Traditional Markovian Modeling vs. Cumulative Damage Modeling

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Abstract: Recent work has advocated cumulative damage modeling (CDM) of passive component reliability in some contexts. Specifically, CDM may be useful in "Reliability Integrity Management" (RIM) (ASME Boiler and Pressure Vessel Code Section XI Division 2). CDM is an application of event-driven simulation performed to understand the interplay between component degradation mechanisms and various management strategies that may be implemented to manage the risk associated with passive component failure. One point of CDM is to model situations in which component functionality is lost only when a particular threshold of degradation is exceeded as a result of the cumulative effect of degradation mechanisms whose severity may fluctuate in time. Constant-failure-rate (e.g., Markovian) modeling has previously been applied to passive component reliability, but if threshold effects dominate in a particular context, there may be limitations to the applicability of a Markovian result in that context. The present paper compares traditional Markov modeling with CDM, with a view to (a) highlighting relative strengths and weaknesses of the two approaches, and (b) assessing what properties CDM needs to have – including what inputs are needed, and how to address uncertainty - to improve meaningfully on Markovian modelling. Probabilistic fracture mechanics (PFM) appears to be a special case of what we mean by CDM, and the existence of PFM work provides a useful way of benchmarking CDM approaches.

1. INTRODUCTION

The purposes of this paper are to illustrate and discuss pros and cons of cumulative damage modeling (CDM) performed to support Reliability and Integrity Management (RIM) [1]. RIM calls for a sophisticated approach to monitoring, nondestructive examination, and component renewal, and CDM may help to establish such an approach.

This paper is not an attempt to reinvent physics-of-failure modeling. It is intended to be a step towards harnessing ongoing developments in instrumentation and materials science to improve the capability to manage risks of passive component failures in novel operating environments.

Significant previous work modeling passive component failure has been done by Fleming and collaborators [2, 3,4,5,6] using Markov modeling. Fleming's Markov model was introduced to support the EPRI risk-informed inservice inspection (RI-ISI) program. The model was developed to evaluate the impact of changes to in-service inspection programs due to changes in the frequency and reliability of inspections for leaks and defects in piping systems. It was reviewed by Martz [5] and approved for use by the U.S. Nuclear Regulatory Commission for use in RI-ISI programs and later applied to 27 plants to implement RI-ISI. The model is currently employed in EPRI's effort to maintain a piping reliability data base for use in internal flooding PRA [3]. In this application it is used to evaluate integrity management strategies to reduce pipe rupture frequencies.

The present paper extends previous work on cumulative damage modeling [4-8] to illustrate the potential benefits of cumulative damage modeling in planning inspection and renewal. The model used in this work is loosely based on the traditional state-transition framework used in Markov modeling, but there are significant differences:

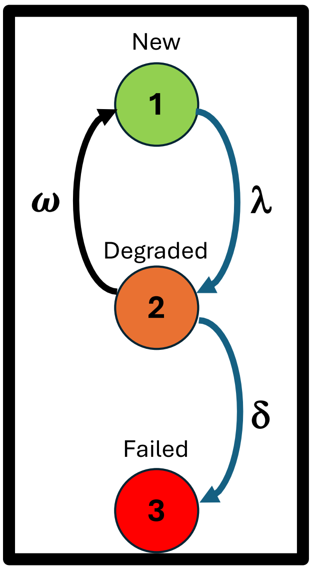
1. Each transition arc in the model can be either Markovian or threshold-driven. The threshold-driven case is aimed at modeling component state transitions explicitly in terms of environmental stressors that degrade the components.

2. In addition, the present model explicitly treats operator action, which can also be threshold-driven, and can be conditioned on whatever information about the plant state the staff is able to obtain during inspections ("perceived") or between inspections ("estimated"). The operator model can account for things like probability of failure to detect flaws, which can be conditioned explicitly on the current level of degradation.

2. MARKOV MODELING

A simple state-transition model of component reliability is illustrated in Figure 1.

Figure 1. Three-State Model Used in the present paper.



This model contains three states: New, Degraded, and Failed. The reliability of the subject component can be treated using traditional concepts of reliability theory. Within a Markovian treatment, the occupancies of states New (p_{New}), Degraded ($p_{Degraded}$), and Failed (p_{Failed}) change with time according to the following equations:

$$\frac{d}{dt} p_{New} = -\lambda p_{New} + \omega p_{Degraded}, \quad (1)$$

$$\frac{d}{dt} p_{Degraded} = \lambda p_{New} - \omega p_{Degraded} - \delta p_{Degraded}, \quad (2)$$

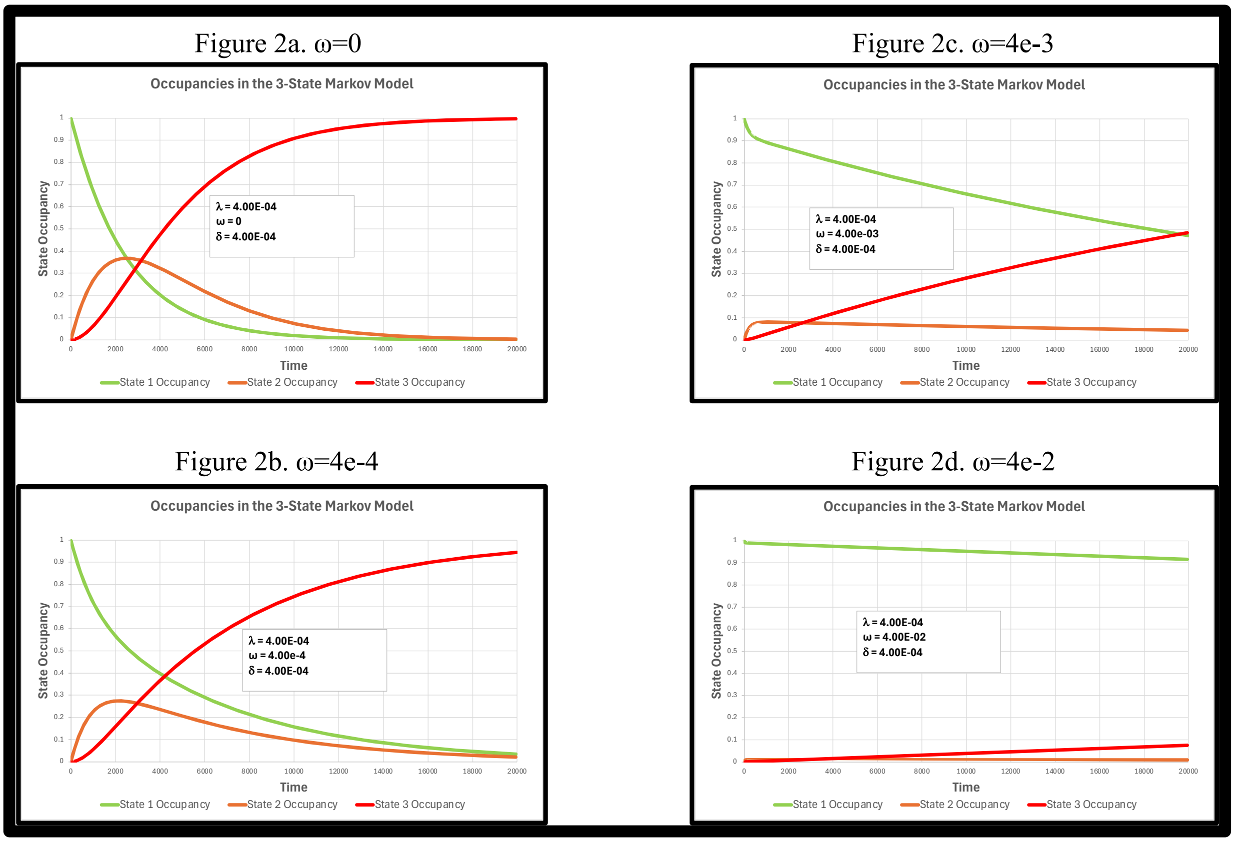
$$\frac{d}{dt} p_{Failed} = \delta p_{Degraded} \;, \quad (3)$$

with transition rates λ, ω, and δ. Together with the requirement that the sum of the p's must be unity, such a model can easily be solved in Excel. For specific rates λ, ω, and δ, and assuming that the component is in state New at t=0, the state occupancies evolve as illustrated in the graphs in Figure 2.

This model contains an important ingredient on which we will focus in the present discussion: the competition between the renewal arc (Arc 2, transition rate ω) and the failure arc (Arc 3, transition rate δ). One way to reduce the rate of transition to the failed state is to increase ω by formulating an approach to monitoring, inspection, and renewal such that if degradation (transition to the "Degraded" state) occurs, renewal occurs <u>on average</u> often enough to reduce the chance of transition to "Failed." Figs. 2a-2d show the results for varying values of ω. The value of ω can reflect not only frequency of surveillance, but also such considerations as probability of detection of degradation. In this particular model, there is no return from State 3 (it is an "absorbing state"), so for λ>0 and δ >0, State 3 occupancy always increases over time; but the *rate* of accumulation in State 3 is very significantly affected by ω.

Let us return to the phrase "on average." By definition, Markov models have no memory; at any particular time (refer to eqs. 1-3 above), the rate of transition of a particular arc depends simply on the assigned rate parameter and on the current occupancy of the state from which the arc originates. Thus, within a Markov model, it is easy to show that increasing ω reduces failure; but the details of when, where, and how inspection might occur are mostly lost.

Figure 2. Markov model calculations for a range of values of ω.



Consider a simulation-based approach that is capable of implementing the cumulative-damage concept discussed previously in [7-11]. Cumulative-damage modeling and Markovian modeling can both be seen as members of a broad spectrum of model types based on a state transition diagram. In Markov modeling, transitions occur at an average rate; the state occupancies plotted reflect an average over time histories in which transition times are random samples from the implied failure-time distribution. In cumulative damage modeling, the transitions occur when some damage threshold is crossed as a result of degradation induced by the operating environment (which needs to be modeled). The two models are distinct, but computationally, the Markov model can be viewed as a special case of the cumulative-damage model, in which "damage" is taken to be proportional to time, and the arc thresholds are sampled from the failure-time distributions implied by the Markov rates.* As an example, the plots in Figure 2 were obtained from the Markov model, while the plots in Figure 3 below were obtained from a cumulative damage model according to the above description, using the parameter values shown above in Figure 2c. The cumulative-damage plots show some jitter owing to the finiteness of the number of histories, but otherwise match Figure 2c.

* In the 3-state model shown, the component may revisit State 1 because of renewal. At that point, it is necessary to resample the threshold of arc 1 in order to obtain Markovian behavior. Put differently: the notional identity of the component being modelled is lost when it re-enters State 1.

Figure 4 shows results in which Arc 2 (the renewal arc) is as it was above, but the other two arcs are no longer Markovian: they are assigned sharp damage thresholds, such that transitions occur when certain levels of damage are exceeded. In this calculation, the threshold values are tightly centered on a specific value, but if the rate of damage accumulation varies from one history to another (which we will examine below), the timing of these transitions will vary from one history to another.

Figure 3. Simulation Results for the Markov Model Shown in Figure 2c.
Figure 3a: 1000 Histories

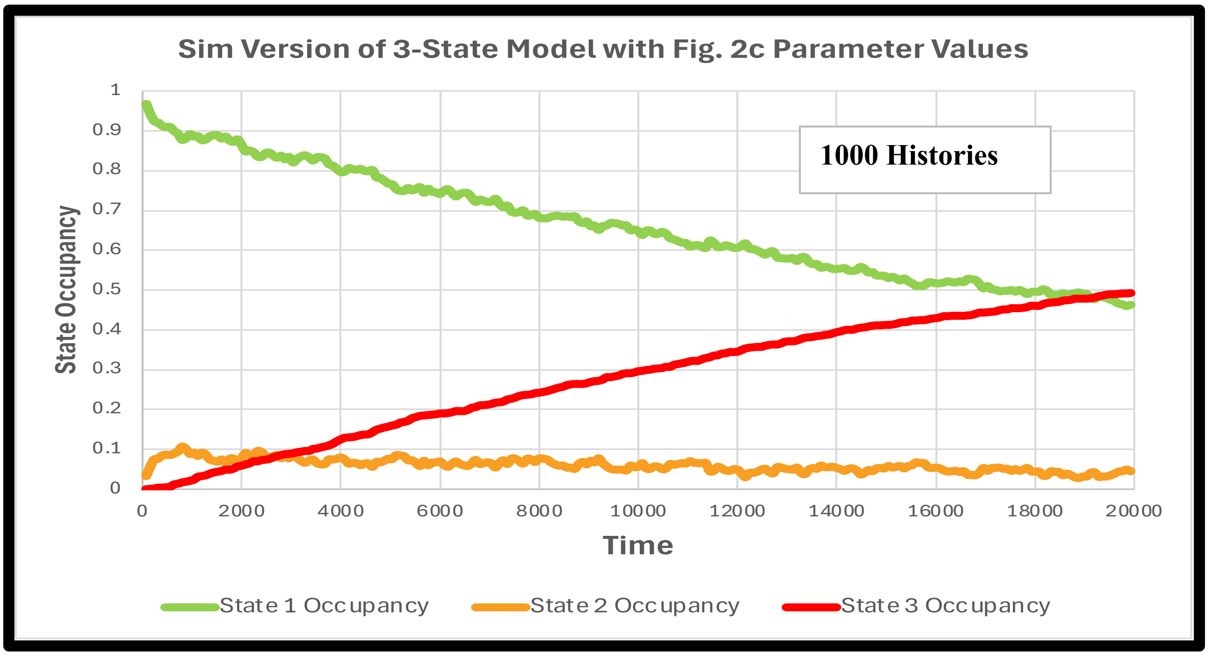
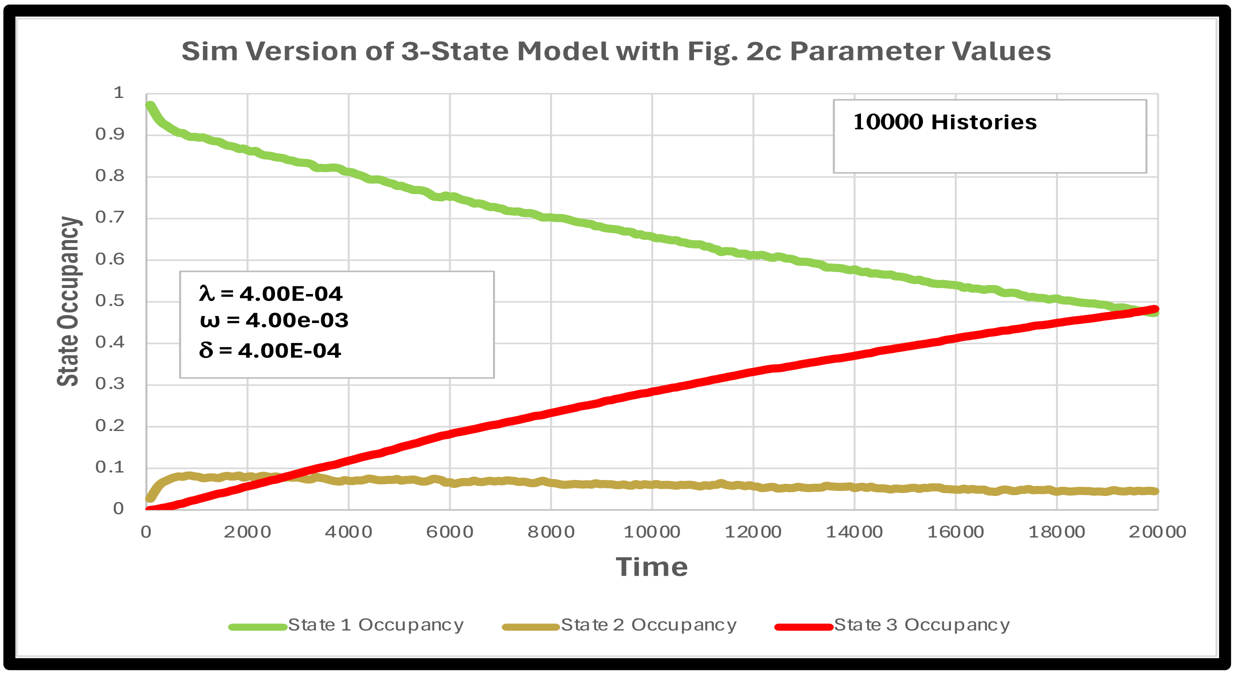


Figure 3b: 10000 Histories



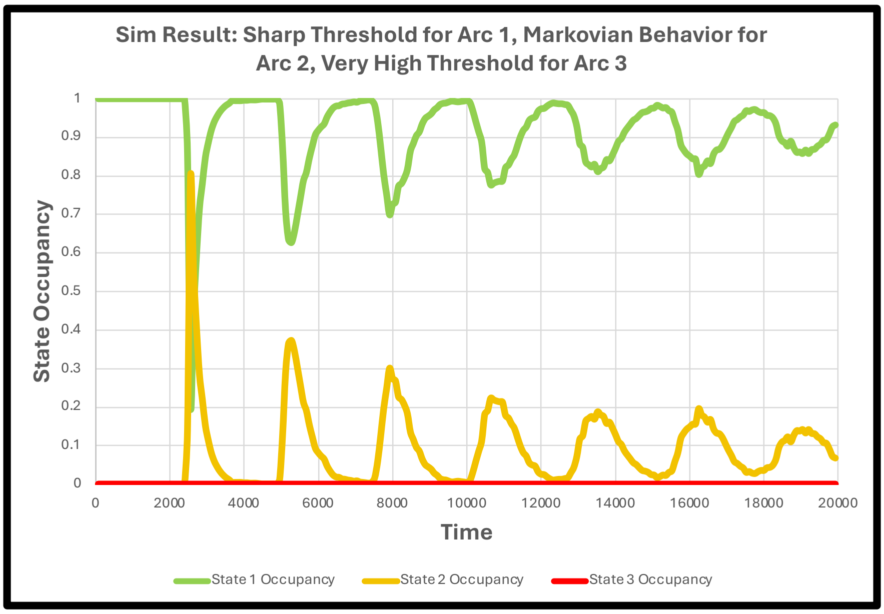
In Figure 4,

- Arc 1 represents a model in which damage is proportional to time (as in earlier figures) but instead of being sampled randomly as in a Markovian treatment, the threshold is sampled from a Gaussian distribution narrowly centered at a level corresponding to Figure 3's *average* transition rate for Arc 1. In the Markovian treatment of Figure 3, State 1 lost occupancy both before and after that average MTBF, but in Figure 4, all histories depart State 1 at around the same time, because that is when accumulated damage crosses the threshold.

- Arc 3 is similar to Arc 1 except that its threshold has been set at a very high level so that State 3 is never entered. This was done for illustration in order to make the "ringing" effect more obvious. The occupancies of States 1 and 2 mirror each other.

In Figure 4, the model eventually loses the synchrony of the departures from State 1 because the Markov renewal arc does not restore all systems at the same time; failures of the renewed components are more spread out in time as a result.

Figure 4. Sharp thresholds have been assigned to Arcs 1 and 3.



Although the notion of "cumulative damage" was introduced earlier, we have so far treated only a simple version of it: one in which accumulated damage has been modeled as being simply proportional to elapsed time. However, degradation mechanisms may fluctuate in time and in severity, which will tend to de-synchronize component failures in different time histories, even if they have similar thresholds. We will return to this point below.

3. FOCUS ON INSPECTION AND RENEWAL

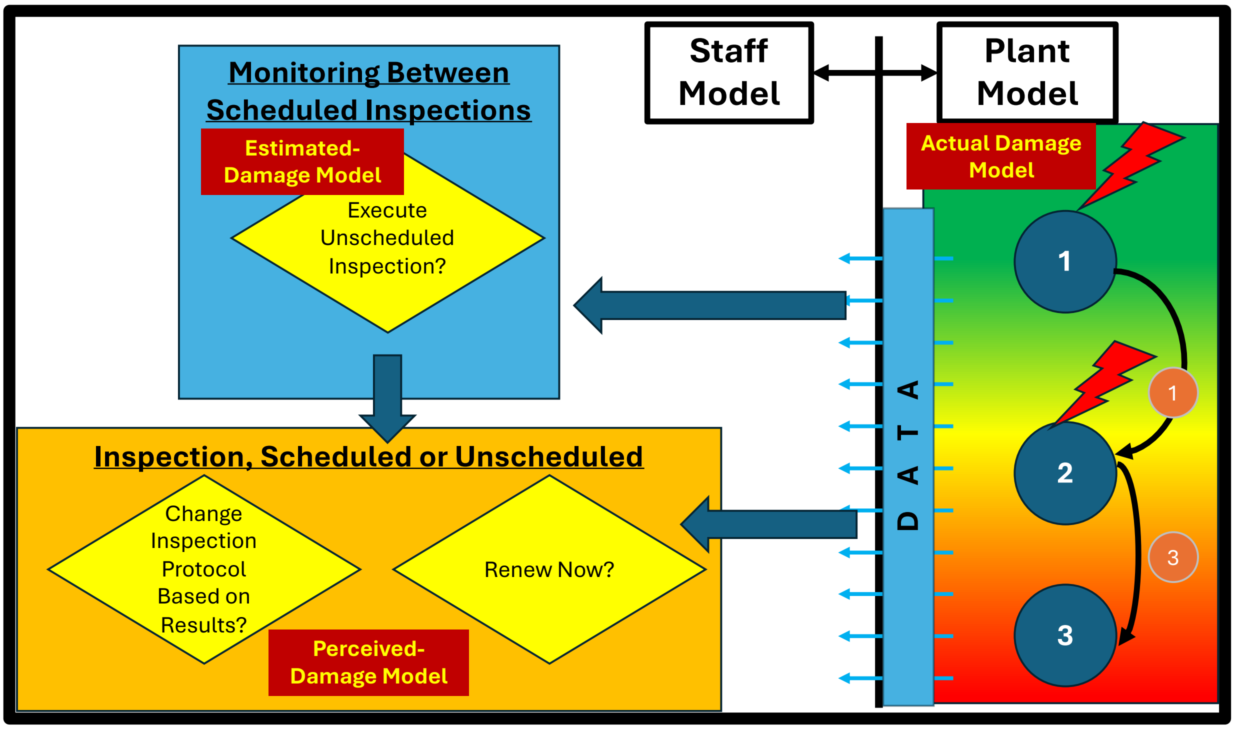
In a typical Markov model, the repair arc addresses inspection and renewal in an average sense. As noted earlier, such a model can qualitatively show the benefits of decreasing the inspection interval. But the present model has been developed with a view to modeling management of risks associated with component degradation in more detail, including being able to evaluate timing and even the content of inspections. To apply RIM, one needs to establish whether reliability targets are satisfied, based on current observations, and the present development is intended to be one step towards being able to do that.

Figure 5 shows the scheme used here. On the right, we see the state transition model, without the renewal arc. Without that arc, everything on the right represents physical phenomena: component degradation driven by plant-imposed stressors, possibly episodic, possibly steady-state.

On the left, we see two additional models that are meant to support more detailed modeling of inspection and repair processes:

- The perceived-damage model is used for two decisions: whether to renew, and whether the current inspection / renewal protocol needs to be modified. (For example, if degradation is occurring more or less rapidly than foreseen, one might either shorten or extend the interval between inspections, based on current observations.). Conceptually, if observations and understanding were always perfect, then "perceived" damage would always equal "actual" damage. But the present intention is to allow for imperfect observation and analysis, including factors such as instrument uncertainty and probability of failure to detect.
- The estimated-damage model is applied between inspections to decide whether or when to inspect. If inspection is done on a fixed schedule, the model is not used; if monitoring occurs between inspections, then the state of knowledge regarding component degradation is improved to some extent, and the estimated-damage model represents the staff's present understanding of component degradation, given knowledge of past history and the results of current monitoring activities.

Figure 5: Coupled Models



For purposes of comparison, the general benefits of the traditional inspection / renewal scheme are illustrated in Figure 6. In Figure 6a, the repair arc used earlier has been suppressed (it has been assigned an extremely low rate). With no inspection or repair, all time histories are failed rather early in the run. In Figure 6b, episodic stressors have been added.[†] As in [7-11], they are modeled as if they are initiating events that cause significant but momentary stress: they are assumed to be Poisson-distributed. The evolution of state occupancies is more chaotic as a result. In Figure 6b, we have retained the relatively high repair rate presumed in earlier figures, but the occupancy of State 3 is seen to grow over time; nearly 10% of the histories are failed over the time interval presented.

If degradation occurred only at a constant rate, and one knew that rate accurately enough, then one could simply plan renewal to occur just before failure. But the chaos in Figure 6b suggests that unless damage accumulation is a simple function of time, a simple approach to scheduling inspection must be either very conservative or not particularly effective. In Figure 7, inspection is not modeled in a Markovian repair arc; instead, it occurs when damage is estimated to have crossed a designated

[†] Episodic stressors are sampled differently in different time histories.

threshold, and renewal then occurs as well. In this plot, only about 1% of the histories have failed over the time window analyzed (nearly too small to see on this plot).

For purposes of illustration, the "estimated damage" model is the same as the actual damage model, but the threshold associated with it is lower than the threshold associated with failure. This is tantamount to evaluating the "value of [near-]perfect information," a point to which we will return later.

Figure 6. Comparison of results without, and with, Markovian renewal arc.

Figure 6a: No renewal.

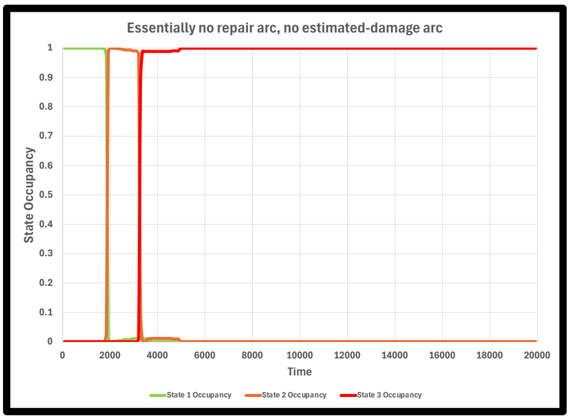


Figure 6b: Markovian renewal Arc

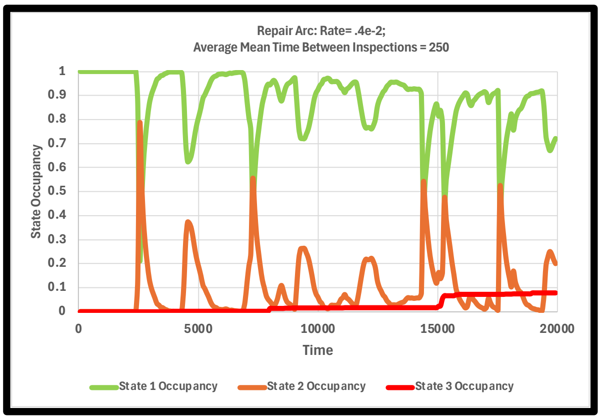
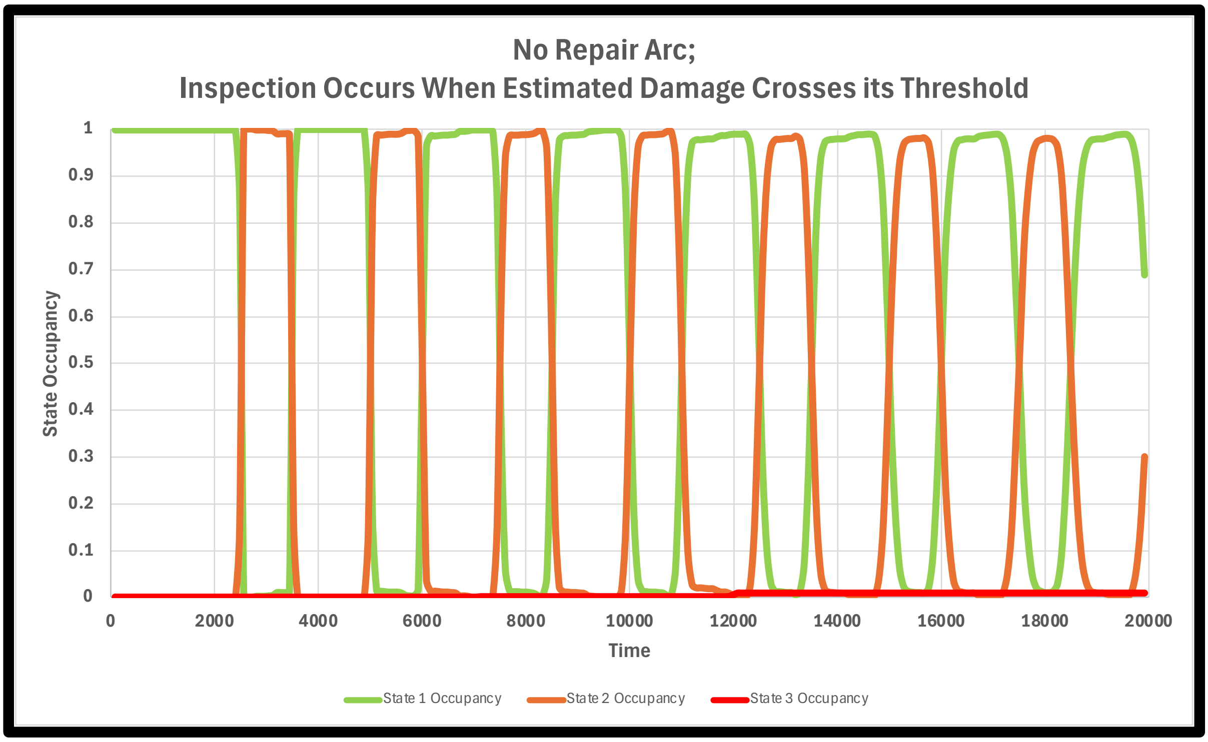


Figure 7. Still no Repair Arc, but
Inspection Occurs when Estimated Damage Crosses a Designated Threshold Value.



Besides noting the reliability performance of the Figure 7 scheme, we need to appreciate that there are many fewer inspections implied in Figure 7 than there were in Figure 4b. In earlier plots, one saw the action of the Markovian repair arc when the system is in State 2, but the plots do not explicitly show all the instances of inspection that occur when the system is in State 1. For the value of ω used, inspection occurred about every 250 time units on average, so inspection was occurring about 80 times in each of those plots; but if the system was in State 1 at the time, no transition occurred, and nothing is visible on the plots. In Figure 7, inspection occurs ONLY in State 2 by construction: inspection occurs only when ongoing monitoring indicates that it is required. If we count the downward steps in the State 2 plot, there are many fewer than 80 (but note that some of them are barely visible hiccups in State 1 occupancy). In short: we are getting much better reliability performance for many fewer inspections. Of course, this result follows because we have assumed that we can know enough about the component's status to proceed in this way: we know when episodic stressors have occurred, and we know how fast the steady-state contribution is accumulating.

4. CONCLUSION

This paper has presented a state-transition model of component reliability. The present application:

- Is simulation-based;
- Treats state transitions as occurring when time-history-specific damage thresholds are crossed, rather than basing transitions on quantified transition rates. (However, Markovian modeling is shown to be a limiting special case of this class of models.)
- Separates out the modeling of inspection and renewal policy.

One reason to consider such an approach is that Reliability and Integrity Management (RIM; see [1]) calls for demonstrating *ongoing* satisfaction of explicit reliability targets for SSCs within its scope; in principle, this calls for being able to make a statement about component reliability, based on current observations. This calls for closely coordinating the monitoring, inspection, and renewal of components. In some applications, especially certain novel operating environments, some components may need renewal well before the end of plant life. It will be important to time renewals so as to maintain the targeted reliability without unnecessary loss of facility production.

For purposes of exposition, the present treatment has highlighted some uncertainties while suppressing others. For example, the randomness of the timing of episodic stressors has been emphasized, while the variability of damage thresholds has been presumed to be small, and possible variability in the effects of episodic stressors has not been displayed. The threshold concept has long been implicit in the concept of Weibull distributions, but for purposes of illustration, the thresholds have been modeled here as being very tightly clustered, corresponding to a very large value of a Weibull shape parameter. It is seen here (and was stressed in [7-11]) that even if thresholds are tightly clustered around particular values, temporal variability in the stressors can spread out failures in *time*. In real applications, there will be significant variabilities and significant epistemic uncertainties that have not been illustrated here, but in other contexts, much work has gone into dealing with them, and much of that work will be straightforwardly applicable in this context.

Earlier, we mentioned "value of information" in connection with Figure 7. The general idea appears to go back to the 1960's [12]: in decision-making under uncertainty, it may be possible to improve the expected utility of decisions significantly by reducing uncertainty, and the benefits of obtaining the necessary information are quantifiable. In the present paper, the "estimated" damage model (with no presumed uncertainty) was shown to lead to significantly improved outcomes (better reliability with less effort). In other words: if monitoring can tell us enough, we can do a very good job of timing our inspections and renewals. Although many treatments focus on the value of "perfect" information, the general idea also applies to assessing the value of imperfect information.

Previous work has considered some of the essential elements of the present modelling approach, but with a different emphasis. The point of departure of Refs. [13-14] was the well-known 4-state Markov model used by Fleming in [2-4] (as was also the case in [7-11]), but [13] and [14] go on to treat the physics of failure in more detail than had been done previously, and thereby improve on some of the simplifications that are inherent in pure Markov models. [13] and [14] showed that more detailed modelling makes an appreciable difference in the results. For purposes of exposition, the present paper has used a simpler state transition model, but the approach has been designed to accommodate the kind of failure modelling done in [13-14], including their recognition of what we have called "episodic stressors," while additionally modelling inspection and renewal in order to support RIM applications.

Acknowledgements

R. Youngblood acknowledges numerous useful discussions with collaborators in previous works cited [7-11]. K. Fleming acknowledges the efforts by EPRI to improve the reliability of passive systems as most recently reflected in [4].

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