Study on Operator Reliability of Digital Control System in Nuclear Power Plants Based on Boolean Network

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Abstract: The current human reliability analysis method of analyzing system operator's reliability, carried out from the perspective of operators themselves, is relatively static, for it hasn't taken the effect of system evolution on the operators' performance into consideration. In view of operator reliability in digital control system in nuclear power plant, this paper, based on boolean network theory, tries to explore the operators' behavior in the dynamic logic process of system evolution, aiming at finding out the dynamic evolution process of human-system interaction. A new technique, called the semi-tensor product of matrices, can convert the logical systems into standard discrete-time dynamic systems, and then the discrete-time linear equation and reliability analysis model are established. Data collected from simulation experiments carried out in full-size simulator in LingDong Nuclear Power Plant is found to be in consistence with the operator reliability model constructed before.

Keywords: Boolean network, Digital control system, Reliability analysis, Semi-tensor product of matrices.

1. INTRODUCTION

With the development of science and technology, the safety and efficiency of system and equipment have been improving, but the reliability of human-machine system has been depending on man. According to statistics, over 60% of fatal casualties and over 80% of serious casualties at home and abroad are due to human errors [1,2]. The serious consequences caused by operators have been fully realized after the accidents at the Chernobyl nuclear power plant and the America Three Mile Island nuclear power plant accidents. Therefore, research on the relationship between operators and system, on qualitative analysis and quantitative assessment on human operations, has become increasingly important in the engineering field [3].

Ever since digital control system was adopted in nuclear power plant, operators have been enjoying the conveniences it has brought about, in the meantime, they have also been facing risks of operation reliability caused by enormous and centralized information. In the main control room of a digital control system, the central display of system alarming, parameters and pictures has formed a keyhole effect with enormous information and limited display [4,5], for the operators, since in a traditional control room, the operators can take in everything at a glance while in the main control room of a digital control system, they have to use a computer to carry out interface management tasks to find information promptly and efficiently. This shows that the adoption off digital technology has brought some new risks for operators, and whether the reliability of them can meet the safe and economic requirements has become one of the urgent problems a plant has to solve.

The method of Fault Tree Analysis and Event Tree Analysis are two of the common ways in probabilistic safety assessment. However, in case of accidents at a nuclear power plant, the response of the system or the behavior of an operator changes with the process of the accident, so an operator's behavior at the next time node is closely related to the situation of the system and the operation at the previous time node. The traditional methods of Fault Tree Analysis and Event Tree Analysis are static analysis technology based on Boolean Logic [6], without taking the dynamic development between

man and system into full consideration, so it is of significance to probe into the dynamic relationship and applying it to reliability analysis both in theoretical research and industrial application.

On the basis of Boolean network, now a powerful tool in system control, Cheng Daizhan put forward a new method of matrix calculation--- semi-tensor product of matrix [7,8], with which logic variable can be expressed as vector form, and logic function as multiple linear mapping form. However, under algebraic expression, Boolean network equations, having all the information of Boolean network, are represented by general discrete-time linear equations. With the method, Boolean network equations can be established to analyze the operations in the digital control system at a nuclear power plant by determining the relationship between operations with data collected through analog experiment and video analysis. The 2nd section of this paper is about basic knowledge of semi-tensor product of matrix, some basic properties needed in derivation, matrix expression of logic and Boolean network model. The 3rd section is about the establishment of Boolean network of operations, introducing the obtainment of experiment data and the specific process of model construction. The 4th section is conclusion and discussion, analyzing on experiment results and discussing future work.

2. PRELIMINARIES

First, we give some notations for the statement ease.

1)
$$D_k := \{0, \frac{1}{k-1}, \dots, \frac{k-2}{k-1}\}, k \ge 2; D := D_2 = \{0, 1\}.$$

- 2) Let δ_n^i be the *i* th column of the identity matrix I_n .
- 3) $f: D^n \to D$ are logical functions.
- 4) $\Delta_n : \Delta_n = \{\delta_n^i | i = 1, 2, ..., n\}$, when n = 2, simply use $\Delta := \Delta_2$.
- 5) Denote by COL(A) the set of columns of A.
- 6) Assume a matrix $M = [\delta_n^{i_1}, \delta_n^{i_2}, ..., \delta_n^{i_s}] \in M_{n \times s}$, its columns, $COL(M) \subset \Delta_n$. We call M a logical matrix, and simply denote it as $M = \delta_n[i_1, i_2, ..., i_s]$.
- 7) \otimes is Kronecher product.

2.1. Definition and Proposition of the Semi-tensor Product of Matrices [7,8,9,10]

Definition (1): (i) Let $X = [x_1, ..., x_s]$ be a row vector, $Y = [y_1, ..., y_t]^T$ be a column vector. (1): If $s = t \times n$. Then

$$\langle X, Y \rangle_L \coloneqq \sum_{k=1}^t X^k y_k \in \mathbb{R}^n$$
 (1)

Where $X = [X^1, ..., X^t], X^i \in \mathbb{R}^n, i = 1, ..., t$. We call $\langle X, Y \rangle_L$ a semi-tensor product. (2): If $t = s \times n$. Then

$$\langle X, Y \rangle_L \coloneqq (\langle Y^T, X^T \rangle_L)^T \in \mathbb{R}^n$$
 (2)

 $\langle X, Y \rangle_L$ also called a semi-tensor product.

(ii) Assume $M \in M_{m \times n}$, $N \in M_{p \times q}$, if *n* is the divisor of *p* or *p* is the divisor of *n*. We call $C = M \ltimes N$ is the semi-tensor product of *M* and *N*.

If C is composed of $m \times q$ blocks, $C = (C^{ij})$, meanwhile

$$C^{ij} = \langle M^{i}, N_{j} \rangle_{L}, i = 1, ..., m, j = 1, ..., q.$$
 (3)

Where M^{i} is a row of M, N_{i} is a column of N.

Throughout this paper, the matrix product is assumed to be the semi-tensor product. In the following, the symbol \ltimes is omitted.

The semi-tensor product has the following properties.

Proposition (1): (i) If $A \in M_{m \times np}$, $B \in M_{p \times q}$. Then

$$A \ltimes B = A(B \otimes I_n). \tag{4}$$

(ii) If $A \in M_{m \times n}, B \in M_{np \times q}$. Then

$$A \ltimes B = (A \otimes I_n)B. \tag{5}$$

Proposition (2): Let $X \in \mathbb{R}^m, Y \in \mathbb{R}^n$ be two columns. Then

$$W_{[m,n]} \ltimes X \ltimes Y = Y \ltimes X; \tag{6}$$

$$W_{[n,m]} \ltimes Y \ltimes X = X \ltimes Y. \tag{7}$$

Where $W_{[m,n]}$ is an $mn \times mn$ matrix, called the swap matrix.

Proposition (3): Let $x \in \Delta$. Then

$$x^2 = M_r x \tag{8}$$

Where $M_r = \delta_4[1, 4]$ is called the power-reducing matrix.

2.2. Matrices Expression of Logic [7,8,9,10]

A logical variable means a proposition. When the proposition is true, we say that the logical variable takes value "T" or "1", and when it is false, the logical variable takes value "F" or "0". In classical logic a logical variable can only take values from $\{0,1\}$. We note

$$T := 1 \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad F := 0 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \tag{9}$$

Four fundamental operators usually be used are Conjunction $P \wedge Q$, Disjunction $P \vee Q$, Conditional $P \rightarrow Q$, and Biconditional $P \leftrightarrow Q$. A conventional way to depict the values of an operator is using a table, called the truth table. We can have truth table for "conjunction", "disjunction", "conditional", and "biconditional", respectively as in Table 1.

р	q	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$	$p \lor q$
1	1	1	1	1	1
1	0	0	0	0	1
0	1	0	1	0	1
0	0	0	1	1	0

 Table 1: Truth Table

Definition (2): Let σ be an r-ary logical operator. $M_{\sigma} \in M_{2 \times 2^r}$ is called the structure matrix of σ , if in the vector form we have

$$\sigma(p_1, \dots, p_r) = M_{\sigma} \ltimes p_1 \ltimes p_2 \ltimes \dots \ltimes p_r = M_{\sigma} p_1 \dots p_r$$
(10)

Theorem (1): Assume $f(x_1, ..., x_n)$ is a logical function, and in the vector form we have $f: \Delta_{2^n} \to \Delta$. Then there exists a unique logical matrix M_f , called the structure matrix of f, such that following equation(11) holds.

$$f(x_1, ..., x_n) = M_f x.$$
(11)

Where $x = \ltimes_{i=1}^{n} x_i$.

Using Theorem (1), the structure matrix of four fundamental operators are obtained as

$$\begin{split} \boldsymbol{M}_{\wedge} &\coloneqq \boldsymbol{M}_{c} = \boldsymbol{\delta}_{2} \left[1, 2, 2, 2 \right]; \\ \boldsymbol{M}_{\vee} &\coloneqq \boldsymbol{M}_{d} = \boldsymbol{\delta}_{2} \left[1, 1, 1, 2 \right]; \\ \boldsymbol{M}_{\rightarrow} &\coloneqq \boldsymbol{M}_{i} = \boldsymbol{\delta}_{2} \left[1, 2, 1, 1 \right]; \\ \boldsymbol{M}_{\leftrightarrow} &\coloneqq \boldsymbol{M}_{e} = \boldsymbol{\delta}_{2} \left[1, 2, 2, 1 \right]. \end{split}$$

2.3. Boolean Network

Boolean network was firstly introduced by Kaufman to formulate the cell networks. Then, it becomes a powerful tool in describing, analyzing, and simulating the cell networks, and also be used as models of some complex systems such as neural networks. A Boolean network is a directed network graph, consists of a set of nodes, and a set of edges.

Definition (3): A Boolean network is a set of nodes $x_1, x_2, ..., x_n$, which interact with each other in a synchronous manner. At each given time t = 0, 1, 2, ... a node has only one of two different values: 1 or 0. Thus the network can be described by a set of equations:

$$\begin{cases} x_{1}(t+1) = f_{1}(x_{1}(t),...,x_{n}(t)) \\ x_{2}(t+1) = f_{2}(x_{1}(t),...,x_{n}(t)) \\ \dots \\ x_{n}(t+1) = f_{n}(x_{1}(t),...,x_{n}(t)) \end{cases}$$
(12)

Where $f_i: D^n \to D, i = 1, ..., n$ are n-ary logical functions, $x_i(t) \in D$ are state variables.

3. MODEL CONSTRUCTION

In digital control system, an operator's work involves monitoring, situation assessment, response planning and response implementation [11]. Suppose $x_1(t), x_2(t), x_3(t), x_4(t) \cdot x_i(i = 1, 2, 3, 4) \in D$ represent the operations at "t" (a certain time), $x_1(t), x_2(t), x_3(t), x_4(t)$ represent monitoring, situation assessment, response planning and response implementation respectively, 1 indicates that an operator takes some action, while 0 no action. For instance, if at a certain time "t", $x_1(t), x_2(t), x_3(t), x_4(t)$ has the values (1,1,0,0), that means the operator take the actions of monitoring and situation assessment at this moment.

Thus, the operator's behavior at any moment can be expressed in a four-dimensional array, the evolution of the operator's behavior is equal to that of the array. With this abstract method, the operator's behavior at different time can be arranged using the method of moment, so as to analyze the dynamic process of the operator's behavior in the evolution of the system.

3.1. Experiment Data Source and Explanation

The experiment data of this paper are from the Steam Generator Tube Rupture experiment in Lingdong Nuclear Power Plant carried out on full-scale simulator. The reason why SGTR is chosen is that it is a typical problem of reliability related to operations after initial accident at a nuclear power plant, and they are crucial human's operations to be considered in PSA analysis [12]. 8 sets of data are obtained after observation and analysis:

1) t = 0, (0,0,1,0); t = 1, (0,0,0,1); t = 2, (1,0,0,0); t = 3, (0,1,0,0); t = 4, (1,0,0,1); t = 5, (1,1,0,0); t = 6, (1,1,1,1); 2) t = 0, (1,0,1,0); t = 1, (0,1,0,1); t = 2, (1,0,0,1); 3) t = 0, (0,1,1,1); t = 1, (1,0,0,1); t = 2, (1,1,1,1); 4) t = 0, (0,0,1,1); t = 1, (1,1,0,1); t = 2, (1,1,1,1); 5) t = 0, (1,1,1,0); t = 1, (1,1,0,1); 6) t = 0, (0,1,1,0); t = 1, (1,0,0,1); 7) t = 0, (1,0,1,1); t = 1, (1,1,0,1); 8) t = 0, (0,0,0,0); t = 1, (0,0,0,0);

As 17 experiment data are needed to determine a four-noded Boolean network model, 8 different time

nodes are chosen as starting observation points to avoid being special, and 24 data are obtained.

In the above 8 sets of data, all the time nodes are discrete, but the time nodes of each set of experiment data are successive. The interval is the time needed by an operator to change his operation from one moment to another. As there are additional accidents planned in the experiment, four operation models happen simultaneously at some time nodes. As for the 8th set of data, it indicates that when an operator takes no action, the situation will be better, and the same will happen at next time node.

3.2. Algebraic Form of Boolean Network

From equation (12), the key to build the dynamic relationship between these four variables is to determine the four logic function.

Define $x(t) = \ltimes_{i=1}^{4} x_i(t)$, from equation (11) and (12), we have

$$\begin{cases} x_{1}(t+1) = M_{1}x(t) \\ x_{2}(t+1) = M_{2}x(t) \\ x_{3}(t+1) = M_{3}x(t) \\ x_{4}(t+1) = M_{4}x(t) \end{cases}$$
(13)

Where $M_i \in M_{2 \times 2^r}$, called the structure matrix of f_i . Equation (13) is called the component-wise algebraic form of (12).

Then,

$$x(t+1) = \ltimes_{i=1}^{4} x_i(t+1) = M_1 x(t) M_2 x(t) M_3 x(t) M_4 x(t)$$
(14)

Refer to [7,8,10], (13) can further be converted as

$$c(t+1) = Lx(t) \tag{15}$$

Where $L \in L_{2^n \times 2^n}$ is called the transition matrix of the system. Equation (15) is called the algebraic form of (12).

Refer to [8], it was proved that (12),(13),(15) are equivalent to each other. While building model (12) directly seems much more difficult, we prefer to construct model (13) or (15) by calculating the structure matrix of M_i or the transition matrix L.

Next step, we use the experimental data collected in section 3.1 to construct model (13) and (15).

3.3. Dynamic Model Construction of An Operator's Performance

For the first experimental data, in the vector form [8,13], we have

$$(0,0,1,0) = X^{1}(0) = \delta_{2}[2,2,1,2], \text{ and } x^{1}(0) = \delta_{2}^{2} \ltimes \delta_{2}^{1} \ltimes \delta_{2}^{1} \ltimes \delta_{2}^{2} = \delta_{16}^{14}.$$

Similarly, we can calculate

 $x^{1}(1) = \delta_{16}^{15} \quad ; \quad x^{1}(2) = \delta_{16}^{8} \quad ; \quad x^{1}(3) = \delta_{16}^{12} \quad ; \quad x^{1}(4) = \delta_{16}^{7} \quad ; \quad x^{1}(5) = \delta_{16}^{4} \quad ; \quad x^{1}(6) = \delta_{16}^{1}.$

The following proposition can help us to determine the column of the transition matrix L.

Proposition (4)[13]: If
$$x(t) = \delta_{2^n}^i$$
 and $x(t+1) = \delta_{2^n}^j$, the *i* th column of the transition matrix *L* is
$$Col_i(L) = \delta_{2^n}^j$$
(16)

Using the Proposition, it is known that

$$Col_{14}(L) = \delta_{16}^{15}; Col_{15}(L) = \delta_{16}^{8}; Col_{8}(L) = \delta_{16}^{12};$$

$$Col_{12}(L) = \delta_{16}^{7}; Col_{7}(L) = \delta_{16}^{4}; Col_{4}(L) = \delta_{16}^{1}.$$

The 6 columns of L have been determined.

Using the same procedure to the other groups of data, certain values of column of L can be figured out.

Finally, we can obtain

$$L = \delta_{16}[1, 1, 1, 1, 3, 11, 4, 12, 3, 7, 7, 7, 3, 15, 8, 16]$$
(17)

Refer to [13], the corresponding retrievers are

$$\begin{split} S_1^4 &= \delta_2[1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2]\,;\\ S_2^4 &= \delta_2[1,1,1,1,2,2,2,2,2,1,1,1,1,2,2,2,2,2]\,;\\ S_3^4 &= \delta_2[1,1,2,2,1,1,2,2,1,1,2,2,1,1,2,2]\,;\\ S_4^4 &= \delta_2[1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2]\,. \end{split}$$

Then

$$\begin{split} M_1 &= S_1^4 L = \delta_2 [1,1,1,1,1,2,1,2,1,1,1,1,1,2,1,2]; \\ M_2 &= S_2^4 L = \delta_2 [1,1,1,1,1,1,1,1,2,2,2,1,2,2,2]; \\ M_3 &= S_3^4 L = \delta_2 [1,1,1,1,2,2,2,2,2,2,2,2,2,2,2,2,2]; \\ M_4 &= S_4^4 L = \delta_2 [1,1,1,1,1,2,2,1,1,1,1,1,2,2]. \end{split}$$

Consider the logical expression of $x_1(t)$

$$x_1(t+1) = M_1 x(t) = \delta_2 [1,1,1,1,1,2,1,2,1,1,1,1,1,2,1,2] x(t).$$

It is easy to verify that

$$\begin{split} M_1(M_n - I_2) &= 0; \\ M_1W_{[2,2]}(M_n - I_2) \neq 0; \\ M_1W_{[2,4]}(M_n - I_2) &= 0; \\ M_1W_{[2,8]}(M_n - I_2) \neq 0. \end{split}$$

So $x_1(t)$, $x_3(t)$ are fabricated variables in the dynamic equation of $x_1(t+1)$.

Setting $x_1(t) = x_3(t) = \delta_2^1$, then

$$\begin{aligned} x_1(t+1) &= M_1 x(t) = M_1 x_1(t) x_2(t) x_3(t) x_4(t) \\ &= M_1 x_1(t) W_{[2,2]} x_3(t) x_2(t) x_4(t) \\ &= M_1 (I_2 \otimes W_{[2,2]}) x_1(t) x_3(t) x_2(t) x_4(t) \\ &= M_1 (I_2 \otimes W_{[2,2]}) (\delta_2^1)^2 x_2(t) x_4(t) \\ &= \delta_2 [1, 1, 1, 2] x_2(t) x_4(t) . \end{aligned}$$

Hence its logical expression is

$$x_1(t+1) = (\neg x_2(t)) \to x_4(t)$$
.

The same procedure can be used to construct the logical expression of $x_2(t)$, $x_3(t)$, $x_4(t)$. Finally, the logical expression of the dynamics of the operators' performance is obtained as

$$\begin{cases} x_{1}(t+1) = (\neg x_{2}(t)) \rightarrow x_{4}(t) \\ x_{2}(t+1) = x_{1}(t) \lor [x_{3}(t) \land x_{4}(t)] \\ x_{3}(t+1) = x_{1}(t) \land x_{2}(t) \\ x_{4}(t+1) = x_{2}(t) \lor x_{3}(t) \end{cases}$$
(18)

Its network graph depicted in Fig.1



The above Boolean Network equations are the constructed dynamic model of an operator. It can be seen from equation (18) that if monitoring and situation assessment are carried out at a time node, response planning will be carried out at the next; that if situation assessment or response planning is carried out at a time node, a specific operation will be done at the next; that if monitoring and /or response planning and specific operation are carried out at one time node, situation assessment on the system will be carried out at the next; and that if situation assessment or specific operation is carried out at a time node, monitoring will be carried out at the next.

4. CONCLUSION AND DISCUSSION

In the past, analysis on human's operation was carried out through static analysis technology on the basis of Boolean algebra to work out human's error probability [3], focusing on analysis from an operator's cognitive behavior, failing to show how his operation changed with the situation of the system, seldom considering the dynamic relationship between operations at different time nodes. This paper, define the operator's behavior in the system process as a state variables, using an array of expression, the evolution of operator's behavior with process of incident could be abstract, then the logical process could be changed into algebraic expression and construct the logical equation of operations. This method has shed new light on analyzing the reliability of operations at a nuclear power plant.

For example, a set of data as $(1,0,0,0) \rightarrow (0,1,0,0) \rightarrow (0,0,1,0) \rightarrow (0,0,0,1)$ can be deduced from model (18), which shows that monitoring, situation assessment, response planning and response implementation, which can be overlapping, have to be carried out repeatedly at a nuclear power plant under the State Oriented Procedure(SOP).

Though the analysis method put forward in this paper is of significance in making up the inadequacy of the traditional method of static analysis on human reliability on the basis of Boolean Logic, the Boolean network models are special since they are deduced according to data obtained from one analog experiment. If different analog experiments can be carried out over and over again with this method to analyze the relationship between operations in different experiments, the general dynamic relationship between operations may be deduced in a digital control system.

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