A convex multi-objective optimization method for reliability allocation in fault tree analysis

Sebastian Jovancic[a]
[a] Vysus Sweden, Stockholm, Sweden, Sebastian.jovancic@vysusgroup.com

Abstract: Reliability allocation aims to distribute system-level performance requirements across components or barrier functions while satisfying safety acceptance criteria. At early design stages, when system realisations are not yet defined, fault tree models describe functional relationships between initiating events, barriers, and consequences. In this setting, component-level cost information is generally unavailable, and reliability serves as a proxy for implementation effort under an assumed monotonic relationship. The allocation problem is inherently underdetermined as many combinations of function reliabilities can satisfy a given top-level constraint. A principled approach to selecting among these feasible solutions can therefore provide a risk-informed basis for design decisions.

We present a multi-objective optimization method in which the feasible set is rendered convex through a logarithmic reparametrisation of cutset contributions. Convex surrogates for entropy-like objectives are constructed, enabling the systematic exploration of trade-offs between risk distribution across cutsets and reliability dispersion across barrier functions, generating structured families of optimal solutions.

1. INTRODUCTION

Nuclear power plant (NPP) design must satisfy both regulatory and stakeholder constraints. Regulatory constraints include acceptance criteria such as core damage frequency and balanced risk profiles, while stakeholder constraints are largely associated with implementation cost. At early design stages, detailed cost models are typically unavailable, however increased reliability generally implies increased implementation effort and cost.

Probabilistic Safety Assessment (PSA) complements deterministic design by quantifying how design decisions affect risk. In PSA, consequences are represented by combinations of initiating event frequencies and associated barrier failure sequences, represented as cutsets. At early design stages, these barriers are typically represented at a functional level rather than at a detailed systems or component level.

Given initiating events and defined plant responses, cutsets and their associated consequence frequencies can be generated from assigned initiating event frequencies and barrier failure probabilities. Determining barrier failure probabilities that satisfy regulatory constraints therefore becomes an optimization problem. In this work, the problem is framed within convex optimization since any local optimum is globally optimal and efficient numerical algorithms exist with well-understood convergence properties [1]. Objective functions for reliability minimization, risk-spread and reliability spread are proposed. An optimization strategy is then employed on the results from a simplified PSA model to generate feasible barrier failure probabilities as a proof of concept.

2. THEORY

2.1. Problem formulation

From the perspective of PSA, an NPP can be seen as a function $T(\boldsymbol{x})$ that maps the space of possible initiating events I to the space of possible consequences C

$$T(\boldsymbol{x}): I \mapsto C, \quad (2.1.1)$$

where $\boldsymbol{x}$ is a vector of barrier function failure probabilities. Regulatory requirements define a subset of acceptable consequences $C_R \subseteq C$. The reliability allocation problem is therefore to determine the set of barrier probabilities $\boldsymbol{x_D}$ such that initiating events map only to acceptable consequences. Since many feasible reliability allocations may exist, the problem is formulated as a constrained optimization problem with a feasible set defined as

$$\mathcal{F} = \{\, \boldsymbol{x_D} \mid g(\boldsymbol{x_D}) \leq c \,\}, \quad (2.1.2)$$

where g is the constraint function representing a consequence for the given design and c the associated acceptance criteria. At early design stages, detailed cost models are typically unavailable. Since increased reliability generally implies increased implementation cost, reliability may instead be used as a surrogate optimization metric. The methodology is formulated within the framework of convex optimization, i.e. minimization of convex objective functions over a convex feasible set, allowing globally optimal solutions to be identified efficiently.

2.2. Cutset representation

2.2.1. Posynomial form

The computed result of a PSA model is a cutset list for a given consequence. Each cutset is the product of an initiating event frequency λ_i and a set of basic event probabilities x_k that represent barrier failures; component failures and human failure events at the lowest level and function failures at the highest level. The total frequency computed can be conservatively approximated as the sum of the frequencies for all the corresponding cutsets, meaning that in general, the top result for a cutset list, as a function of the barrier failure probabilities, is a posynomial. For simplicity, we will omit the subscript from $\boldsymbol{x_D}$ from now on, writing the top frequency in posynomial form as

$$F(\boldsymbol{x}) = \sum_{n=1}^{N} \lambda_n \prod_{k=1}^{K} x_k^{a_{nk}}, \quad (2.2.1)$$

where $\boldsymbol{x} = [x_1, \dots, x_K]$ is a vector of K barriers, each parameter x_k representing the probability that a barrier fails and a_{nk} are elements of a matrix A that are either 1 if the barrier is credited in the nth cutset or 0 if it is not credited in the cutset. Posynomials are generally neither convex nor concave, meaning any objective function or feasible set that is based directly on a cutset list in a posynomial form will also be neither convex nor concave. Every posynomial function can however, through an appropriate change of variables, be made convex [2].

2.2.2. Log-Sum-Exponential form

Let $\boldsymbol{y} = [y_1, \dots, y_n]$ such that

$$\boldsymbol{x} = [e^{y_1}, \dots, e^{y_n}] = \exp(\boldsymbol{y}) \quad (2.2.2)$$

where $y_n \leq 0$ since $x_n \in [0,1]$. Introducing vector $\boldsymbol{\Lambda} = [\ln(\lambda_1), \dots, \ln(\lambda_N)]$ the scalar valued top frequency function can be expressed in vector form as

$$F(\boldsymbol{y}) = \boldsymbol{I}^T \exp(\boldsymbol{\Lambda} + A\boldsymbol{y}) \quad (2.2.3)$$

We introduce the vector $\boldsymbol{z} = [z_1, \dots, z_N]$

$$\boldsymbol{z} = \boldsymbol{\Lambda} + A\boldsymbol{y} \quad (2.2.4)$$

such that

$$F(\boldsymbol{z}) = \boldsymbol{I}^T \exp(\boldsymbol{z}) = \sum_n \mathrm{e}^{z_n} \quad (2.2.5)$$

Each exponential term e^{z_n} represents the frequency of the nth cutset. Taking the logarithm of both sides yields the Log-Sum-Exponential (LSE) function, which we will from now on define as

$$L(\boldsymbol{z}) := \ln\big(F(\boldsymbol{z})\big) \qquad (2.2.6)$$

which is convex. Since $\boldsymbol{z}$ is an affine in $\boldsymbol{y}$ then the feasible set over $\boldsymbol{y}$ is also convex.

2.2.3. Constraint function

The constraint can be expressed as a function in vector form as a function of $\boldsymbol{z}$

$$g(\boldsymbol{z}) = L(\boldsymbol{z}) - \ln(c) \leq 0 \qquad (2.2.7)$$

Since c is independent of $\boldsymbol{z}$**,** the Hessian of the constraint function $g(\boldsymbol{z})$ is the Hessian of $L(\boldsymbol{z})$. Since $L(\boldsymbol{z})$ is convex it follows that the constraint function $g(\boldsymbol{z})$ is also convex.

2.3. Objective functions

In the choice of objective functions, we can establish the following properties we wish to optimise for:

1. The overall cost → minimizing total reliability
2. A balanced risk profile → spread risk across cutsets such that there are no extreme cutsets with disproportionate contribution.
3. No extreme barriers → avoid single barriers being excessively reliable or redundant.

In the following chapters objective functions for each property will be presented and a means to produce a pareto efficient set of solutions for designers to choose an acceptable trade-off between property 2 and 3 while minimizing property 1.

2.3.1. Reliability minimization objective function

The square of the L_2 norm of the vector $\boldsymbol{y}$ is strictly convex

$$f_1 = \|\boldsymbol{y}\|_2^2, \qquad (2.3.1)$$

and since $\lim_{y_k \to 0} x_k \to 1$ its optimum produces a feasible $\boldsymbol{y}$ that minimizes the reliability across all barriers. Such an objective function allows for trivial solutions where some x_k can become either 0 or 1 and the objective is agnostic to how risk is spread across cutsets. It is therefore not a suitable objective function by itself, however it is a useful auxiliary objective function since it is strictly convex and by definition minimizes $\boldsymbol{y}$**.**

2.3.2. Risk-spread objective function

The gradient of the log-sum-exp function is the softmax distribution

$$\frac{\partial}{\partial z_n} L(\boldsymbol{z}) = \frac{e^{z_n}}{\sum_m e^{z_m}}, \qquad (2.3.2)$$

which, in the PSA context, corresponds to the fractional contribution of the nth cutset to the total top-event frequency. A concentrated softmax distribution therefore corresponds to risk being dominated by a small number of cutsets, while a flatter distribution corresponds to a more balanced risk profile. A natural way to encourage such balance is therefore to minimize the spread of the vector **z**. A suitable convex measure is the squared L^p mean deviation

$$\|\boldsymbol{z} - \bar{\boldsymbol{z}}\|_p^2 = \left(\sum_{i=1}^{N} |z_i - \bar{z}|^p\right)^{\frac{2}{p}}, \quad \bar{\boldsymbol{z}} = \bar{z}\boldsymbol{1} = \frac{1}{N}(\boldsymbol{1}^T\boldsymbol{z})\boldsymbol{1}, \qquad (2.3.3)$$

which is minimized when all elements of $\boldsymbol{z}$ are equal. For $p = 2$, this is proportional to the variance of $\boldsymbol{z}$

$$\|\boldsymbol{z} - \bar{z}\|_2^2 = \sum_{i=1}^{N} |z_i - \bar{z}|^2 = N \cdot \mathrm{Var}(\boldsymbol{z}), \tag{2.3.4}$$

while larger p increasingly penalize the largest deviations from the mean. The resulting risk-spreading objective function is therefore defined as

$$f_2(\boldsymbol{y}, p) = \frac{1}{2N} \|\boldsymbol{z}(\boldsymbol{y}) - \bar{\boldsymbol{z}}\|_p^2. \tag{2.3.5}$$

2.3.3. Reliability-spread objective function

The risk-spreading objective distributes risk across cutsets but does not prevent trivial solutions in which some barriers are assigned extreme reliabilities. A second objective is therefore introduced to distribute reliability more uniformly across barriers. As for the risk-spreading objective, a convex measure based on the squared L^p mean deviation is used. Choosing $p = 2$ yields a function proportional to the variance of $\boldsymbol{y}$, which is minimized when all barrier reliabilities are equal. The reliability-spreading objective function is therefore defined as

$$f_3(\boldsymbol{y}) = \frac{1}{2K} \|\boldsymbol{y} - \bar{\boldsymbol{y}}\|_2^2, \quad \bar{\boldsymbol{y}} = \bar{y}\mathbf{1} = \frac{1}{K}(\mathbf{1}^T \boldsymbol{y})\mathbf{1}, \tag{2.3.6}$$

2.3.4. Normalised weighted-average objective function

There are three proposed objective functions given by equations (*2.3.1*), (*2.3.5*) and (*2.3.6*) where f_2 and f_3 are of primary interest and f_1 is an auxiliary objective function used to find unique solutions for f_2 and f_3 since neither are strictly convex. Since $f_2$and $f_3$are incommensurate, they are normalized using their utopia and nadir values [3],

$$\hat{f}_i(\boldsymbol{y}) = \frac{f_i(\boldsymbol{y}) - f_i^U}{f_i^N - f_i^U}, \tag{2.3.7}$$

The final objective function is then defined as

$$f(\boldsymbol{y}) = \alpha \hat{f}_2(\boldsymbol{y}) + (1 - \alpha)\hat{f}_3(\boldsymbol{y}). \tag{2.3.8}$$

where α controls the trade-off between risk-spreading and reliability-spreading. The utopia and nadir points of $f_2$and f_3 are computed subject to the lexicographic constraint

$$\min_{\boldsymbol{y}} f_i(\boldsymbol{y}) \quad s.t\; f_1(\boldsymbol{y}) \le f_1^\star(1 + \delta), \tag{2.3.9}$$

where $f_1^\star$is the minimum feasible value of $f_1$and δ is a relative tolerance parameter controlling the allowable deviation from the minimum-norm solution.

2.4. Optimization strategy

2.4.1. Algorithm

Since the problem has been posed with a convex constraint and a convex objective function, it can be optimized with a convex optimization solver. The optimization strategy is done as follows:

1. Solve

$$\min_{\boldsymbol{y}} f_1(\boldsymbol{y}) \quad s.t. \quad g(\boldsymbol{y}) \le 0, \tag{2.4.1}$$

   and store $f_1^\star$.

2. Solve $y_2^{k,\star}$ and $y_3^{k,\star}$ for each kth consequences

$$\min_{\boldsymbol{y}} f_i^k(\boldsymbol{y}) \quad s.t \quad f_1(\boldsymbol{y}) \le f_1^\star(1 + \delta) \;\text{ and }\; g(\boldsymbol{y}) \le 0, \tag{2.4.2}$$

3. Compute utopia and nadir estimates per kth consequence as

$$f_2^{k,U} = f_2^k\left(y_2^{k,\star}\right),\quad f_2^{k,N} = f_2^k\left(y_3^{k,\star}\right),\quad f_3^{k,U} = f_3^k\left(y_3^{k,\star}\right),\quad f_3^{k,N} = f_3^k\left(y_2^{k,\star}\right), \tag{2.4.3}$$

and compute the per consequence normalised functions

$$\widehat{f_i}^k(\boldsymbol{y}) = \frac{f_i^k(\boldsymbol{y}) - f_i^{k,U}}{f_i^{k,N} - f_i^{k,U}}, \tag{2.4.4}$$

4. Compute the weighted sums of the normalised functions over the kth consequences

$$\hat{f}_i(\boldsymbol{y}) = \sum\nolimits_k \beta \widehat{f_i}^k(\boldsymbol{y}),\quad \sum\nolimits_{\boldsymbol{k}} \beta = 1, \tag{2.4.5}$$

5. If more than one consequence is analysed, repeat step 2 and 3 for the normalised weighted sums over all consequences instead of per consequence.
6. Define the final objective function as

$$\hat{f}(\boldsymbol{y}) = \alpha \hat{f}_2(\boldsymbol{y}) + (1-\alpha)\hat{f}_3(\boldsymbol{y}), \tag{2.4.6}$$

and sweep $\alpha: 0 \rightarrow 1$ to store the pareto set of efficient solutions as a function of α.

$$\min_{\boldsymbol{y}} \hat{f}(\boldsymbol{y}) \quad s.t.\quad f_1(\boldsymbol{y}) \leq f_1^\star(1+\delta) \ \text{ and } \ g(\boldsymbol{y}) \leq 0. \tag{2.4.7}$$

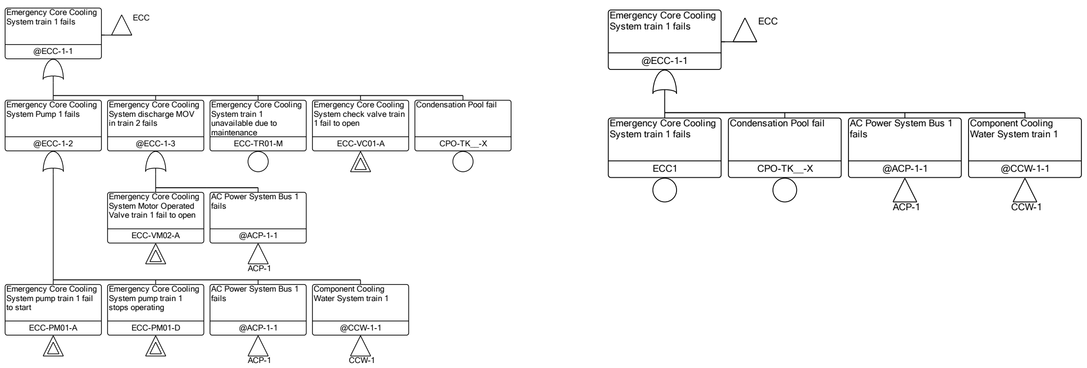
7. If $\boldsymbol{y}(\alpha = 0) \approx \boldsymbol{y}(\alpha = 1)$, increase δ and repeat from step 2 until a kth consequences frequency $F_k\big(\boldsymbol{y}(\alpha)\big)$ at any α is deemed too far below the acceptance criteria of for the kth consequence c_k. If the most contributing cutsets have a relatively high fractional contribution, increase p for objective f_2 and repeat.

3. RESULTS

3.1. Analysis assumptions

RiskSpectrum PSA was used to generate cutsets for optimization. The method is however not designed to be used at a components-level optimization of barriers. The example model EXPSA was therefore modified. In each fault tree representing a system, all basic events representing system-specific failure modes were flattened to a single event, while dependencies to other systems are kept, see Figure 1.

Figure 1. Fault tree EEC-1 from EXPSA before and after event flattening



All basic events that are not initiating events were assigned probability $p = 10^{-1}$ so that no cutsets end up below cutoff. A frequency analysis was performed with CCF disabled. The basic event OFFSITE-POWER is the probability of a loss of offsite power (LOOP) during mission time. The yearly frequency for LOOP in EXPSA is 10^{-1}. With a mission time of 24 hours, the constant probability assigned to OFFSITE-POWER is

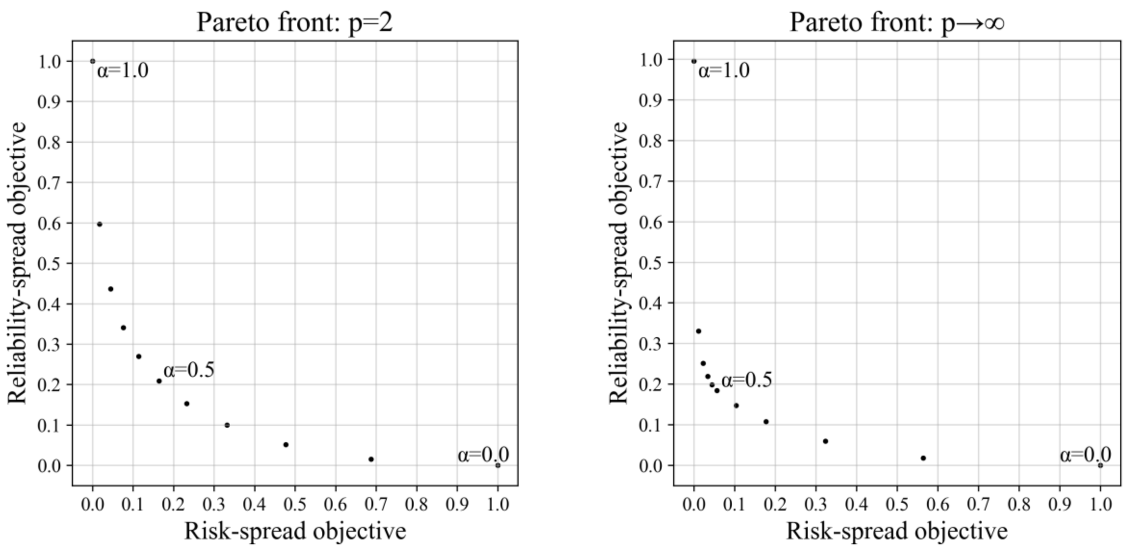
$$P(\text{OFFSITE-POWER}) = \frac{24}{8760} \cdot 10^{-1} \approx 2.74 \cdot 10^{-3}. \tag{3.1.1}$$

A constraint is added to the optimization to fix this value. The optimization algorithm was implemented using CVXPY with Clarabel chosen as the solver. The lexicographic relative tolerance parameter was set to $\delta = 0.5$ and the acceptance criteria for CD was set to $c_{CD} = 10^{-5}$.

3.2. Pareto frontier

In Figure 2 the normalised pareto front $p = 2$ shows moderate curvature near $\alpha = 0.65$. The pareto front for $p \rightarrow \infty$ shows very strong curvature near $\alpha = 0.65$. The implication is that the most decisive tradeoffs are found in the neighbourhood of this region.

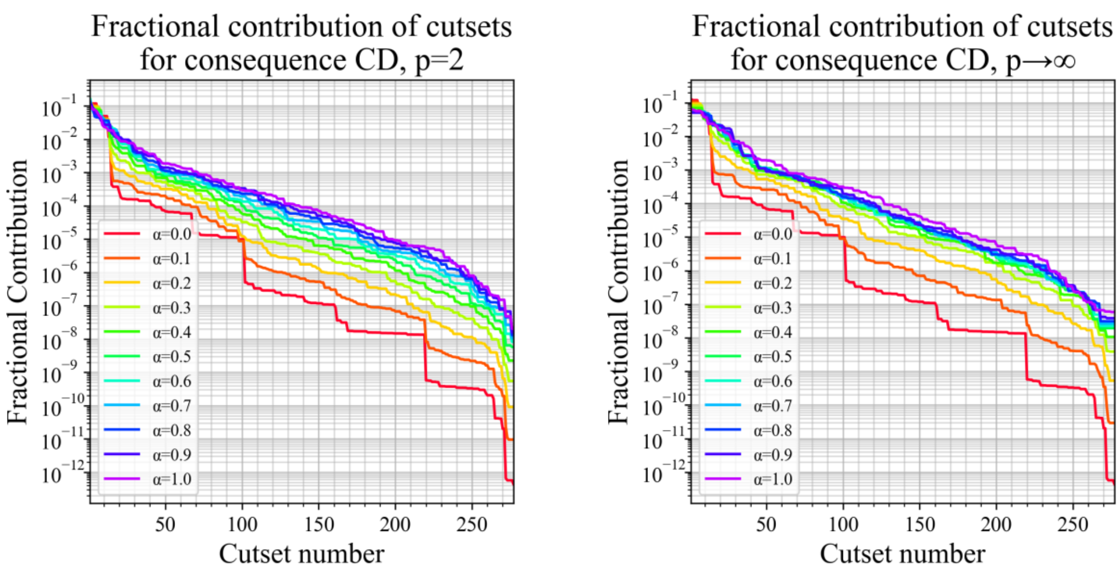
Figure 2 Comparison of the pareto fronts for $p = 2$ and $p \rightarrow \infty$



3.3. Fractional contribution

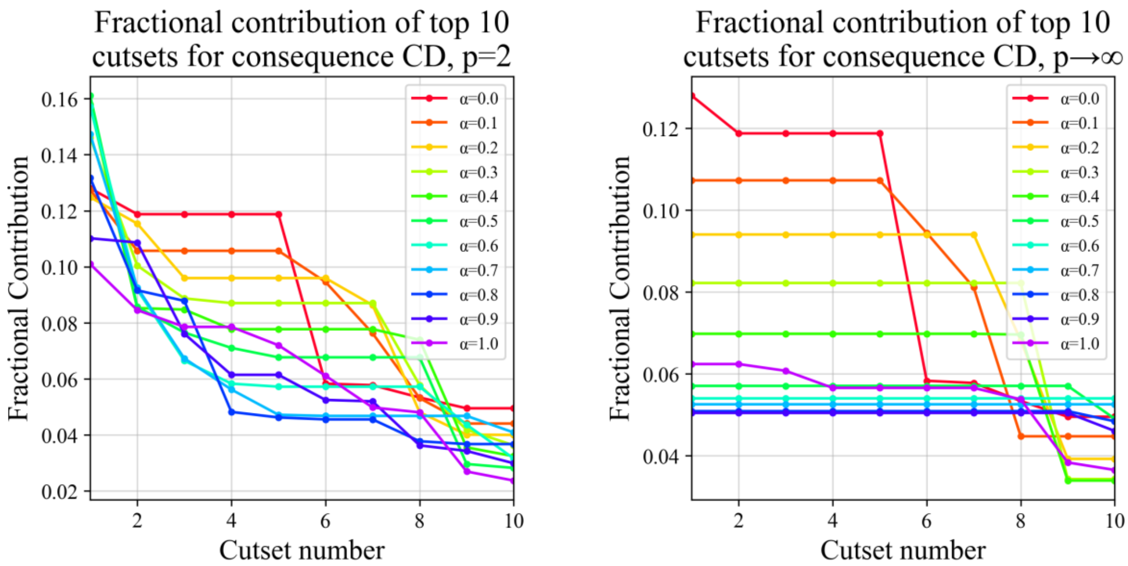
In Figure 3 the fractional contributions of all cutsets for α swept from 0 to 1 are presented, where $\alpha = 1$ yields the highest spread of risk across all cutsets. For the case $p \rightarrow \infty$ the difference between $\alpha = 0.5$ and $\alpha = 1$ is smaller across all cutsets.

Figure 3. Comparison of fractional contributions of cutsets $p = 2$ and $p \rightarrow \infty$



In Figure 4 the top 10 cutsets for α swept from 0 to 1 are presented, where $\alpha = 1$ yields the highest spread of risk across all cutsets. For the case where $p \to \infty$ the risk associated with the cutsets with the highest fractional contribution has been reallocated to the cutsets further down the cutset list as compared to the $p = 2$ case. Notably, for $p \to \infty$ it can be seen that $\alpha = 0.9$ yields lower fractional contributions for the top 8 cutsets than $\alpha = 1$. This is explained by the lexicographic tolerance δ being too large for the chosen optimization parameters.

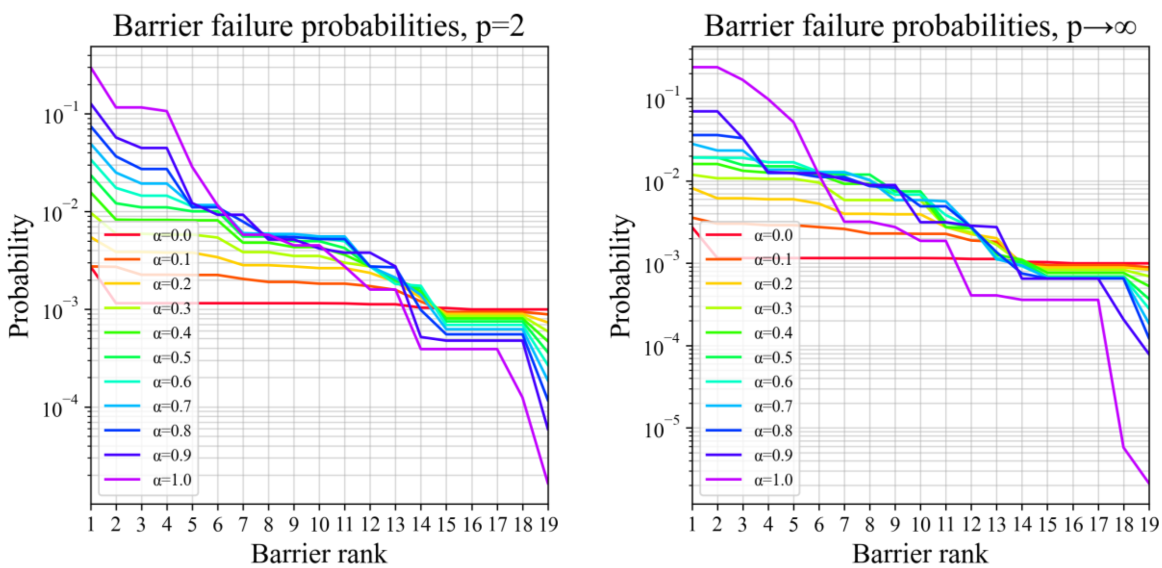
Figure 4. Comparison of fractional contributions of the top 10 cutsets for p=2 and p→∞



3.4. Barrier failure probabilities

In Figure 5 the barrier failure probabilities for α swept from 0 to 1 are presented, where $\alpha = 0$ yields the highest spread of reliability across all barriers. For $\alpha = 0$, the curve is close to flat, the only exception being OFFSITE-POWER which is fixed at $p = 2.74 \cdot 10^{-3}$. For $\alpha = 1$, there are four orders of magnitude difference between the most and the least reliable barriers. When $p \to \infty$ the difference between the most and least reliable barriers grows to 6 orders of magnitude at $\alpha = 1$ but stays roughly the same at $\alpha = 0.5$.

Figure 5. Comparison of barrier failure probabilities ranked by probability for p=2 and p→∞



In **Table *1*** the computed barrier probabilities for $\alpha = 0, 0.5, 1$ and $p = 2$ and $p \to \infty$ are presented. For each α and p the probabilities were imported to RiskSpectrum® PSA and the consequence analysis was reperformed for each set of probabilities. The computed frequency is presented at the bottom of the table.

Table 1 Barrier failure probabilities for $p = 2$ and $p \to \infty$ for $\alpha = 0, 0.5, 1$

	$p = 2$			$p \to \infty$		
Barrier	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
CCW-1	9,95E-04	7,51E-04	3,91E-04	9,95E-04	6,89E-04	3,59E-04
CCW-2	9,95E-04	7,51E-04	3,91E-04	9,95E-04	6,89E-04	3,58E-04
CPO-TK__-X	1,03E-03	3,62E-04	1,66E-05	1,03E-03	3,67E-04	2,13E-06
DG-1	1,15E-03	4,97E-03	4,51E-03	1,15E-03	1,19E-02	3,19E-03
DG-2	1,15E-03	4,97E-03	4,51E-03	1,15E-03	1,19E-02	3,18E-03
DPS-1	1,12E-03	1,97E-03	1,25E-04	1,12E-03	1,03E-03	5,78E-06
DWS-TK__-X	1,04E-03	1,64E-03	1,15E-02	1,04E-03	1,11E-03	1,18E-02
ECC-1	1,15E-03	5,55E-03	1,59E-03	1,15E-03	7,44E-03	4,06E-04
ECC-2	1,15E-03	5,55E-03	1,59E-03	1,15E-03	7,44E-03	4,06E-04
EFW-1	1,15E-03	1,10E-02	1,16E-01	1,15E-03	1,50E-02	2,40E-01
EFW-2	1,15E-03	1,10E-02	1,16E-01	1,15E-03	1,50E-02	2,39E-01
FEED&BLEED	1,12E-03	4,23E-03	2,86E-02	1,12E-03	3,10E-03	9,81E-02
GT	1,15E-03	2,37E-02	3,01E-01	1,15E-03	1,55E-02	1,68E-01
MFW	1,15E-03	1,21E-02	1,07E-01	1,15E-03	1,24E-02	5,15E-02
OFFSITE-POWER	2,74E-03	2,74E-03	2,74E-03	2,74E-03	2,74E-03	2,74E-03
RHR-1	1,15E-03	1,01E-02	5,79E-03	1,15E-03	1,92E-02	1,86E-03
RHR-2	1,15E-03	1,01E-02	5,79E-03	1,15E-03	1,92E-02	1,86E-03
SWS-1	9,95E-04	7,51E-04	3,91E-04	9,95E-04	6,89E-04	3,59E-04
SWS-2	9,95E-04	7,51E-04	3,91E-04	9,95E-04	6,89E-04	3,58E-04
F_{CD} in RS PSA	**1,000E-05**	**1,001E-5**	**9,99E-06**	**1,000E-05**	**9,998E-06**	**2,728E-6**

For the case $p \to \infty, \alpha = 1$ it is seen that the relative tolerance δ for the lexicographic minimization is too high, yielding a computed frequency that is 73% lower than the acceptance criteria 10^{-5}. It is interesting to note that the relationship between allocated reliability to a given system and the value of α is not a monotonic relationship. For example, while the feed & bleed barriers allocated reliability decreases (failure probability increases) when risk-spread is maximized, the residual heat removal system is allocated its lowest reliability (highest failure probability) at $\alpha = 0.5$, where risk-spread and reliability-spread is even.

4. CONCLUSION

A method for early-stage reliability allocation using Probabilistic Safety Assessment (PSA) cutset representations has been presented. By expressing the top-event frequency in log-sum-exp form through a logarithmic reparametrisation, the feasibility constraints become convex, allowing the allocation problem to be formulated within a convex optimization framework.

Three objective functions were proposed: minimization of total reliability, spreading of risk across cutsets, and spreading of reliability across barriers. Convex surrogate functions based on squared mean deviations were introduced for the latter two objectives, enabling systematic exploration of trade-offs between balanced risk profiles and balanced barrier reliabilities through Pareto optimization. An optimization strategy was then employed as a proof of concept to evaluate the results of a simplified PSA model.

The results demonstrate that the methodology produces structured families of feasible solutions satisfying the specified acceptance criteria while significantly altering both the fractional contribution profiles of cutsets and the resulting barrier reliability allocations. Increasing the risk-spreading objective was shown to flatten the cutset contribution distribution, redistributing risk away from dominant sequences. Increasing the norm order further emphasized suppression of the most contributing cutsets, though at the expense of increasingly uneven barrier reliability allocations.

The methodology can therefore be used to derive risk-informed reliability targets for systems or barriers during conceptual design. These targets satisfy specified PSA acceptance criteria while allowing plant designers to explore trade-offs between overall reliability effort, balanced risk profiles, and balanced reliability allocations prior to detailed system realization. Such allocations can serve as reliability targets when verifying the designs reliability with system-specific fault tree models as the design matures.

The presented method does not account for common cause failures between events and as such is not conservative in this regard. Explicit convex treatment of CCFs remains to be studied as it would allow convex optimization of more complex models. Extending the methodology to more complex PSA models would enable optimization of reliability targets for individual systems, groups of systems, or safety functions while treating the remaining portions of the design as fixed.

The treatment of uncertainties must be considered in the selection of the acceptance criteria being optimised for, for example through Monte-Carlo sampling i.e. repeating the allocation processes with different sampled values of initiating event frequencies and uncertainties associated with barrier events.

The optimization strategy relies on heuristics for the selection of norm order and the lexicographic tolerance parameter which influences the resulting Pareto-efficient solutions and the robustness of the method. Different optimization strategies should be explored within this convex optimization framework to improve the robustness of the results.

Despite these limitations, the results demonstrate that convex optimization provides a practical and computationally tractable framework for risk-informed reliability allocation directly from PSA cutset representations. The methodology therefore offers a systematic approach for establishing and comparing reliability targets during the early stages of system design.

ACKNOWLEDGEMENTS

The author would like to acknowledge the support of Blykalla, and Sergey Galushin for providing valuable discussions and feedback.

References

[1] S. Boyd and L. Vandenberghe. "*Convex Optimization*", Cambridge University Press, 2004, Cambridge

[2] S. Boyd, S.J. Kim, L. Vandenberghe and A. Hassibi. "*A tutorial on geometric programming*". Optimization and Engineering, volume 8, pp. 67–127, (2007).

[3] O. Grodzevich and O. Romanko. "*Normalization and other topics in multi-objective optimization*", Proceedings of the Fields-MITACS Industrial Problems Workshop (2006).