to reduce the number of samples, we apply the subset simulation method [4]. The idea is to express the failure domain as a subset of multiple larger failure domains. If the intermediate failure domain is chosen properly, the intermediate probability  $p_j$  can be large (e.g.  $p_j = 0.1$ ).

$$\mathbb{R}^d = F_0 \supset F_1 \supset \dots \supset F_m = F \quad (17)$$

The probability of the system failure then is determined as the product of the conditional probabilities of each subset.

$$p_I = \prod_{j=1}^m P(F_j | F_{j-1}) = \prod_{j=1}^m p_j \quad (18)$$

This ensures that it is easier for samples to reach the incident domain. The first subset can be obtained by applying straightforward Monte-Carlo. A Markov Chain is used to generate the samples for each the following subset levels:

$$\begin{aligned} \text{Subset } j = 1 & \quad p_1 = P(F_1 | F_0) \text{ (Plain Monte - Carlo)} \\ \text{Subset } j \geq 2 & \quad p_j \text{ estimation according to } \pi(\cdot | F_{j-1}) \end{aligned} \quad (19)$$

The intermediate failure domains are defined adaptively [4]. For that purpose a fixed conditional probability  $p_j = p_0$  is defined (here:  $p_0$  equals 0.2). Based on the conditional probability the number of samples in the subset failure domain is determined as:

$$n_F = N \cdot p_0 \quad (20)$$

In other words, an incident (failure) would occur if the SM is not less than zero but less than a certain threshold value  $y_i$ . So if the stop margin of a sample is less than the intermediate threshold this sample lies in the intermediate failure domain.

$$\theta_{n+1} = \begin{cases} SM < y_i & \text{intermediate failure domain} \\ SM > y_i & \text{with the probability } 1 - a(\theta_n, y) \end{cases} \quad (21)$$

### 3.2. Metropolis Algorithm

In the original subset simulation method, the authors used the Metropolis Hastings algorithm [4] in order to create a Markov Chain. Due to the fact, that we have to consider multiple contributing factors with different domains and different distribution types, we perform the sampling in the standard normal space. This simplifies the sampling since we do not have to consider for multiple proposal distributions. In order to generate new samples, we can use the original Metropolis algorithm [8].

This means that we can use a symmetric proposal distribution that further simplifies the sampling. Given a sample  $\theta_n$  in the failure domain of subset  $j$ , we draw a sample candidate  $y$  according to  $q(y|\theta_i)$ . The next step is to compute the acceptance ration a:

$$a = (\theta_n|y) = \min \left\{ 1, \frac{\pi(y) q(\theta_n|y)}{\pi(\theta_n) q(y|\theta_n)} \right\} \quad (22)$$

Then we assign new sample  $\theta_{n+1}$  as follows

$$\theta_{n+1} = \begin{cases} y & \text{with the probability } a(\theta_n, y) \\ \theta_n & \text{with the probability } 1 - a(\theta_n, y) \end{cases} \quad (23)$$

Now, we have to evaluate if a new sample  $\theta_{n+1}$  lies in the failure of the current subset, otherwise we have to reject this new sample.

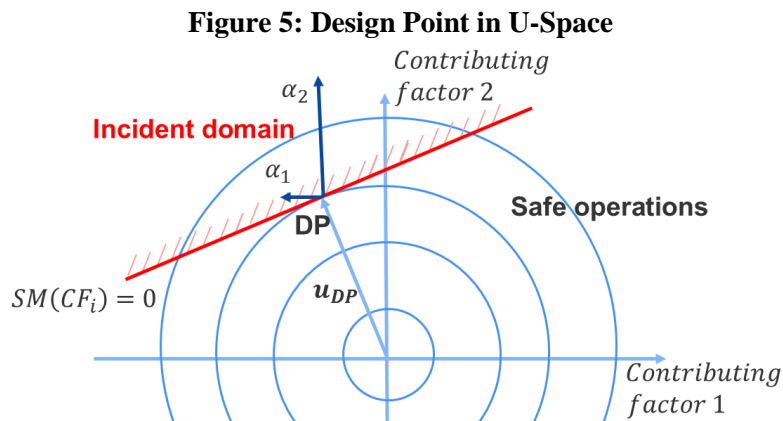
$$\theta_{n+1} = \begin{cases} \theta_n & \text{with the probability } a(\theta_n, y) \\ \theta_n & \text{with the probability } 1 - a(\theta_n, y) \end{cases} \quad (24)$$

We apply the steps from Eq. (22) until Eq. (24) until we have obtained N samples which are distributed according to  $\pi(\cdot | F_{j-1})$  and lie within the next failure domain. In this paper, we used the uniform distribution as the proposal distribution  $q$  with a spread of 1.4.

## 4. DESIGN POINT AND SENSITIVITIES

### 4.1. Method

The design point DP is the most likely point in the failure domain, i.e. the distance from the design point to the origin in the standard normal space is the shortest. The following Fig. 5 illustrates the concept.

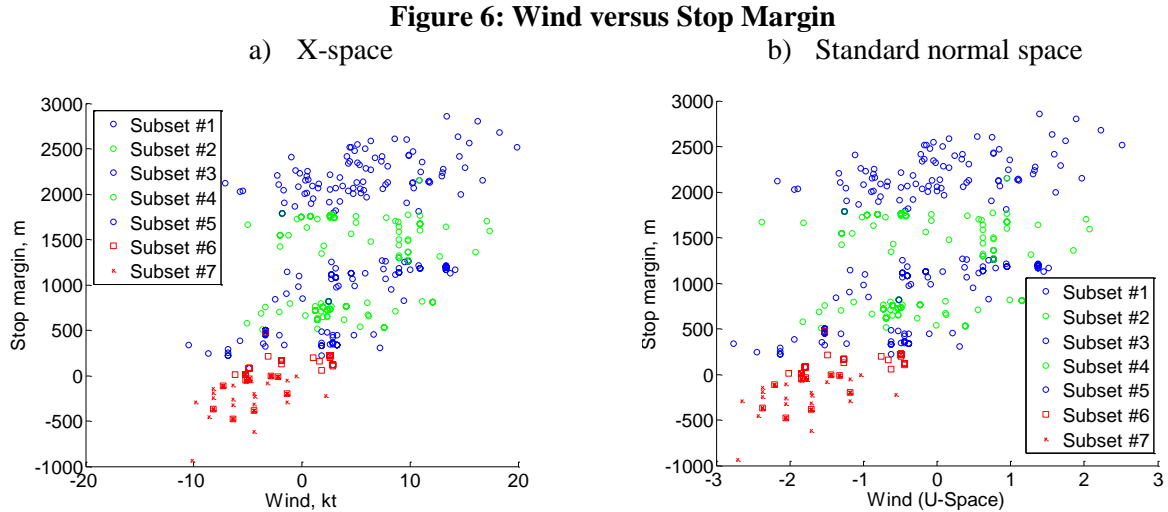


Here, the DP is the shortest distance of the failure domain to the origin of the U-space. For illustration, only two contributing factors are shown. The sensitivities are indicated by the component of the vector  $\mathbf{u}_{DP}$ .

In order to approximate the DP we use the samples obtained from the subset simulation. Therefore, we look for all samples that lie in the failure domain. Then we transform the samples into the standard normal space using the following transformation:

$$\mathbf{u} = U^{-1}(F(x)) \quad (25)$$

Here,  $\mathbf{u}$  represents the sample in the standard normal space ( $U$ -space),  $U$  is the cumulative distribution function (CDF) of the standard normal distribution, and  $F$  equals the CDF of the contributing factor with its value  $x$ . The following figure (Fig. 6) shows the samples of the wind component vs. the stop margin in the  $X$ -space as well as the corresponding samples in the  $U$ -space.



Each sample is a vector with  $n$  elements corresponding to  $n$  contributing factors. We use these samples and compute their distance to the origin of  $U$ -space by computing their norm.

$$\|\mathbf{u}\| = \sqrt{u_1^2 + \dots + u_n^2} \quad (26)$$

The sample with the smallest norm equals the design point  $\mathbf{u}_{DP}$ . It has the greatest joint probability of the contributing factors [9].

$$\mathbf{u}_{DP} = \operatorname{argmin}\|\mathbf{u}\| \quad (27)$$

By applying the First-Order Reliability method (FORM), we are able to estimate the sensitivities for each contributing factor. The sensitivities are obtained by normalizing the negative gradient vector:

$$\boldsymbol{\alpha} = -\frac{\nabla G(\mathbf{u}_{DP})}{\|\nabla G(\mathbf{u}_{DP})\|} \quad (28)$$

The relationship between the sensitivity vector  $\boldsymbol{\alpha}$ , the design point  $\mathbf{u}_{DP}$ , and the reliability index  $\beta$

$$\mathbf{u}_{DP} = \boldsymbol{\alpha} \cdot \beta \quad (29)$$

By solving Eq. (29) for the sensitivity vector  $\boldsymbol{\alpha}$ , we obtain the sensitivity factor  $\alpha_i$  for each contributing factor. The sensitivity factor takes values from -1 until 1. The greater the absolute value of the sensitivity factor of a contributing factor, the greater its influence on the incident probability. A sensitivity factor close to zero indicates almost no impact on the incident probability. If the sensitivity factor of a contributing factor is positive, the incident probability increases with higher values of the contributing factors and vice versa.

## 4.2. Application Example

Now, we tag the contributing factors with the following stakeholders (flight operation, ground operation, training, and environment) and/or postholder (human, maintenance, environment, and organization). This allows us to quantify the contribution of stakeholders to the incident runway overrun. For the runway-overrun example, we allocate the following stakeholders to the following contributing factors (Table 2):



**Table 2: Postholder Allocation**

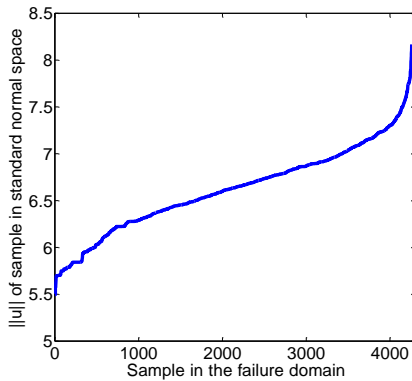
| Contributing Factor        | Postholder        |                |                         |
|----------------------------|-------------------|----------------|-------------------------|
|                            | Environment (ENV) | Training (TRA) | Flight Operation (FOPS) |
| Headwind                   | X                 | X              |                         |
| Landing weight             |                   |                | X                       |
| Air pressure (QNH)         | X                 |                |                         |
| Temperature                | X                 |                |                         |
| Approach speed deviation   | X                 | X              |                         |
| Time of spoiler deployment | X                 |                |                         |
| Start of braking           |                   | X              |                         |
| Reverser deployment        |                   | X              |                         |
| Touchdown distance         | X                 | X              |                         |

Due to the fact that we did not have any operational flight available, our analysis is based on artificial flight data. This means, that we generated samples of fictive flights performed to various airports by one aircraft type. So, the values for the sensitivities of the contributing factors are based on artificial distributions. The distribution type for each of the contributing factor is shown in the appendix A.1.

For this application example, we used 130000 samples out of which 4259 samples fell in the incident domain. The following Fig. 7 shows the norm of each sample that lies in the failure domain sorted by their length. The norm of the vector of the design point  $u_{DP}$  equals 5.49. However, multiple samples had the same length, and these samples were not concentrated at single point but spread (widely) within the domain. By using the sample with the lowest distance to the origin of the standard normal space some values for the sensitivity of some contributing factors were not reasonable from a physical point of view. Therefore, we included more samples. We used hundred samples with the lowest value of the norm.

Consequently, we obtain for each contributing factor one hundred values for its sensitivity. Figure 8 shows a histogram of sensitivities that are calculated by using the one hundred samples closest to the origin of the standard normal space. The values vary within a range of -0.366 to 0.0045.

**Figure 7: Norm of u-Vectors**



**Figure 8: Headwind Sensitivity**

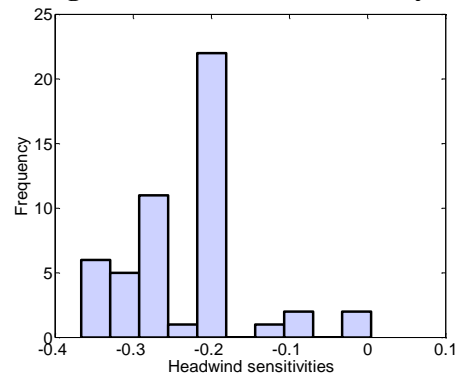


Table 3 shows the average sensitivity of each contributing factor.

**Table 3: Sensitivities**

| Contributing Factor        | Sensitivities |
|----------------------------|---------------|
| Headwind                   | -0.2531       |
| Landing weight             | 0.1710        |
| Air pressure (QNH)         | 0.0071        |
| Temperature                | 0.0852        |
| Approach speed deviation   | 0.1848        |
| Time of spoiler deployment | 0.0235        |
| Start of braking           | 0.2462        |
| Reverser deployment        | 0.0258        |
| Touchdown distance         | 0.8347        |

These sensitivities values are reasonable. For example, the headwind's sensitivity is negative that means smaller values for the headwind increase the likelihood of an overrun. This is plausible as a negative headwind equals tailwind that increases the kinetic energy of the aircraft at touchdown, which has to be absorbed. In contrast, the approach speed deviation has a positive sensitivity that means the greater the approach speed the greater the distance the aircraft requires to stop. Due to the fact, that the runway length in our analysis was quite low, the touchdown distance in this scenario is the main driver.

In order to calculate the individual sensitivity  $\alpha_j$  of a stakeholder  $j$ , we sum up the absolute values of each contributing factor that belongs to the individual contributing factor  $CF_j$ .

$$\alpha_j = \sum_i |\alpha_i(CF_j)| \quad (30)$$

The following table shows the result of the contribution of each stakeholder.

**Table 4: Sensitivities**

| Stakeholder $j$ | Sensitivity $\alpha_j$ | Contribution |
|-----------------|------------------------|--------------|
| TRA             | 1.7671                 | 44.2%        |
| ENV             | 1.5788                 | 49.5%        |
| FOPS            | 0.2257                 | 6.3%         |

In Table 4, the stakeholder contribution  $SC_j$  to runway overrun is computed as

$$SC_j = \frac{\alpha_j}{\sum_i \alpha_{i,j}} \quad (31)$$

The stakeholder ENV contributes most to the incident type runway overrun, based on the contributing factors being analysed.

## 5. SUMMARY AND CONCLUSION

This paper studies the quantification of stakeholder contribution to runway overrun. Therefore, a model tailored to the incident type runway overrun is presented. Then, based on data from a fictive airline, we fitted probability distributions that represent the contributing factors. The occurrence probability is estimated by using the subset simulation method. We used the samples that we obtained through subset simulation method to identify the design point.

By applying FORM, we derive estimates for the sensitivities for each individual contributing factors that include the functional dependencies as well as the deviations based on their distribution. By tagging each contributing factor with its stakeholders, we were able to quantify the contribution of the various stakeholders to the incident runway overrun. However, all the obtained sensitivities are based on artificial flight operation data. This means, that dependencies within the data, which might exist within an airline are not captured at all. Furthermore, the distributions of each contributing factor might not be close to reality. Nevertheless, this paper shows that, if such information would be available, the stakeholder's contribution can be quantified.

However, we also found out, that using only the sample with the lowest distance to the origin of the standard normal space does not necessarily provide sensitivities that are reasonable, especially from a physical point of view. Therefore, we used multiple samples that are close to the origin of the standard normal space and calculated sensitivities using an average. The results are much more reasonable and are in line with the physical and probabilistic knowledge of the contributing factors and their functional relationship. Furthermore, it also becomes clear, that the stakeholder contribution heavily relies on a proper allocation of the stakeholder to the contributing factors and all relevant factors have to be taken into account.

Consequently, the next step is to apply this approach to data obtained from a real flight operation of an airline. Then, the allocation of stakeholder might change as well as additional stakeholder needs to be included. In addition, other contributing factors, such as flap setting or runway condition have to be taken into account.

## Appendix

### A.1

**Table A.1 Distribution of the Contributing Factors**

| Contributing Factor        | Distribution Type         | Distribution Parameter                         |
|----------------------------|---------------------------|--|
| Headwind                   | Generalized Extreme Value | $k = 0.0243, \sigma = 3.4585, \mu = -5.4383$   |
| Landing weight             | Generalized Extreme Value | $k = -0.6716, \sigma = 4.7701, \mu = 67.4481$  |
| Air pressure (QNH)         | Burr                      | $\alpha = 1025.38, c = 235.38, k = 4.38$       |
| Temperature                | Weibull                   | $a = 21.6713, b = 3.7698$                      |
| Approach speed deviation   | t Location-Scale          | $\mu = 0.0207, \sigma = 3.1942, \nu = 36.6177$ |
| Time of spoiler deployment | Generalized Extreme Value | $k = -0.0633, \sigma = 0.4288, \mu = 0.5848$   |
| Start of braking           | Generalized Extreme Value | $k = -0.0007, \sigma = 1.0430, \mu = 1.9173$   |
| Reverser deployment        | Generalized Extreme Value | $k = -0.0123, \sigma = 0.8448, \mu = 3.6858$   |

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