NA226

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Uncertainty Importance Evaluation in Probabilistic Risk Assessment Using Wasserstein Distance with Bias–Variance Decomposition

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Uncertainty in Probabilistic Risk Assessment (PRA)

- **Uncertainty**
  - **Aleatory uncertainty**

    Derived from randomness of the nature of phenomena

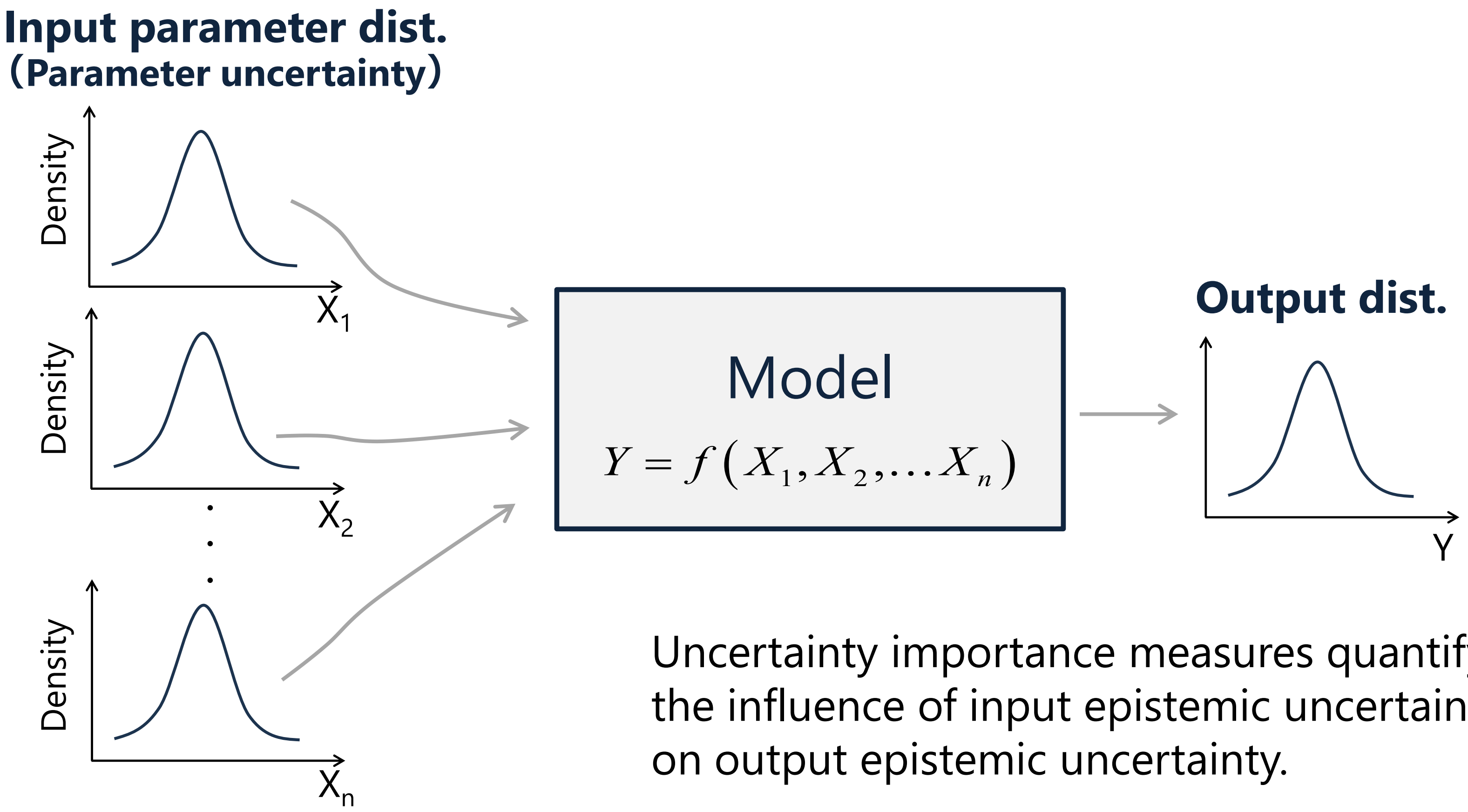
    ⇒ **Irreducible**
  - **Epistemic uncertainty**

    Derived from lack of knowledge

    ⇒ **Reducible**
    - **Parameter**
    - **Model**
    - **Completeness**
      - Known unknowns
      - Unknown unknowns

(Adapted from USNRC, NUREG-1855 Rev. 1 (2017).)

Uncertainty importance evaluation in PRA



Uncertainty importance measures quantify the influence of input epistemic uncertainties on output epistemic uncertainty.

Limitations of existing uncertainty importance measures*

*T. Aven and T. E. Nøkland, RESS, 95, 127–133 (2010).

Non-parametric (Ex. Pearson correlation coefficient)	• Based on correlation btw input and output • Model-dependent (assumption of linearity) • Moment-dependent
Variance-based (Ex. Iman measure)	• Based on change in output variance when eliminated input uncertainty • Moment-dependent
Moment-independent (Ex. Borgonovo index)	• Based on whole output distribution • Directly evaluate the influence of input uncertainty over output uncertainty

None of the existing measures can readily distinguish between changes in the location and shape of the output distribution induced by input uncertainty.

Objective

- Propose an uncertainty importance evaluation method based on the **Wasserstein distance*, an optimal transport distance** btw probability distributions.
  - **The bias–variance decomposition** of the squared Wasserstein distance to separately evaluate distributional differences in terms of bias and variance components

"variance" is defined here in a broad sense as the non-bias component of the distributional difference, arising mainly from discrepancies in distributional shape.

 *C. Villani, Optimal transport old and new, Grundlehren Math. Wiss., 338, Springer (2009).

Wasserstein distance

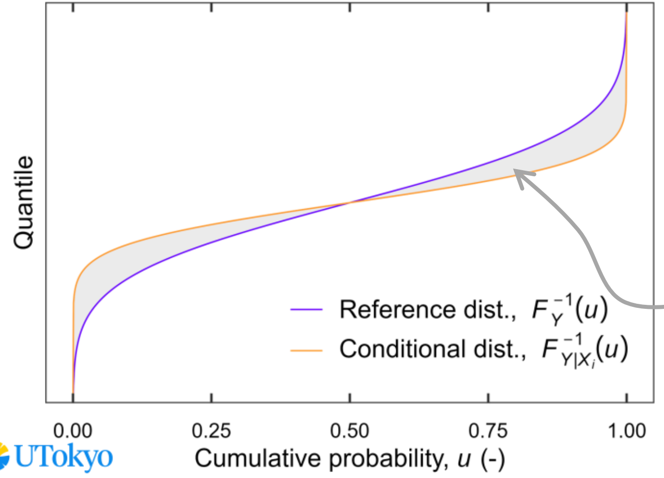
- The minimum cost required to transport the probability mass of the output distribution $f_Y(y)$ to another probability distribution $f_{Y|X_i}(y)$

Wasserstein Distance of *p* order（one dimension）

$$W_p\left(f_Y(y),\, f_{Y|X_i}(y)\right) = \left(\int_0^1 \left| F_Y^{-1}(u) - F_{Y|X_i}^{-1}(u) \right|^p du\right)^{\frac{1}{p}}$$

Quantile function of the output Y

Quantile function of the conditional dist. of output Y given input Xi



In one dimension, the shaded area corresponds to the Wasserstein distance.

Bias-variance decomposition of Wasserstein distance

Bias–variance decomposition of the squared Wasserstein distance

$$W_2^2\left(f_Y(y),\, f_{Y|X_i}(y)\right) = \int_0^1 \left\{F_Y^{-1}(u) - F_{Y|X_i}^{-1}(u)\right\}^2 du$$

$$= \underbrace{\left(\mu_Y - \mu_{Y|X_i}\right)^2}_{\text{Bias}} + \underbrace{\int_0^1 \left\{\delta_Y(u) - \delta_{Y|X_i}(u)\right\}^2 du}_{\text{Variance}}$$

Bias
(Mean difference)

Variance
(Difference in centered quantile functions)

Uncertainty importance measures based on Wasserstein distance

Wasserstein importance

$$W_i := \mathrm{E}_{X_i}\left[W_2^2\left(f_Y(y),\, f_{Y|X_i}(y)\right)\right]$$

Wasserstein–Bias importance

$$W_i^B := \mathrm{E}_{X_i}\left[\left(\mu_Y - \mu_{Y|X_i}\right)^2\right]$$

Wasserstein–Variance importance

$$W_i^V := \mathrm{E}_{X_i}\left[\int_0^1 \left\{\delta_Y(u) - \delta_{Y|X_i}(u)\right\}^2 du\right]$$

Sensitivity study using prescribed probability distributions

- Benchmark

Borgonovo index

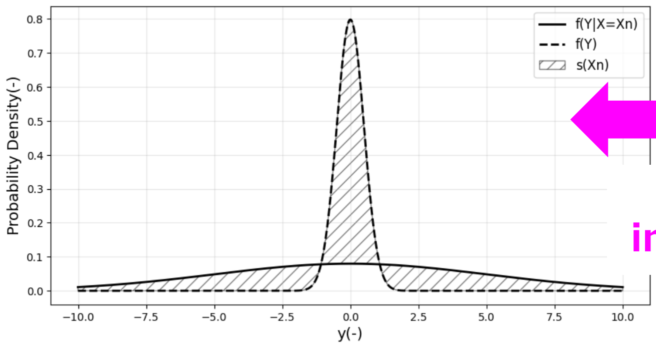
$$s(X_i) = \int \left| f_Y(y) - f_{Y|X_i}(y) \right| dy$$

$$\sigma_i = \frac{1}{2} E_{X_i}[s(X_i)]$$

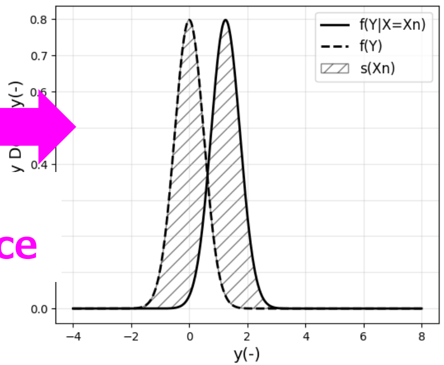
Output Y probability density dist.

Conditional probability density dist. of Y given input X_i

Shape change (variance change)

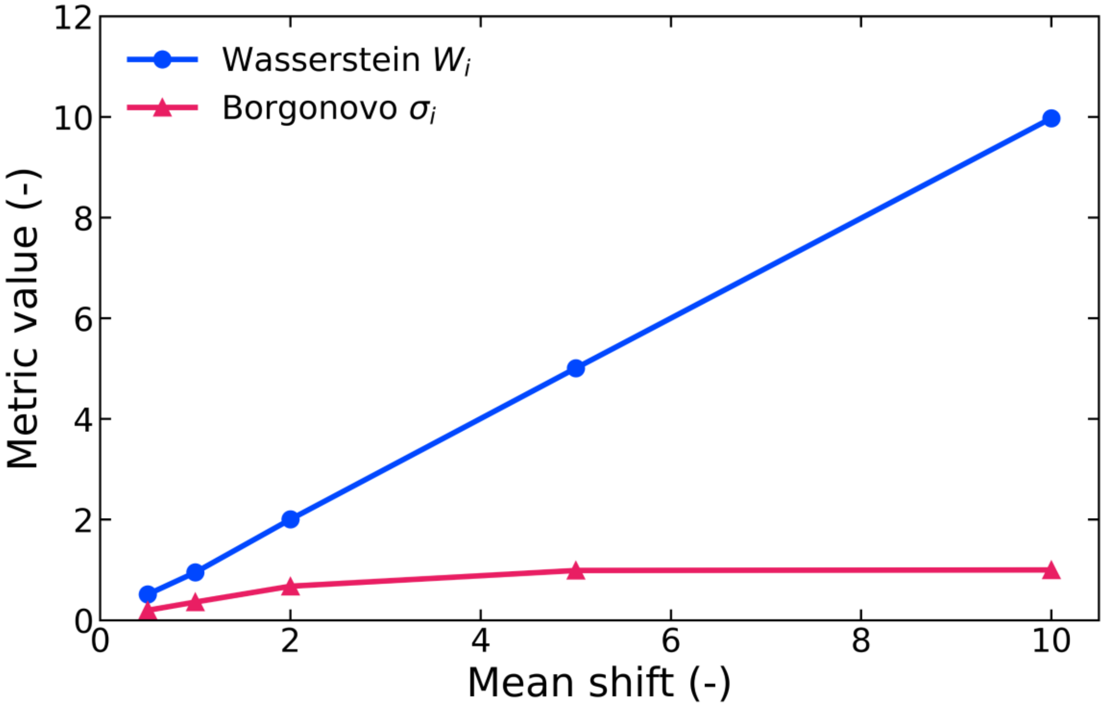


Locational change (Bias)



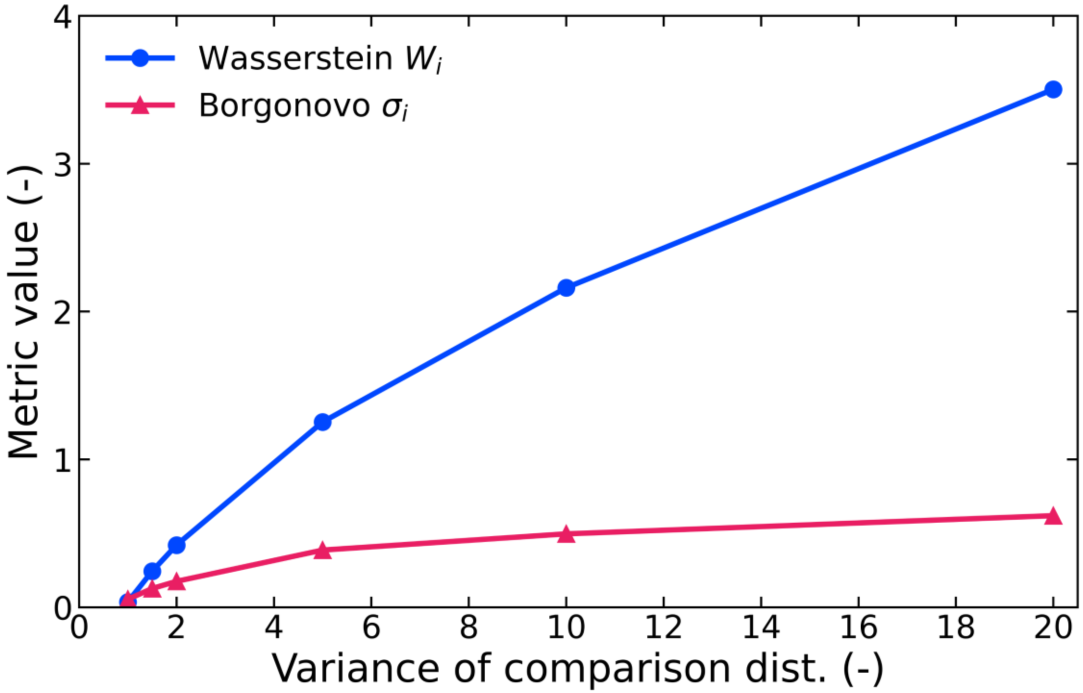
Same importance

Sensitivity of mean shift of normal dist. (with variance=1)



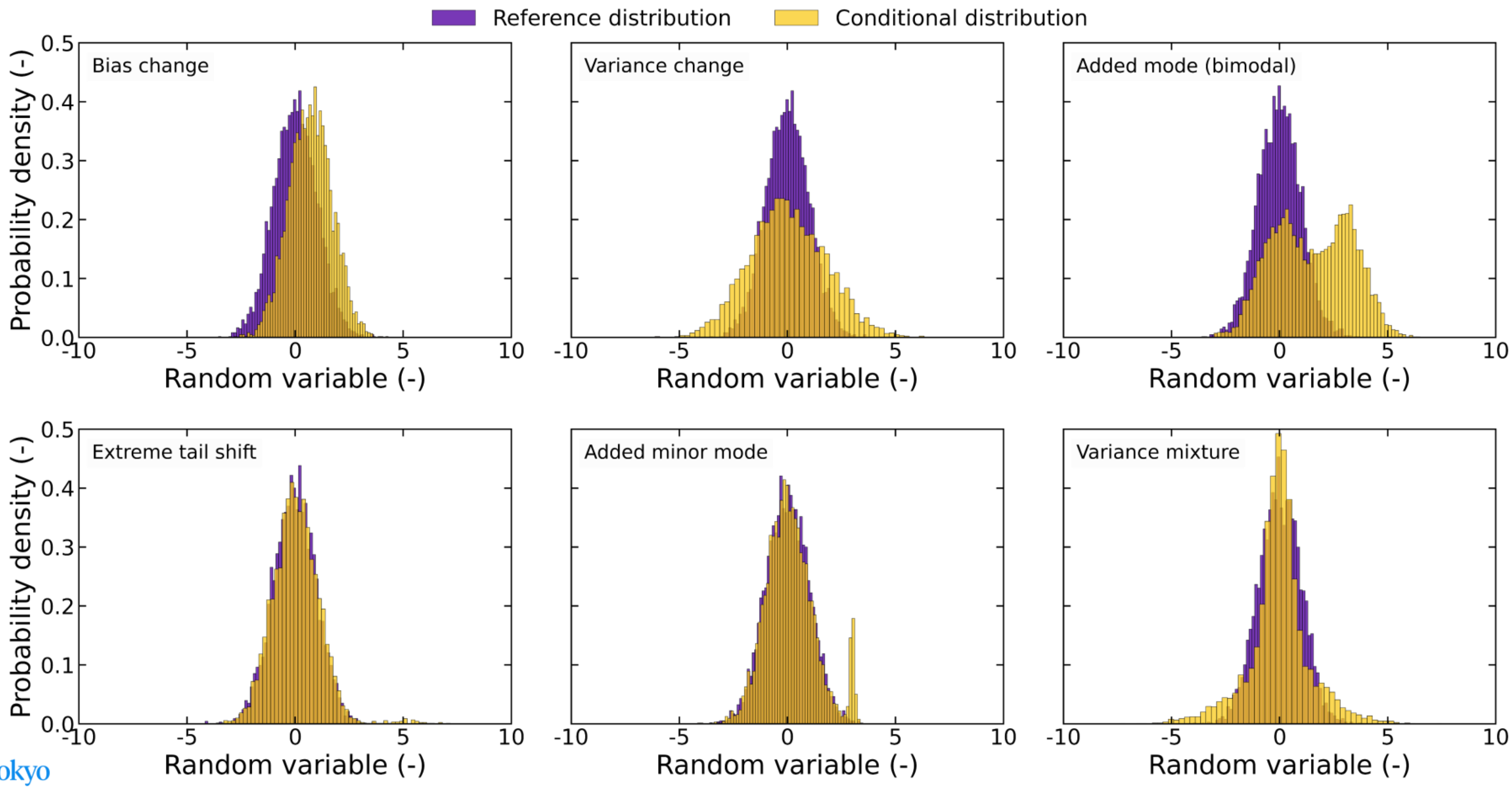
- Wasserstein importance **linearly increases** with increasing mean shifts.
- Borgonovo index increases more gradually and eventually **tends to saturate**.

Sensitivity of variance change of normal dist. (with mean=0)



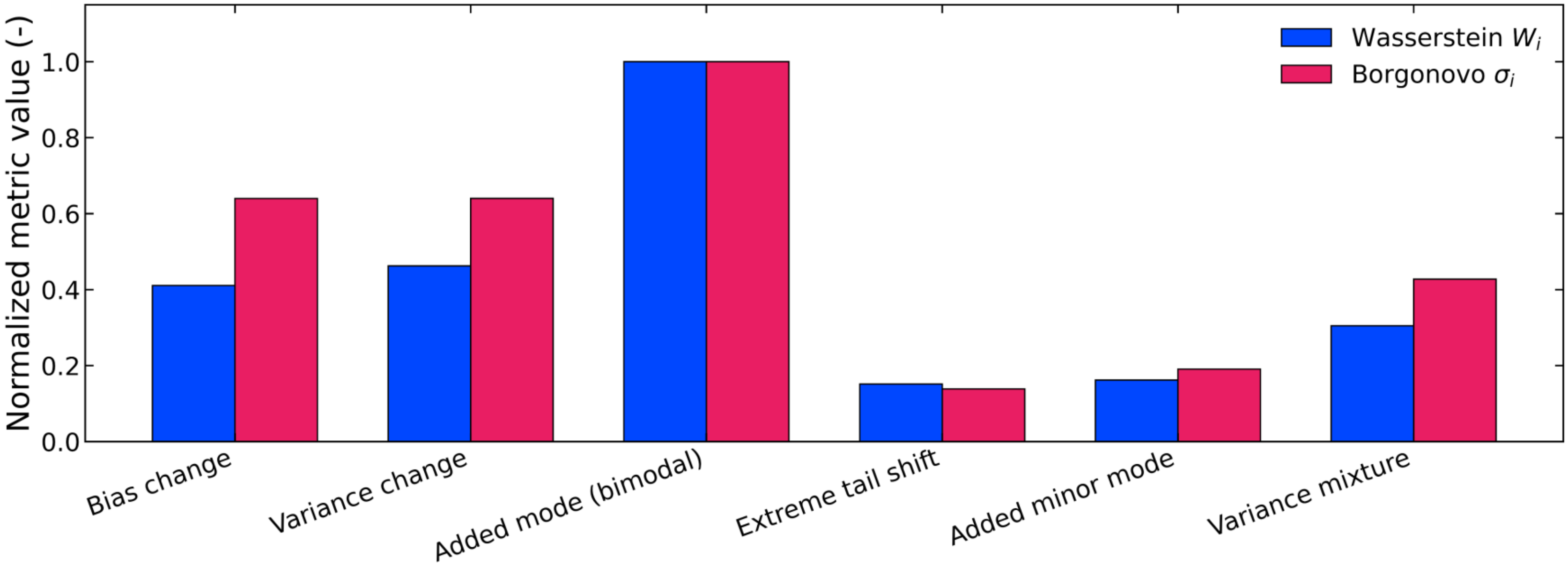
- Wasserstein importance **increases monotonically** as the variance of the conditional distribution increases.
- Borgonovo index increases more gradually, and its **rate of increase decreases for large variance values**.

Sensitivity to various distributional shapes



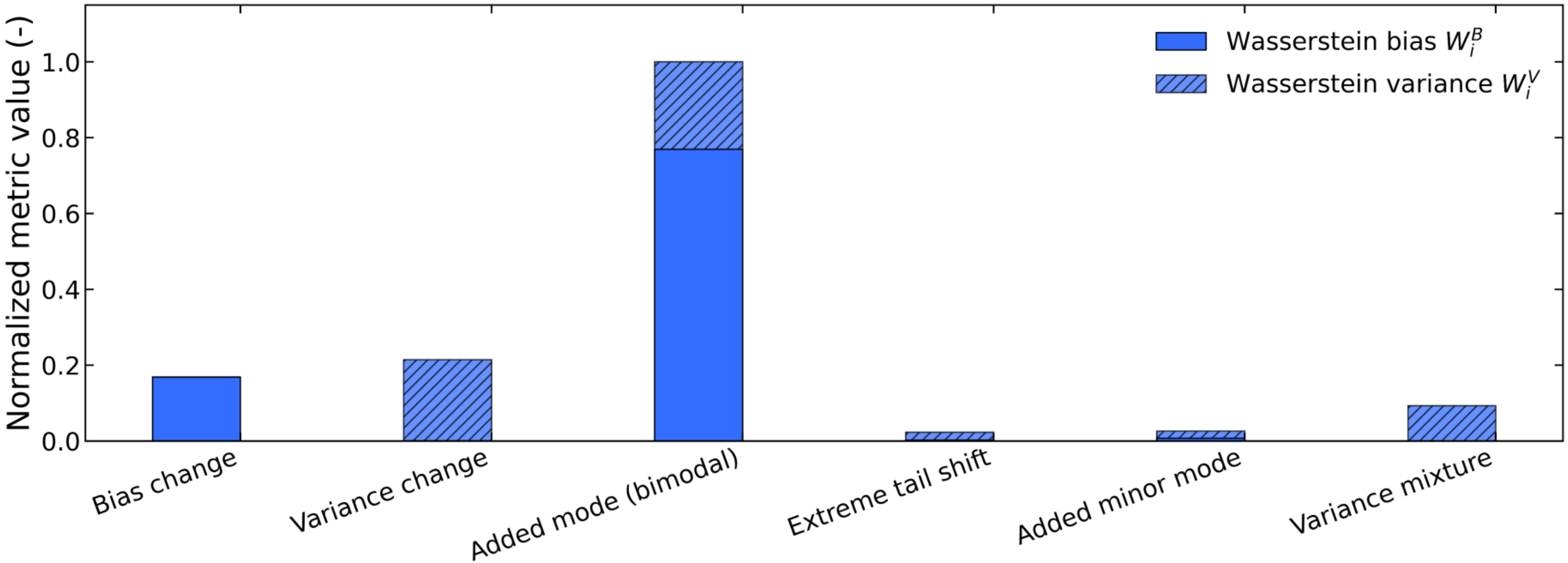
Sensitivity to various distributional shapes

Wasserstein vs. Borgonovo measures



- **The relative importances are generally consistent** btw measures.
- Wasserstein importance tends to be much sensitive to larger change.

Bias–variance decomposition



- The proposed decomposition can separate the effect of distributional changes into the mean change and the variance change after removing the mean.

Conclusions

- **Proposed a Wasserstein-based uncertainty importance evaluation method**
- Sensitivity analysis demonstrated
  - **Importance trends broadly consistent with the Borgonovo measure**
  - **A bias–variance decomposition** that distinguishes mean shifts from mean-centered distributional changes
- The method can be used for **prioritizing data collection and model improvement to reduce epistemic uncertainty**.
- Future work will assess the applicability of the proposed measures to event-tree/fault-tree-based static PRA and dynamic PRA.