Bayesian games for optimal cybersecurity investment with incomplete information on the attacker

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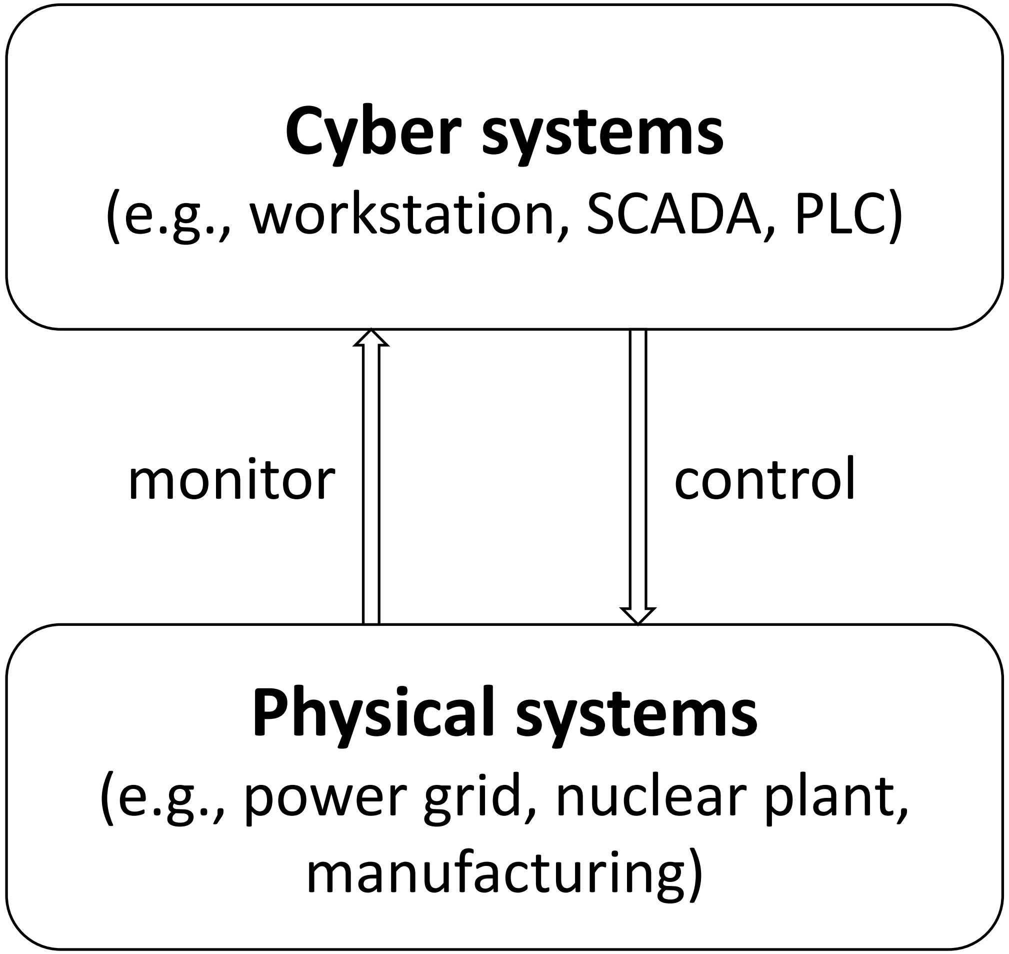
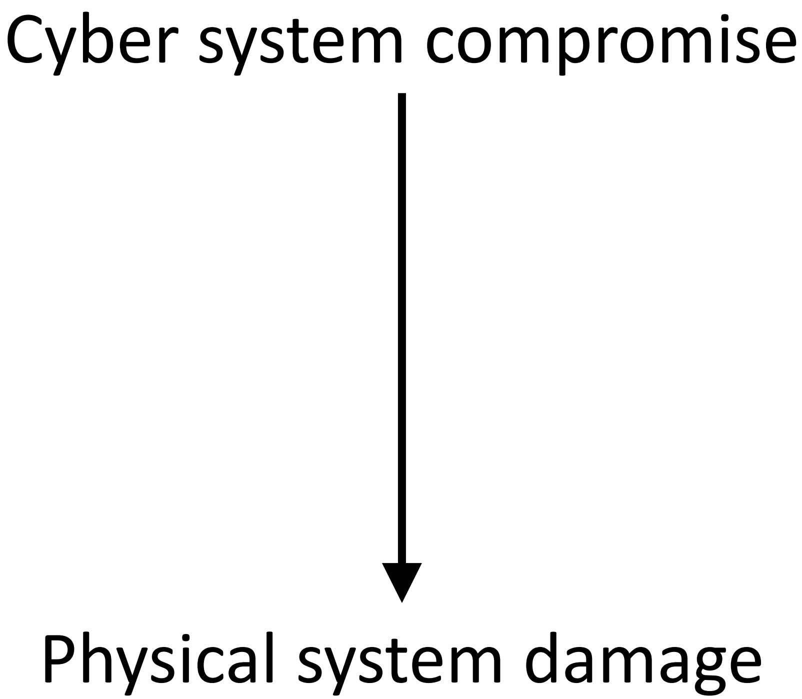
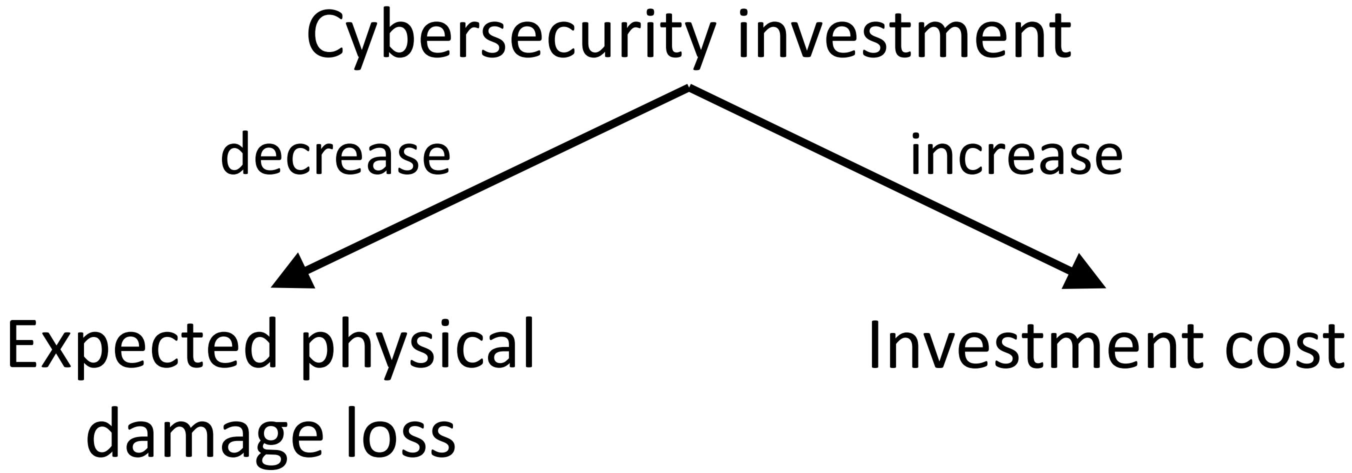
Background


Cyber systems
(e.g., workstation, SCADA, PLC)
monitor
control
Physical systems
(e.g., power grid, nuclear plant, manufacturing)

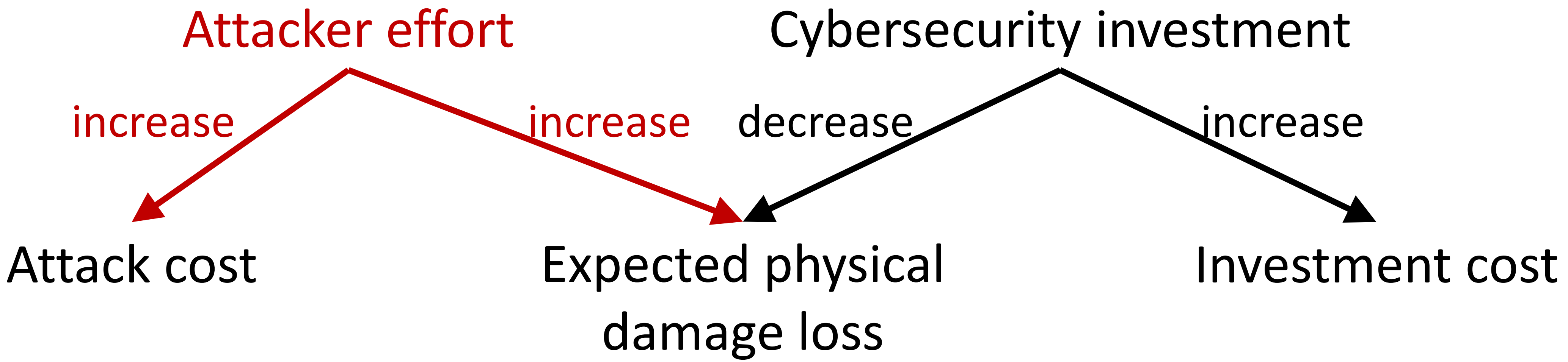

Cyber system compromise
Physical system damage

Single-agent decision-making



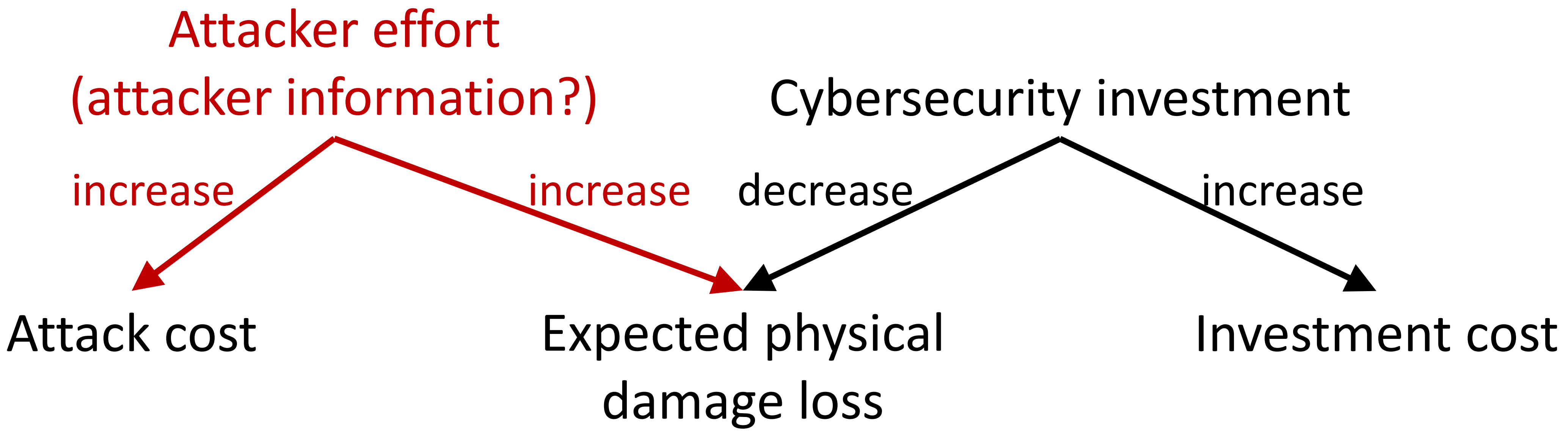
Optimal cybersecurity investment: the level of investment that achieves the **minimum sum** of **physical damage loss** and **investment cost**.

Multi-agent decision-making



Optimal cybersecurity investment: the level of investment that achieves the **minimum sum** of **physical damage loss** and **investment cost** considering **the effort of the attacker**.

Multi-agent decision-making with incomplete information



Optimal cybersecurity investment: the level of investment that achieves the **minimum sum** of **physical damage loss** and **investment cost** considering **the effort of the attacker** and with **incomplete information on the attacker**.

Problem formalization

- Problem
  - A piece of cyber equipment: if compromised, will incur physical loss of C
  - Players: defender – 1; attacker – 2
  - Defender cybersecurity investment: $a_1 \in [0, +\infty)$
  - Attacker type: $\theta_2 \in \Theta_2$
  - Attacker type distribution: $p(\theta_2)$
  - Attacker attack effort: $a_2 = \sigma_2(\theta_2) \in [0, +\infty)$
  - Cyber equipment vulnerability: $v(a_1, \sigma_2(\theta_2), \theta_2)$

Problem formalization (cont.)

- Objective
  - Defender:

Potential physical loss

Vulnerability

$$\max u_1(a_1, \sigma_2) = \sum_{\theta_2 \in \Theta_2} [-C \cdot p(\theta_2) \cdot v(a_1, \sigma_2(\theta_2), \theta_2)] - a_1$$

Defender utility

Attacker types

Attacker type probability

Cybersecurity investment

  - Attacker:

Potential physical loss

$$\max u_2(a_1, \sigma_2(\theta_2), \theta_2) = C \cdot v(a_1, \sigma_2(\theta_2), \theta_2) - \sigma_2(\theta_2)$$

Attacker utility

Vulnerability

Attack effort

Bayesian games for cybersecurity investment

- What we just described is actually a Bayesian game
  - The defender has incomplete information on the attacker
  - This incomplete information is described by the various types of attacker and the probability distribution over the types
- We can solve the game using the solution concept of Bayesian Nash equilibrium
  - i.e., obtain the Bayesian Nash equilibrium (a_1^*, σ_2^*) such that
  - For the defender:

$$u_1(a_1^*, \sigma_2^*) \geq u_1(a_1, \sigma_2^*), \forall a_1 \in [0, +\infty)$$

  - For the attacker of any type:

$$u_2(a_1^*, \sigma_2^*(\theta_2), \theta_2) \geq u_2(a_1^*, \sigma_2(\theta_2), \theta_2), \forall \theta_2 \in \Theta_2, \forall \sigma_2(\theta_2) \in [0, +\infty)$$

Obtain the Bayesian Nash equilibrium

- Obtain and solve the following system of partial differential equations

$$\frac{\partial u_1(a_1, \sigma_2)}{\partial a_1} = \frac{\partial\left[\sum_{\theta_2 \in \Theta_2}[-C \cdot p(\theta_2) \cdot v(a_1, \sigma_2(\theta_2), \theta_2)] - a_1\right]}{\partial a_1} = 0$$

$$\frac{\partial u_2(a_1, \sigma_2(\theta_2), \theta_2)}{\partial \sigma_2(\theta_2)} = \frac{\partial[C \cdot v(a_1, \sigma_2(\theta_2), \theta_2) - \sigma_2(\theta_2)]}{\partial \sigma_2(\theta_2)} = 0, \forall \theta_2 \in \Theta_2$$

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Numerical case study

- Two types of attacker:
  - One with high capability ($\theta_2 = H$)
  - The other with low capability ($\theta_2 = L$)
- Cyber equipment vulnerability
  - For $\theta_2 = H$

$$v(a_1, \sigma_2(H), H) = \frac{\sigma_2(H)}{\alpha_H(a_1 + \sigma_2(H) + \beta)}$$

$\alpha_H \geq 1$ and $\beta > 0$

  - For $\theta_2 = L$

$$v(a_1, \sigma_2(L), L) = \frac{\sigma_2(L)}{\alpha_L(a_1 + \sigma_2(L) + \beta)}$$

$\alpha_L > 1$ and $\alpha_L > \alpha_H$

Numerical case study (cont.)

- Additional information about the problem:

Parameters (unit)	Parameter description	Nominal Value	Value Range
C (in USD)	Potential physical loss	1000	[0, 2000]
$p(H)$ (unitless)	Belief in $\theta_2 = H$	0.6	[0, 1]
α_H (unitless)	Parameter defining vulnerability for $\theta_2 = H$	5	[1, 10)
α_L (unitless)	Parameter defining vulnerability for $\theta_2 = L$	10	(5, 20]
β (in USD)	Parameter defining vulnerability	5	[1, 10]

- For each parameter setting, we can obtain the corresponding (a_1^*, σ_2^*)

Results for the setting with nominal parameter values

Parameters (unit)	Parameter description	Nominal Value	Value Range
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$p(H)$ (unitless)	Belief in $\theta_2 = H$	0.6	[0, 1]
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β (in USD)	Parameter defining vulnerability	5	[1, 10]

- Bayesian Nash equilibrium

$$(a_1^* = 33.97\ USD, \sigma_2^*(H) = 49.31\ USD, \sigma_2^*(L) = 23.46\ USD)$$

- Utility
  - Defender: $u_1(a_1^*, \sigma_2^*) = -116.03\ USD$
  - Attacker of type H: $u_2(a_1^*, \sigma_2^*(H), H) = 62.40\ USD$
  - Attacker of type L: $u_2(a_1^*, \sigma_2^*(L), L) = 14.12\ USD$

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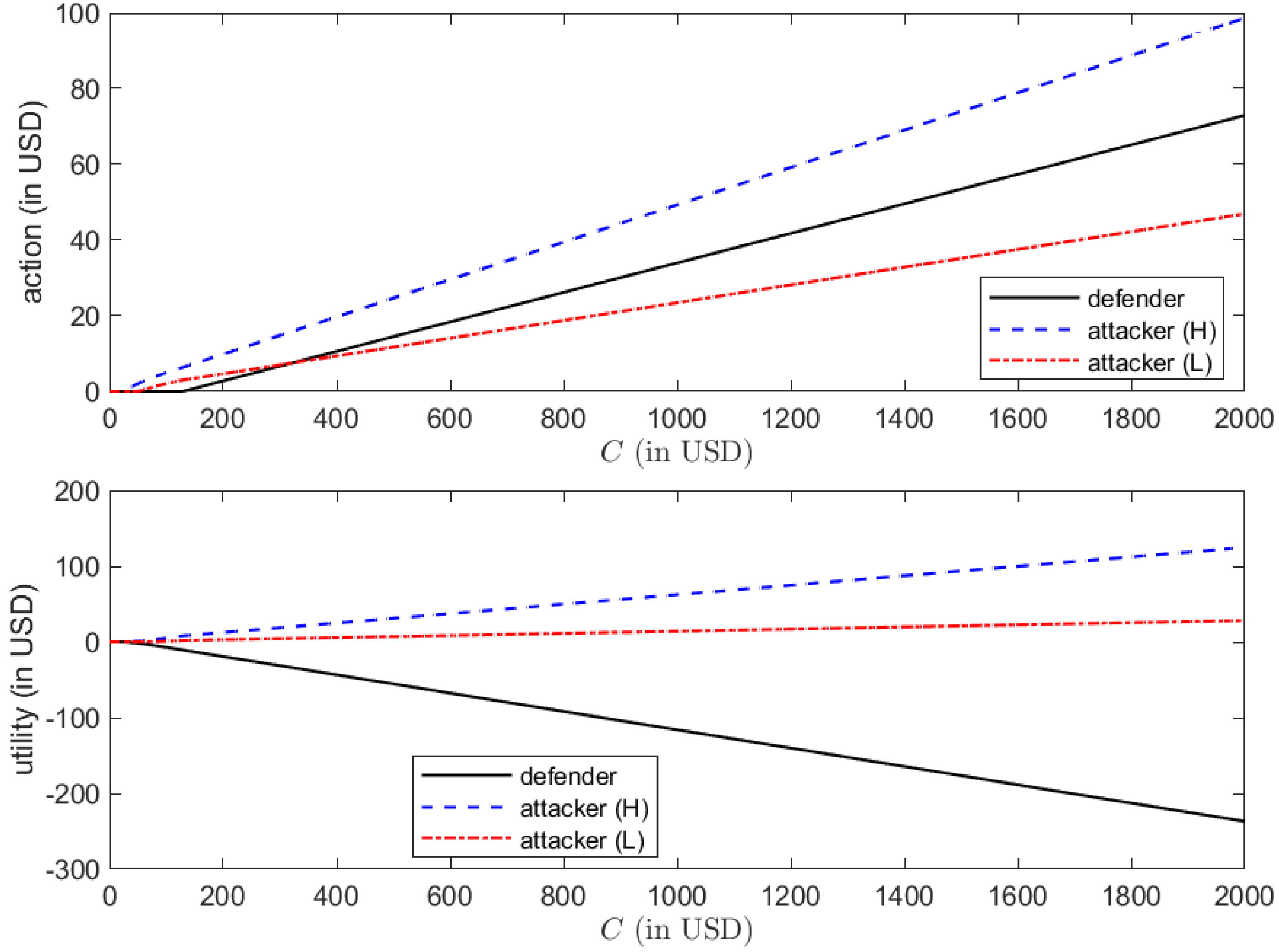
Sensitivity of results to certain parameter values

- The effect of C on the outcome



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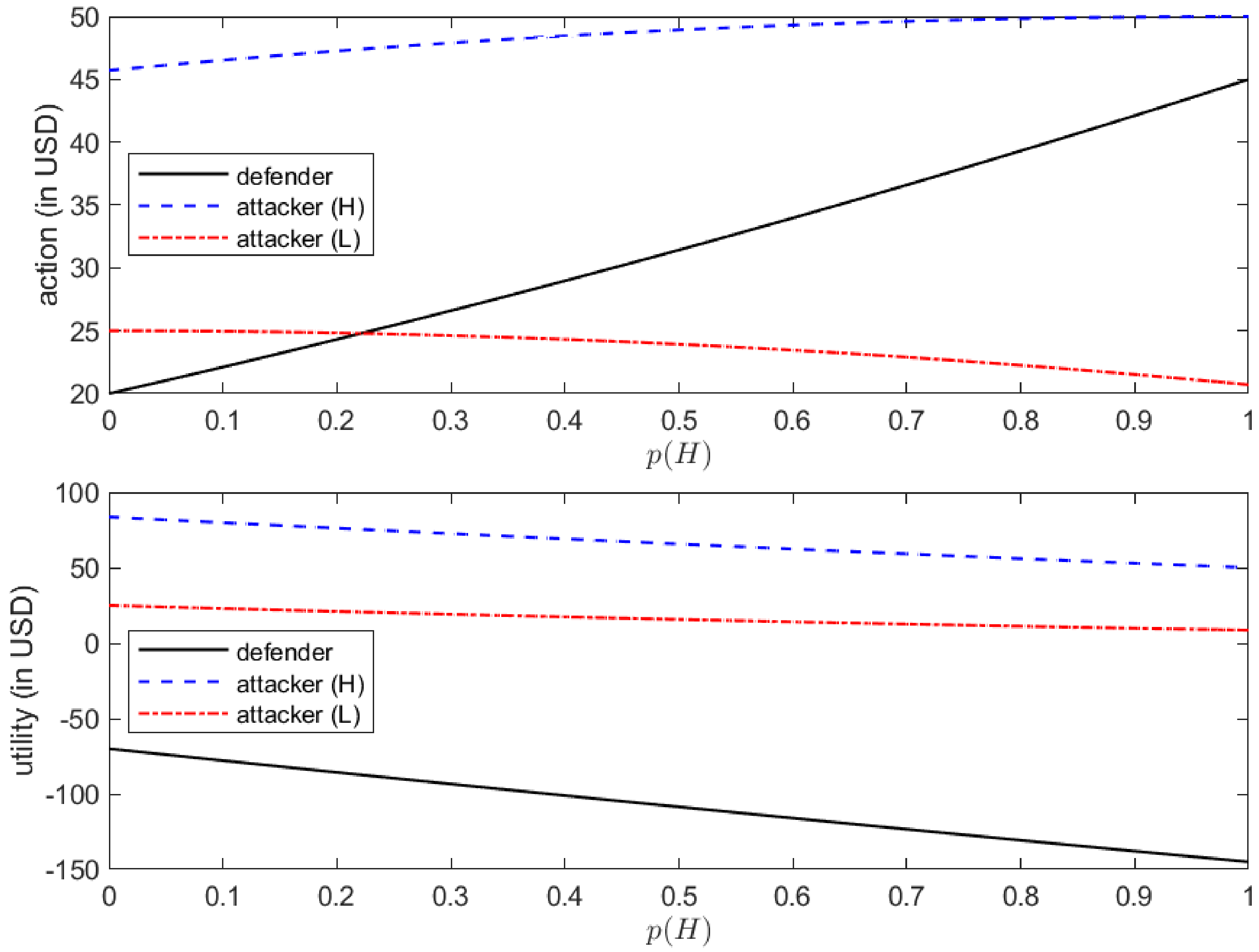
Sensitivity of results to certain parameter values (cont.)

- The effect of $p(H)$ on the outcome



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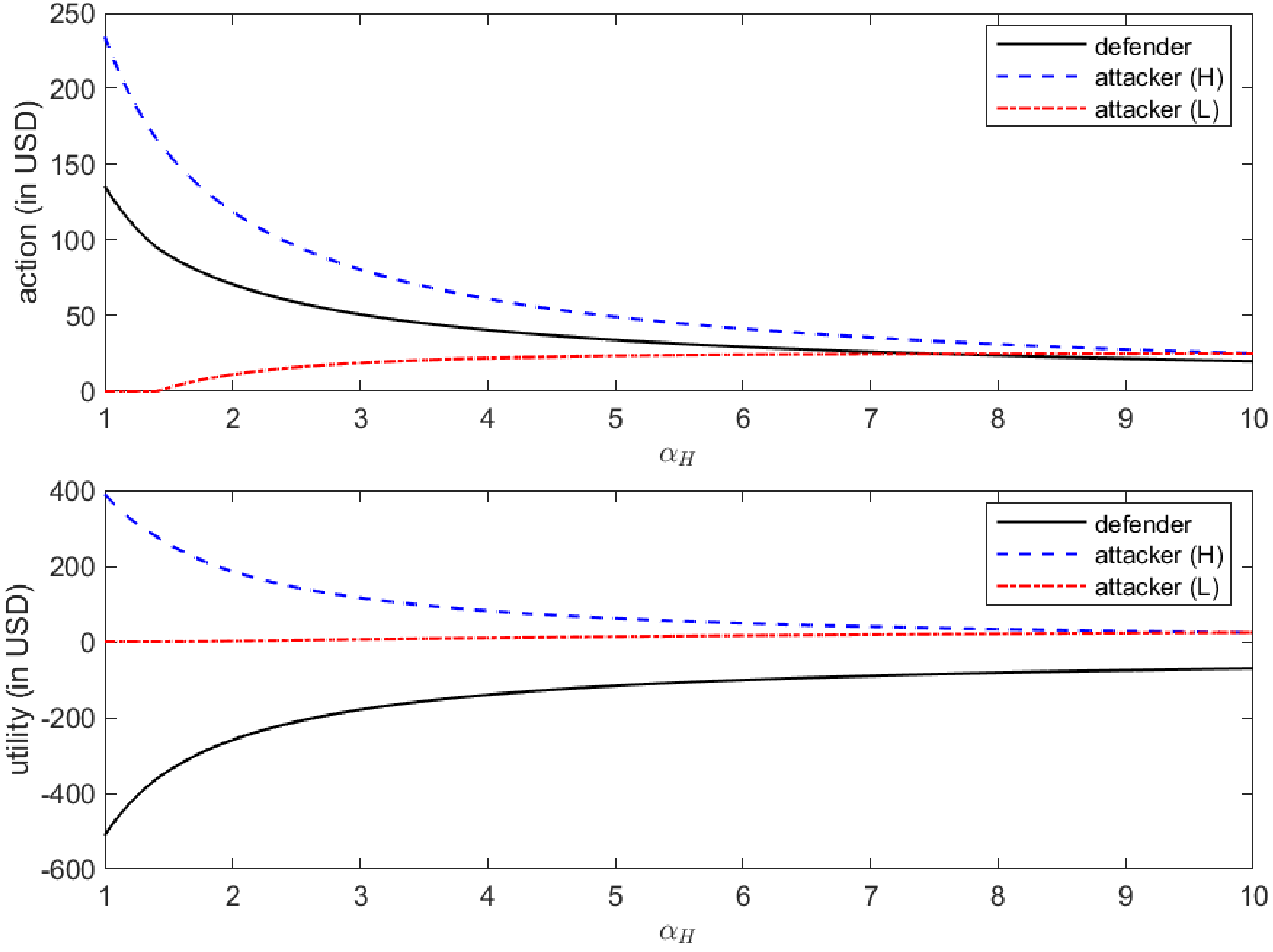
Sensitivity of results to certain parameter values (cont.)

- The effect of α_H on the outcome



Summary and future work

- Cybersecurity investment
  - Defender decision-making while considering the level of attacker effort
  - Incomplete information on the attacker
  - Bayesian games for modeling and solving the cybersecurity investment problem
- Numerical example
  - The outcome for a setting with nominal parameter values
  - Sensitivity of the outcome to various model parameters
- Future work
  - Multiple defenders, multiple attackers
  - Determination of model parameters

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Thank you!