Simple Bayesian Updating by Cross-Entropy Minimization: Application at Leibstadt NPP

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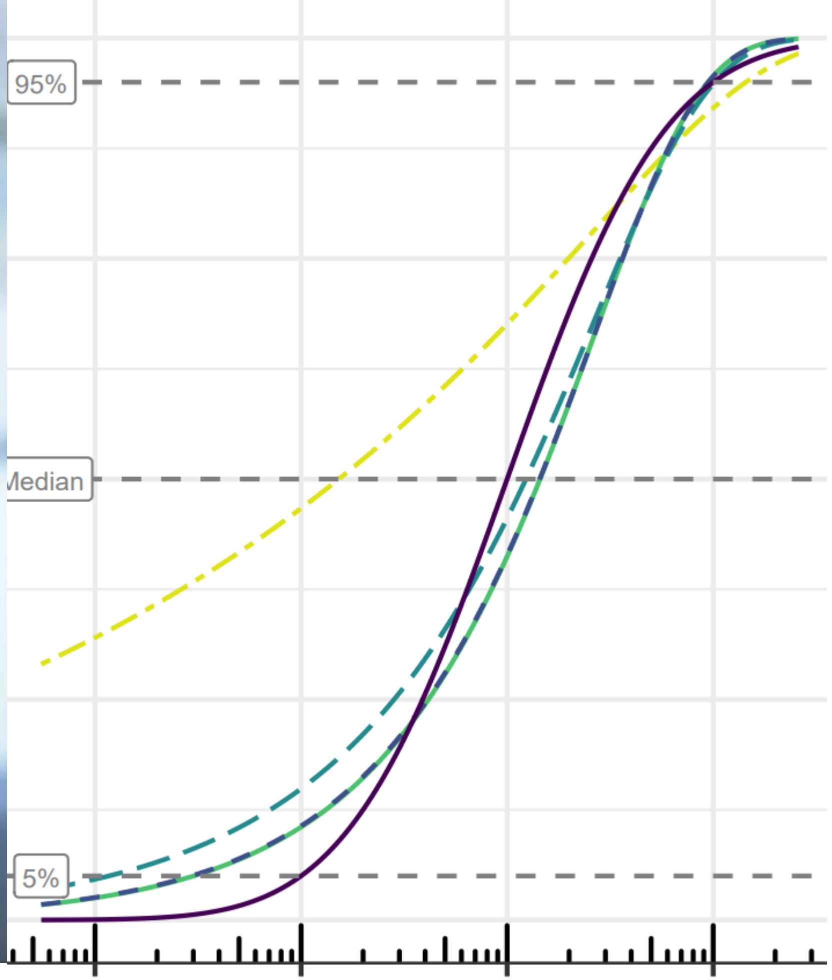

Outline

1. Background & Motivation
2. Theory: Development of New Approach
3. Evaluation
4. KKL PSA Application
5. Extensions & Other Uses

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Bayesian Updating for Leibstadt NPP (KKL) PSA: Background & Motivation

- Purpose: Combine quantitative information from different sources in a systematic way

  → Generic industry data + plant-specific evidence
  - Taking into account the associated uncertainties
  - Obtaining actual probability distributions as a result (as opposed to point estimates & confidence intervals)

- Swiss PSA regulation ENSI-A05 requires Bayesian Updating for:
  - Component Reliability Data: Failure Rates, CCF Parameters
  - Internal Plant Hazards: Frequencies of Internal Fire & Internal Flooding
  - Internal Initiating Events: Frequencies of Transients & LOCAs

- Practical challenges:
  - Generic Distributions in (KKL) PSA often not conjugate → numerical methods
    - Numerical instabilities
    - Transparency: Bayesian Update becomes a «black box»
    - Software dependence → code maintenance, accessibility to all analysts, IT restrictions,…..

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Theory:

Development of New Approach

Simplified Method: Fitting Conjugate Priors to Generic Distributions

- **Well-known**: «*Moment Matching*»

- Equating first and second moments of generic distribution (density f) and conjugate prior (density g):

  - $\int \theta f(\theta) d\theta \;=\; \int \theta g(\theta) d\theta$
  - $\int \theta^2 f(\theta) d\theta \;=\; \int \theta^2 g(\theta) d\theta$

  Equivalent to matching mean and variance

Gives large weight to differences in distribution tails

- **Proposed**: *Minimize Kullback-Leibler Divergence (KLD) & Cross-Entropy (such that generic mean is retained)*

$$D_{KL}(f \parallel g) = \int_\Omega f(\theta) \cdot ln\left(\frac{f(\theta)}{g(\theta)}\right) d\theta$$
$$= \int_\Omega f(\theta) \cdot ln(f(\theta))\, d\theta - \boxed{\int_\Omega f(\theta) \cdot ln(g(\theta))\, d\theta}$$

→ e.g., Optimization problem for Gamma distribution (density g):

$$\max_{\alpha,\beta} \int_0^\infty f(\theta) ln(g_{\alpha,\beta}(\theta))\, d\theta$$
$$\text{such that} \quad Mean_{generic} = \frac{\alpha}{\beta}$$

More robust to differences in tail behavior

Approximate Cross-Entropy Minimization to obtain Gamma Priors

- **Continuous Generic Distributions (General Case)**
  - Requires continuous distribution with support on *subset* of $[0, \infty)$
  - *Exact* cross-entropy minimization: Numerically solve
$$ln(\alpha) - \psi_0(\alpha) \stackrel{!}{=} ln(\mathbb{E}_f(\theta)) - \mathbb{E}_f(ln(\theta))$$
  - Define
$$\Delta = ln(\mathbb{E}_f(\theta)) - \mathbb{E}_f(ln(\theta))$$
  - Bounds for shape parameter α:
$$\boxed{\frac{1}{2\Delta}} < \alpha < \frac{1}{\Delta}$$
    - Optimal α converges to lower bound as $\Delta \rightarrow 0$
      → works well for small Δ
    - Refined Approximation for large Δ:
$$\tilde{\alpha} = \frac{\Delta + 1.55}{\Delta^2 + 3.1\Delta}$$

- **Lognormal Generic Distributions**
  - usual case in (KKL) PSA
  - General Bounds become:
$$\boxed{\frac{2.7}{ln^2(EF)}} < \alpha < \frac{5.4}{ln^2(EF)}$$
    - Lower bound within 19% of numerical solution for EF <= 10
  - Refined approximation for large EF:
$$\tilde{\alpha} = \frac{5.4ln^2(EF) + 45}{ln^4(EF) + 17ln^2(EF)}$$
    - Within 4% of numerical solution for EF <= 100

After obtaining α, calculate the rate parameter
$\beta = \alpha / Mean_{generic}$
→ generic mean always retained

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Equivalence to Maximum Likelihood

- Maximum Likelihood Estimation to fit Gamma distribution on observational data:

$$ln(\alpha) - \psi_0(\alpha) \stackrel{!}{=} ln\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) - \frac{1}{n}\sum_{i=1}^{n} ln(x_i)$$

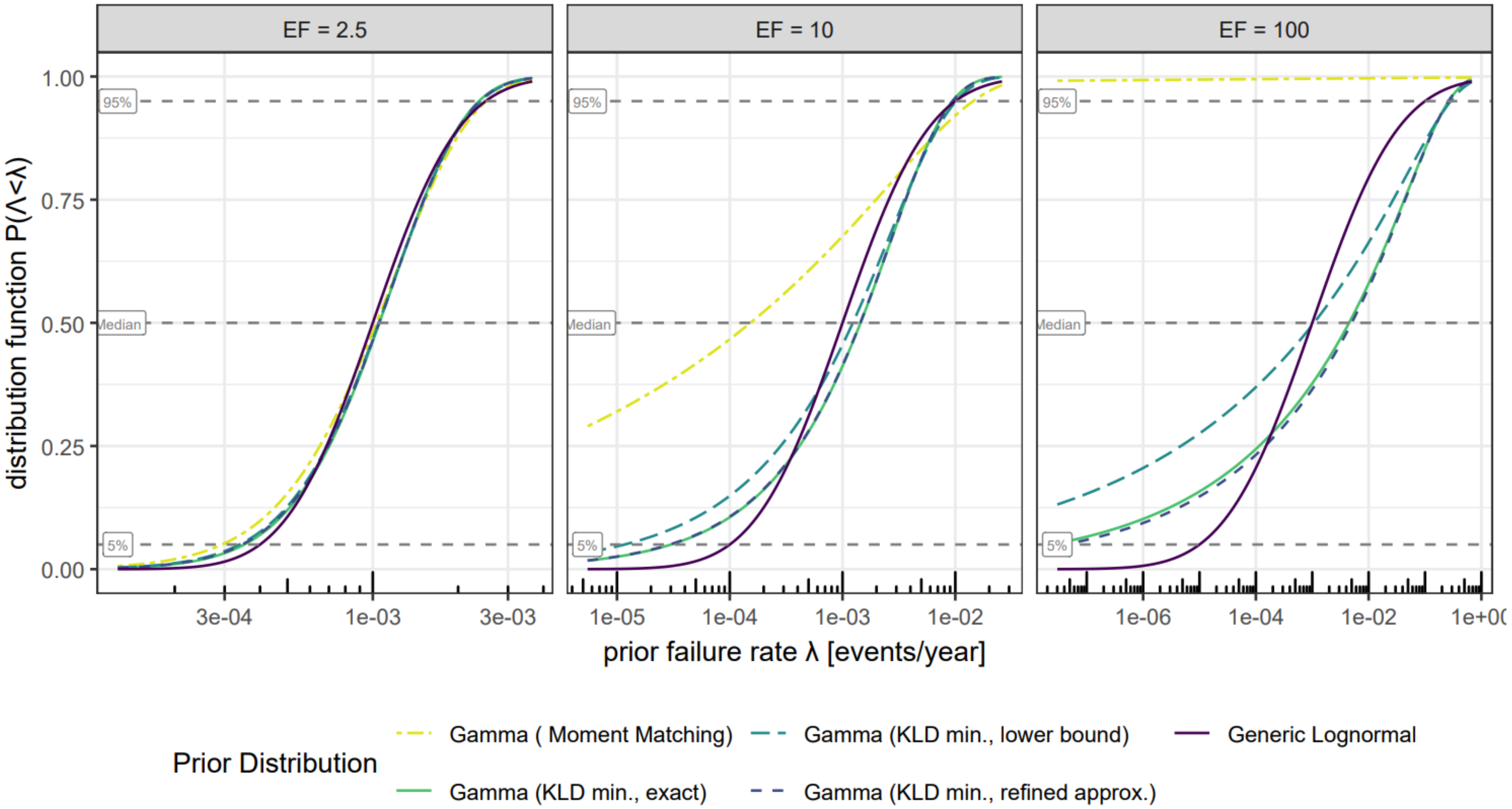
- Cross-Entropy Minimization to obtain approximate Gamma distribution:

$$ln(\alpha) - \psi_0(\alpha) \stackrel{!}{=} ln(\mathbb{E}_f(\theta)) - \mathbb{E}_f(ln(\theta))$$

Evaluation:

Generic Lognormal Distributions

Accuracy of Approximating Gamma Distributions



Comparison of *Posteriors* - Example from NUREG/CR-6823, Sec. 6.2.2.7.2: (Lognormal Prior: Median=1/1000, EF=10 & 0 events in 2102 exposure years)

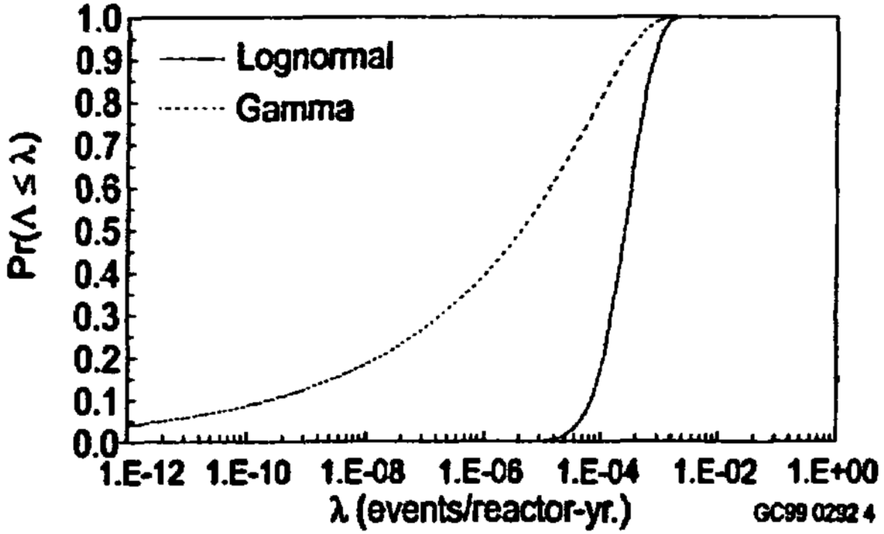
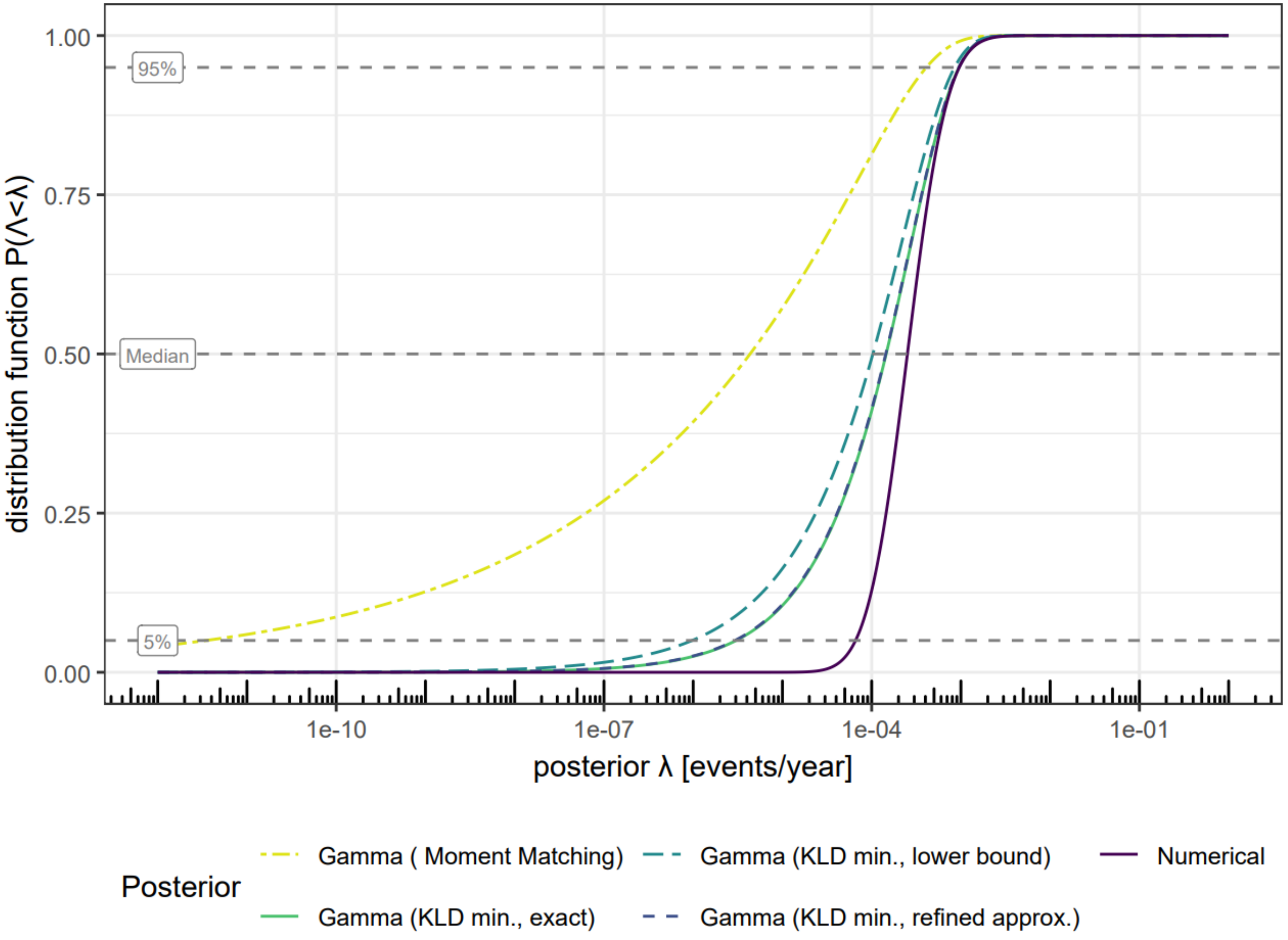


Figure 6.20 The same posterior distributions as in the previous figure, with λ plotted on logarithmic scale.



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General Properties of Posteriors: Evaluation on Synthetic Data

- Synthetic data – constructed from realistic ranges for KKL PSA appplications
  - Lognormal «Generic Data»
    - Mean = 1/100 events per year of exposure

      → *Accuracy of Gamma approximation to lognormal depends only on the error factor*
    - EF = {2.5, 10, 30}
  - Plant-Specific «Experience»
    - k = {0, 1, 2, …, 30} events
    - Log10(T) = {-3, -2, -1, …., 6}, exposure time T in years
- Create a grid of all combinations between generic parameters and evidence
- Filter out combinations which are extremely unlikely → prior-data consistency check
  - *Prior Predictive Distribution* (for Poisson Likelihood): Under the prior and statistical model assumed for plant data, what is the probability of observing exactly k events within exposure time T?

$$\mathbb{P}_{\text{ppd}}(K=k) = \int_0^\infty \mathbb{P}(K=k) f_{\text{gen}}(\lambda) d\lambda$$

    - Filter out combination if $\mathbb{P}_{\text{ppd}}(K \leq k) = \sum_0^k \mathbb{P}_{\text{ppd}}(K=k)$ or $\mathbb{P}_{\text{ppd}}(K \geq k) = 1 - \sum_0^{k-1} \mathbb{P}_{\text{ppd}}(K=k)$ is smaller than 1/1000

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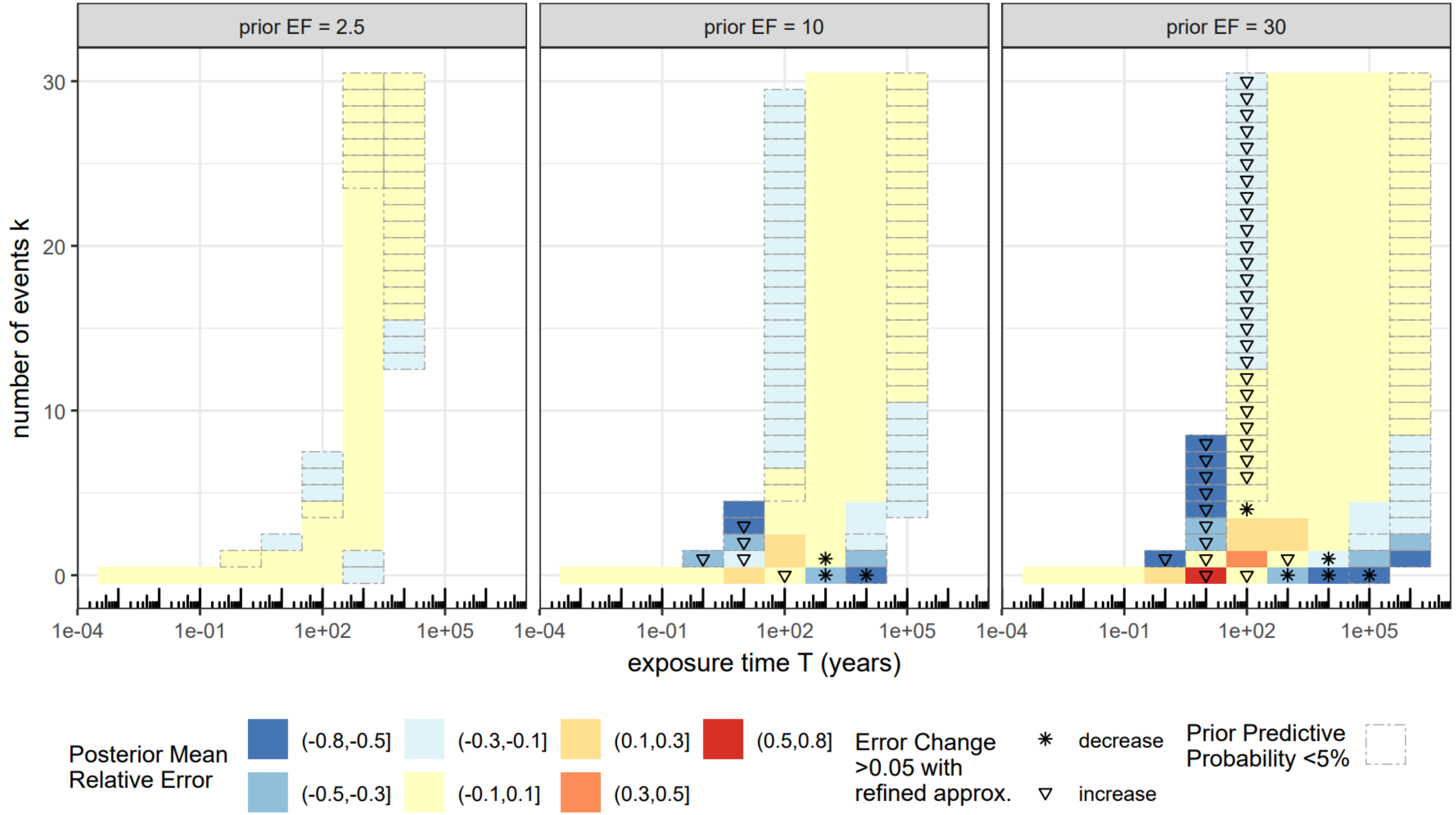
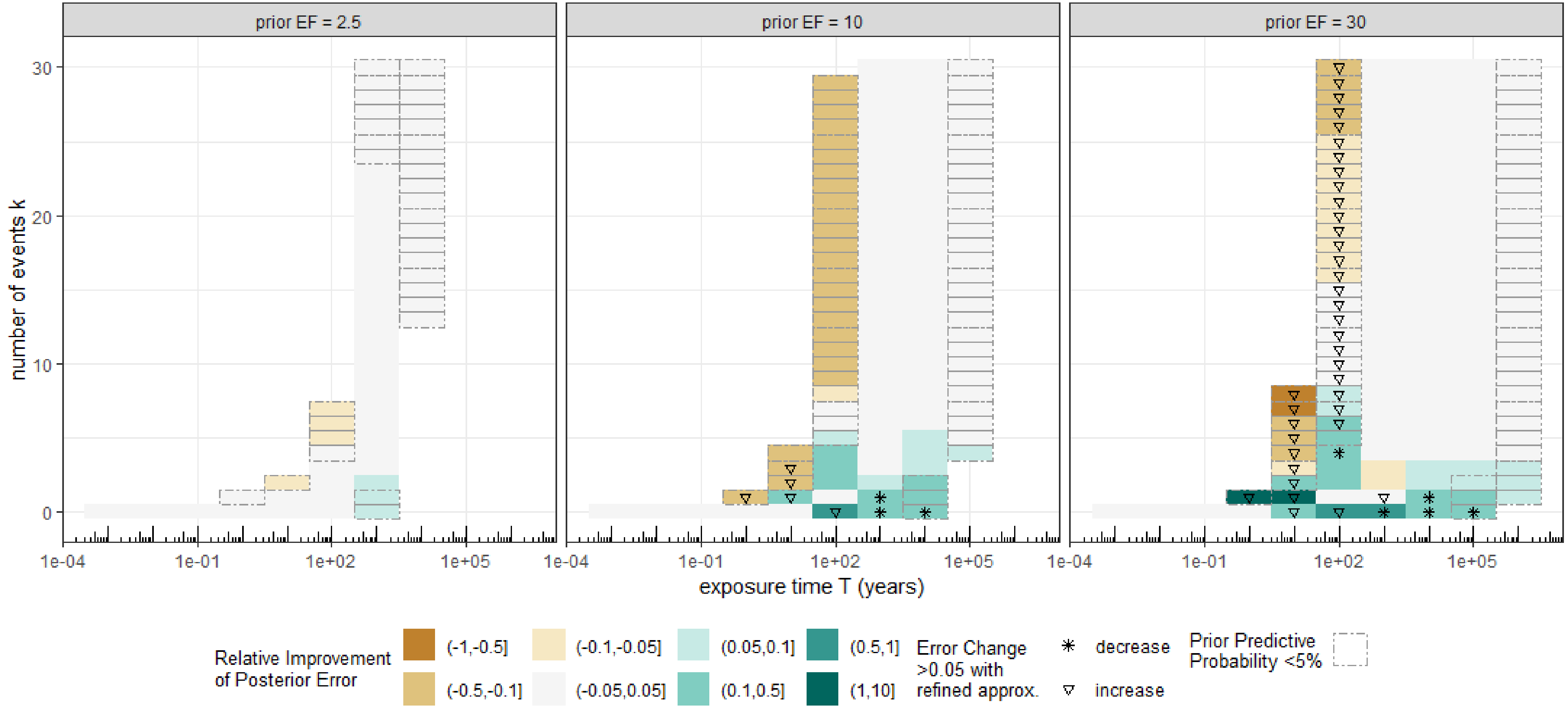
Approximate Cross-Entropy Minimization vs Numerical Updating: Posterior Mean Relative Error



< 10% Change in Posterior Means (more conservative for small k & T, less conservative in case of prior-data conflict)

Approximate Cross-Entropy Minimization vs Moment Matching: Comparison of Posterior Mean Relative Error



Cross-Entropy Min. gives lower / similar Posterior Mean Error (unless event count is inconsistently high vs Generic Data)

Similar pattern for 5th and 95th Posterior Percentiles (see paper)

KKL PSA Application

KKL PSA Application: Component Failure Rates

- KKL PSA Failure Rates:
  - On average, 2.5% decrease in Posterior Mean (q05: -60%, q95: +45%)
  - Order of magnitude changes in 5th Percentiles for 44% of components (mostly decreases)
  - Similar 95th Percentiles: on average +14% change (order of magnitude changes for 0.3% of components)

  → larger changes observed when using Moment Matching

- Core Damage Frequency:
  - Fully integrated PSA model; all plant operating states and internal & external initiators
  - Replaced numerically calculated mean failure rates with approximate cross-entropy minimization (lower bound of α)

    → **+ 3.7% in total mean core damage frequency**

- Differences between new and old failure rates due to 2 effects:
  - Bayesian Updating
  - Preprocessing of prior distributions due to different component boundaries in generic data vs KKL PSA
    - «Apportioning»: Splitting prior failure rates between sub-components

      → *Previously: Inexact apportioning by retaining the generic distribution type (→ 2014 IAEA IPSART mission comment)*

      → *Gamma Distribution: Exact Apportioning due to* $\lambda_i \sim Gamma(\alpha_i,\, \beta) \implies \sum_{i=1}^{n} \lambda_i \sim Gamma\left(\sum_{i=1}^{n} \alpha_i,\, \beta\right)$

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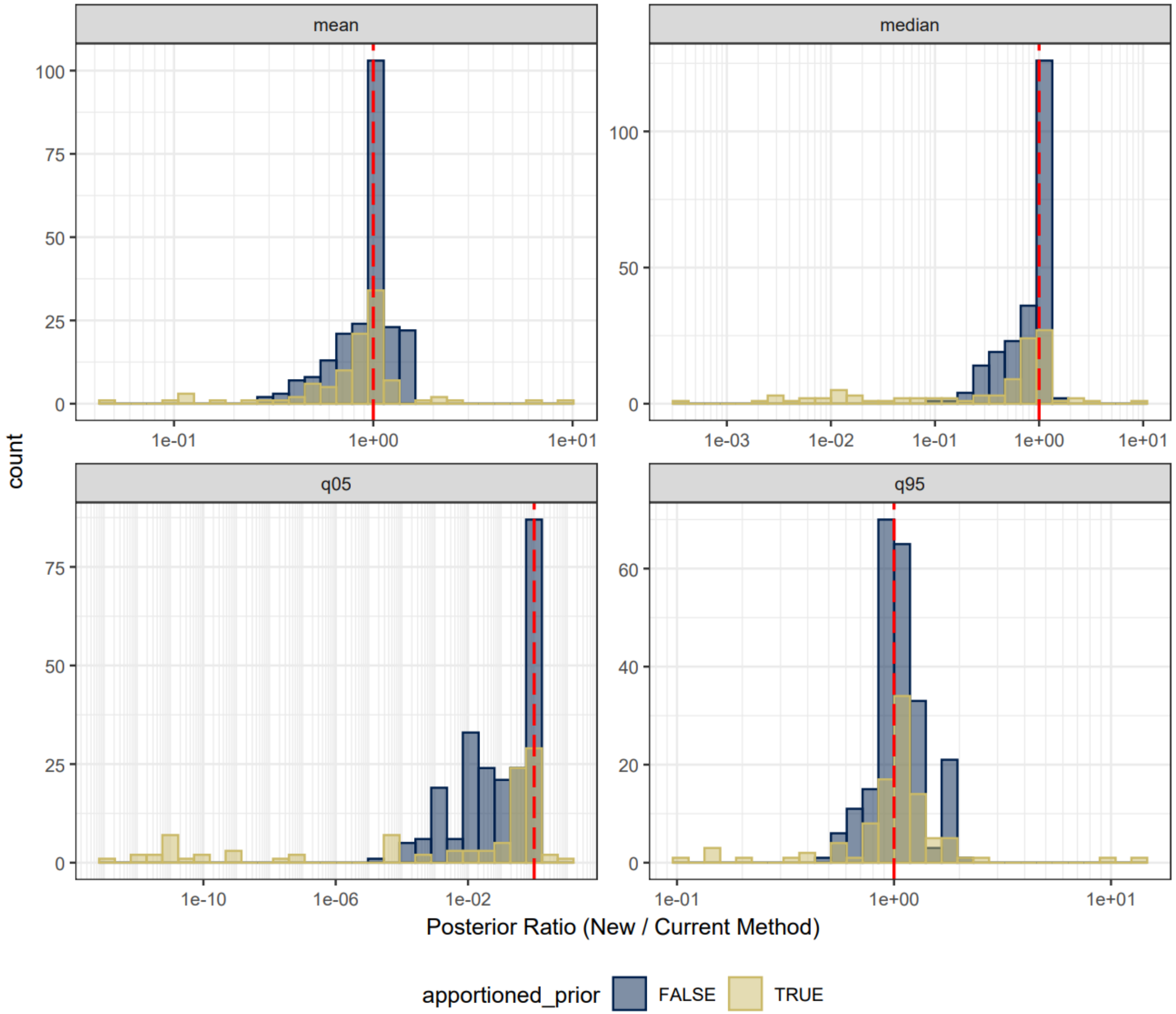
Posterior Differences & «Apportioning»



Extensions & Other Uses

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Other Generic Distributions & Likelihoods

- **Generic Distribution Types other than Lognormal**

- Calculate $\Delta = ln(\mathbb{E}_f(\theta)) - \mathbb{E}_f(ln(\theta))$ for different generic distribution types to approximate them with Gamma distributions
  - Possible for distributions without support on $(-\infty, 0)$
    - Beta, Log-Uniform, Uniform, …..

- **Binomial Likelihood (Probability Parameter)**

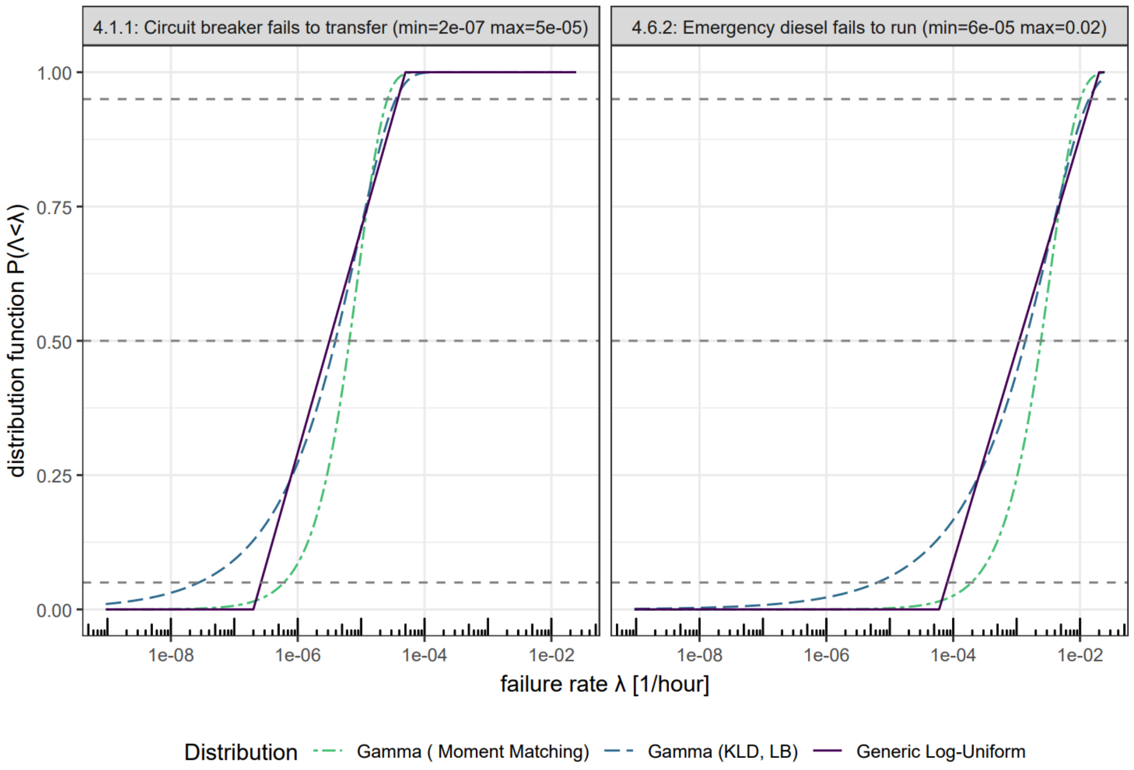
  - Conjugate Prior: Beta Distribution
    - If event probabilities are small, Gamma distributions are well-approximated by Beta distributions
      1. *Convert Generic Distribution into Gamma*
      2. *Convert resulting Gamma Distribution into Beta*

Table 1: Conversion of Common Generic Distributions into Conjugate Priors

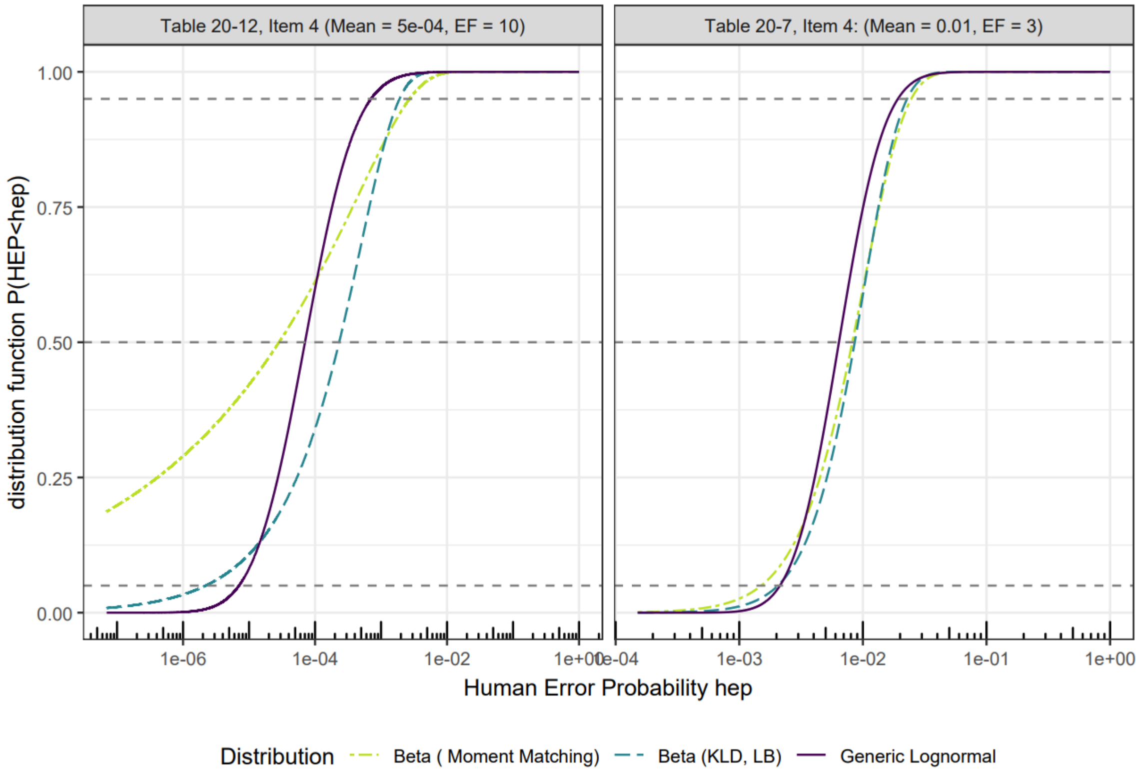
Generic Prior	Likelihood	(First) Shape Parameter, Lower Bound	Remark
$Beta(\varrho, \xi)$	Poisson	ϱ	mismatch of support & units
$Gamma(\alpha, \beta)$	Binomial	α	mismatch of support & units
$LogNormal(Mean,\ EF)$	Poisson	$2.7/ln^2(EF)$	for large EF, see Equation 10
$LogNormal(Mean,\ EF)$	Binomial	$2.7/ln^2(EF)$	mismatch of support & units
$LogUniform(a, b),\ 0 < a < b \leq 1$	Binomial	$1/(2ln(b-a) - 2ln(ln(b/a)) - ln(a) - ln(b))$	mismatch of support
$LogUniform(a, b),\ 0 < a < b$	Poisson	$1/(2ln(b-a) - 2ln(ln(b/a)) - ln(a) - ln(b))$	mismatch of support

Other Conversions: Examples

- Log-Uniform Failure Rates (from NUREG/CR-2815)



- Lognormal Human Error Probabilities (NUREG/CR-1278)



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Further Applications

- Converting Lognormal probability parameters into Beta distributions
  - Matching the bounds for probabilities
  - E.g., Human Error Probabilities in HRA

- Kullback-Leibler divergence as measure for «information gain» from plant experience
  - KKL PSA: Supplements Prior Predictive Probabilities for prior vs data consistency checks (component failure rates)
    - Both criteria have closed-form expressions / approximations due to using Gamma priors

Thank You!

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