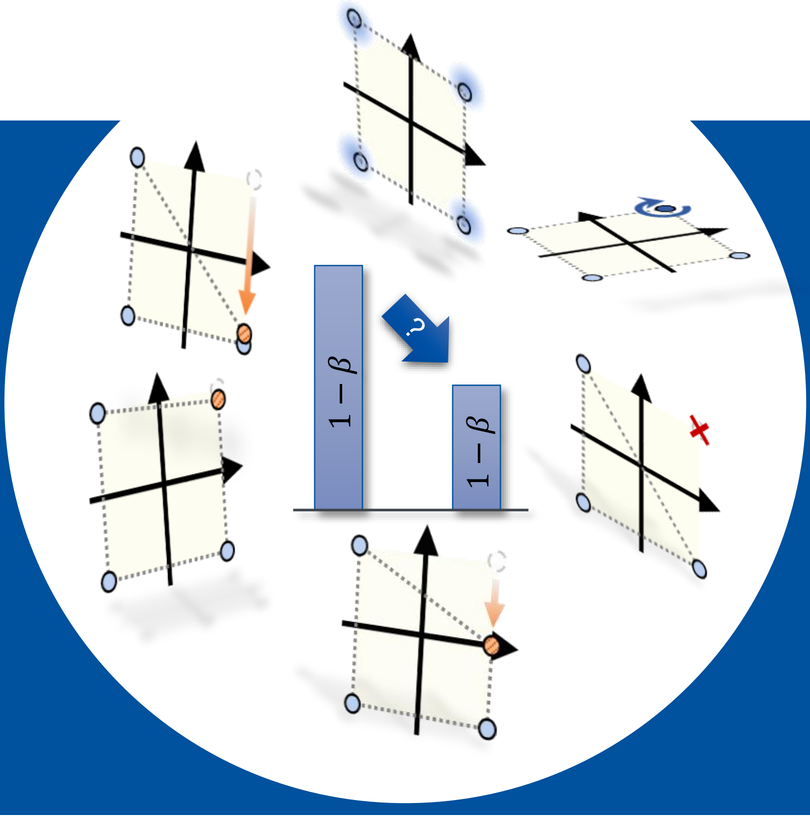
University of Stuttgart
Institute of Machine Components
Reliability Department

Probabilistic Safety Assessment and Management
PSAM 16, June 26–July 1, 2022, Honolulu, Hawaii

# Non-orthogonality in test design

## Practical relevance of the theoretical concept in terms of regression quality and test plan efficiency




Philipp Mell, M.Sc.

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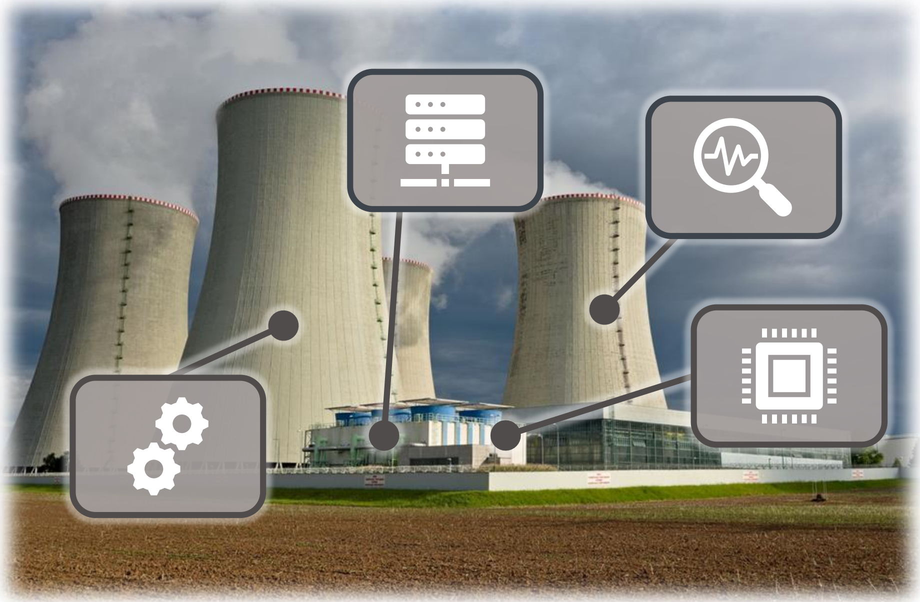
# Motivation

## Typical DoE scenario

- Approval tests or performance sensivity assessment for arbitrary components or (sub-)systems
- Case: „performance“ $y$ depends on several influencing factors; e.g.:
  1. temperature
  2. voltage





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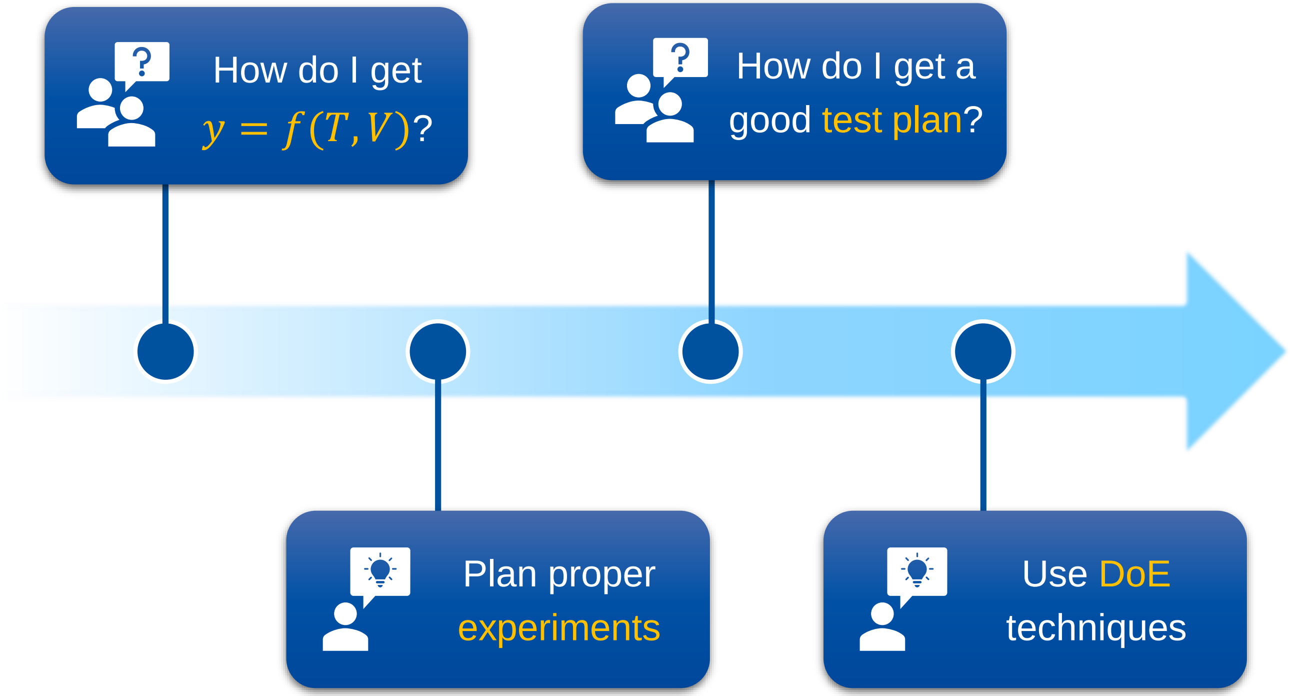
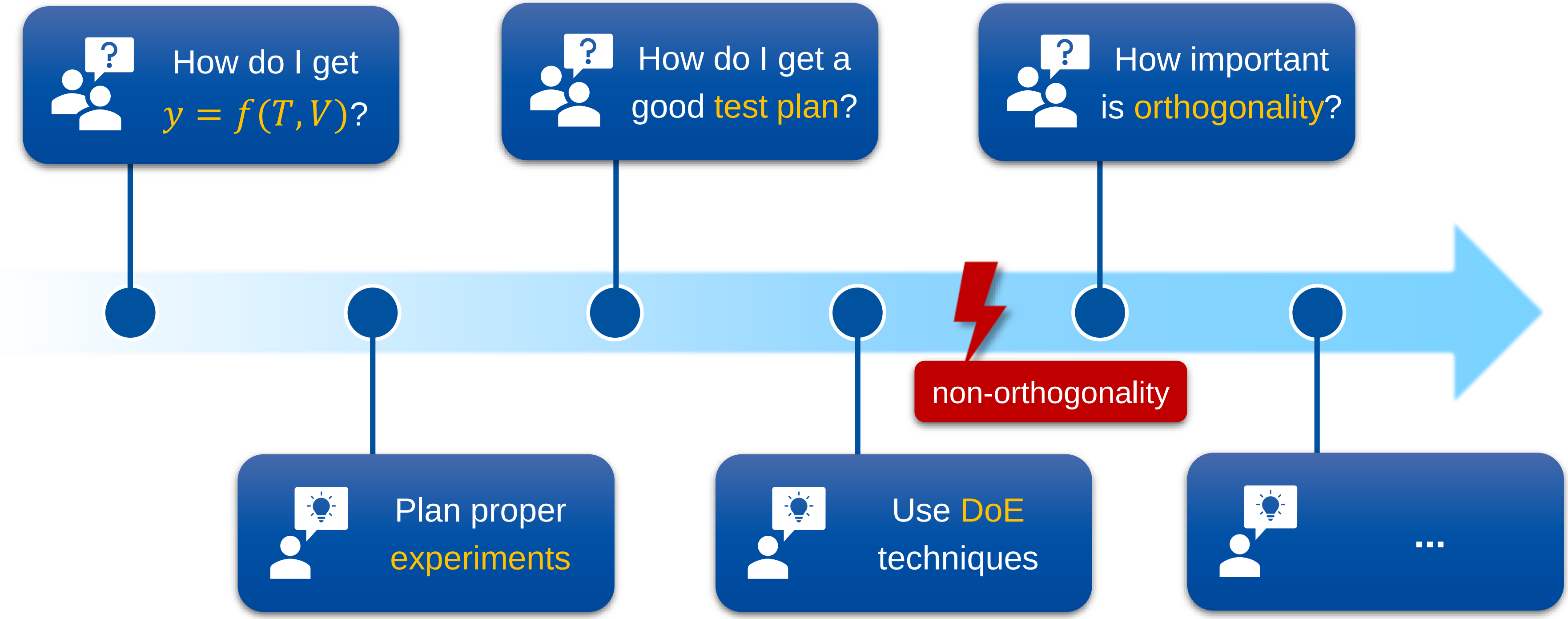
# Motivation

## Typical DoE scenario



# Motivation

## Typical DoE scenario

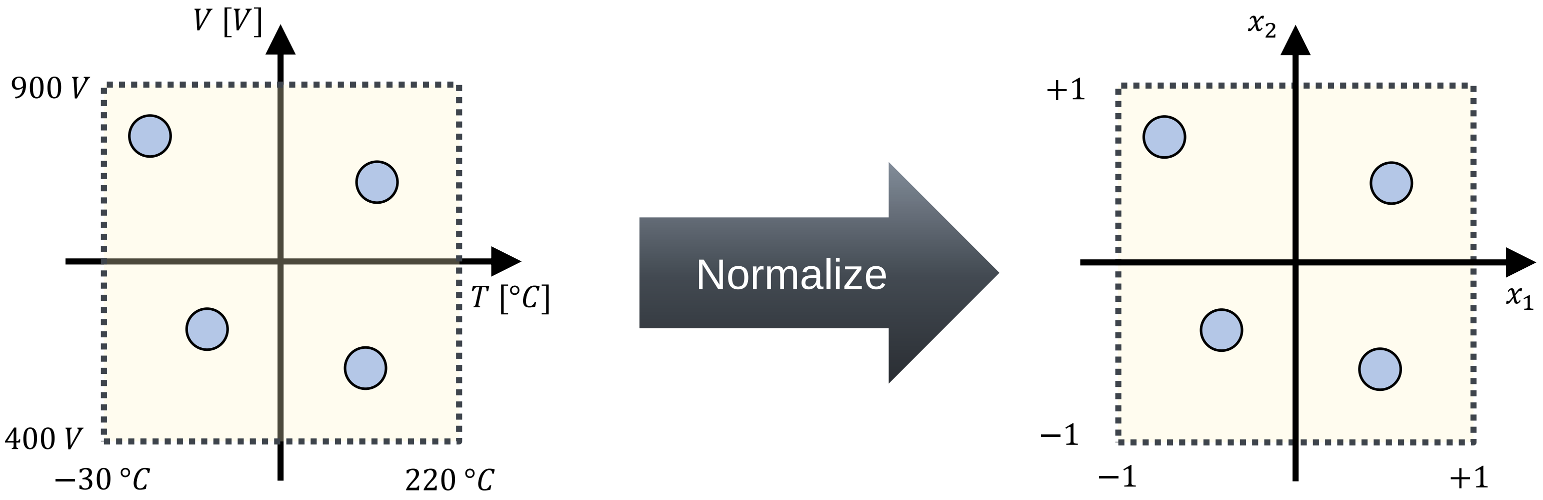


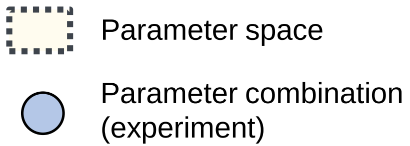
# Overview

- Problem definition
- Theory: orthogonality as a DoE principle
- Application: sources of non-orthogonality
- Simulation study
- Summary & future work

# Problem definition

## Parameter space



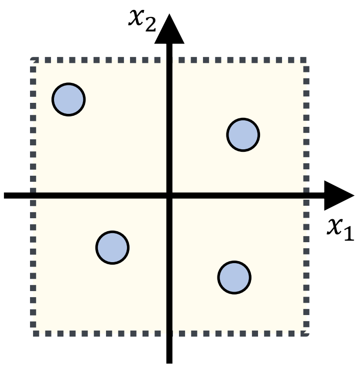




| | Temperature | Voltage |
|---|---|---|
| Minimum | $T_{min} = -30\ °C$ | $V_{min} = 400\ V$ |
| Maximum | $T_{max} = 220\ °C$ | $V_{max} = 900\ V$ |

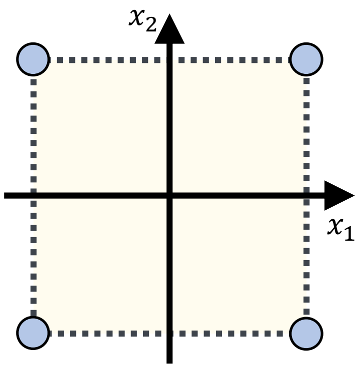
# Theory: orthogonality as a DoE principle

## What are the effects?

### Non-orthogonal test plan vs. Orthogonal (factorial) test plan




| $i$ | $x_1$ | $x_2$ |
|---|---|---|
| 1 | $-0.35$ | $-0.30$ |
| 2 | $0.45$ | $-0.60$ |
| 3 | $0.55$ | $0.40$ |
| 4 | $-.80$ | $0.70$ |




| $i$ | $x_1$ | $x_2$ |
|---|---|---|
| 1 | $-1$ | $-1$ |
| 2 | $1$ | $-1$ |
| 3 | $1$ | $1$ |
| 4 | $-1$ | $1$ |

- In the non-orthogonal case, the variances increase
- Additionally, the estimates are suddenly correlated (not shown)

| **Change in the estimation variance of...** | | | |
|---|---|---|---|
| $c_0$ (mean) | $c_1$ (main effect $x_1$) | $c_2$ (main effect $x_2$) | $c_3$ (interaction $x_1 x_2$) |
| $-70\ \%$* | $+34\ \%$ | $+12\ \%$ | $+284\ \%$ |

*mainly due to narrower parameter space



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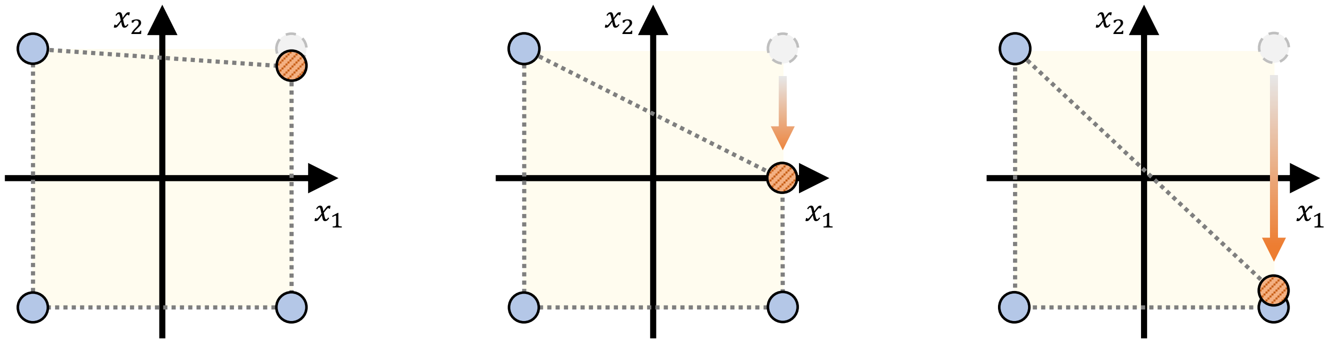
# Application: sources of non-orthogonality

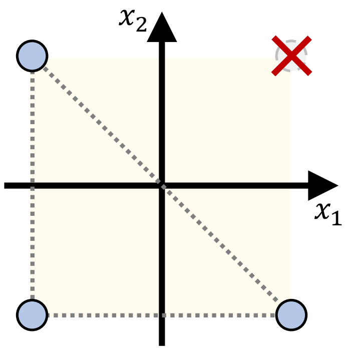
## Example (a): parameter combination shift




- Certain values of a single parameter cannot be set and have to be lowered, e.g. due to ...
  - high cost (high energy consumption, high cooling effort),
  - limited test setup (no high temperatures at low pressure without phase change),
  - physical boundaries (no arbitrary combinations of humidity and temperature),
  - triggering of a different failure mechanism.

# Application: sources of non-orthogonality

## Example (b): parameter combination omitted




- A particular parameter combination has to be omitted due to ...
  - timely constraints (consequences of earlier delays, or management decisions),
  - unplanned unavailability of the test setup (damage, or use for another project),
  - insufficient sample size (costly product, or limited number of prototypes).



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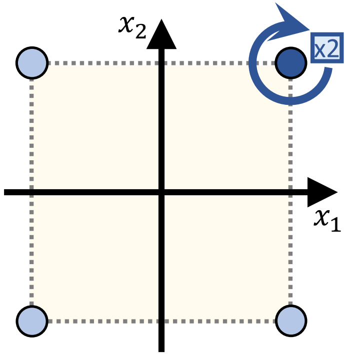
# Application: sources of non-orthogonality

## Example (c): parameter combination repeated




- A particular parameter combination is repeated due to ...
  - an originally planned, but then cancelled repetition,
  - human errors (e.g. miscommunication between different test bench workers),
  - „It can‘t hurt, can it?“



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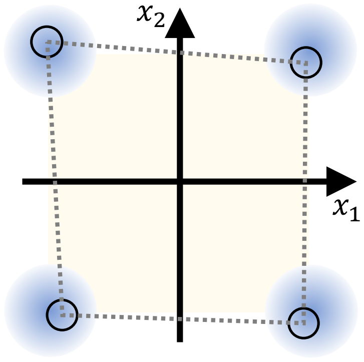
# Application: sources of non-orthogonality

## Example (d): normal scattering of all parameter combinations




- All parameter combinations are slightly off their ideal value due to...
  - inexact test setup (which might be cheaper than high-end machinery),
  - influence of the operator,
  - changes during the experiment (e.g. ambient temperature, pressure, electric fields).

# Application: sources of non-orthogonality

## Take-aways

- Some kind of non-orthogonality is always present, the question is just to which degree
- This especially applies to the last case (parameters being slightly & unsystematically off)

While perfect non-orthogonality is impossible in the evaluation, perfect orthogonality is impossible in the execution

# Application: sources of non-orthogonality

## What next?

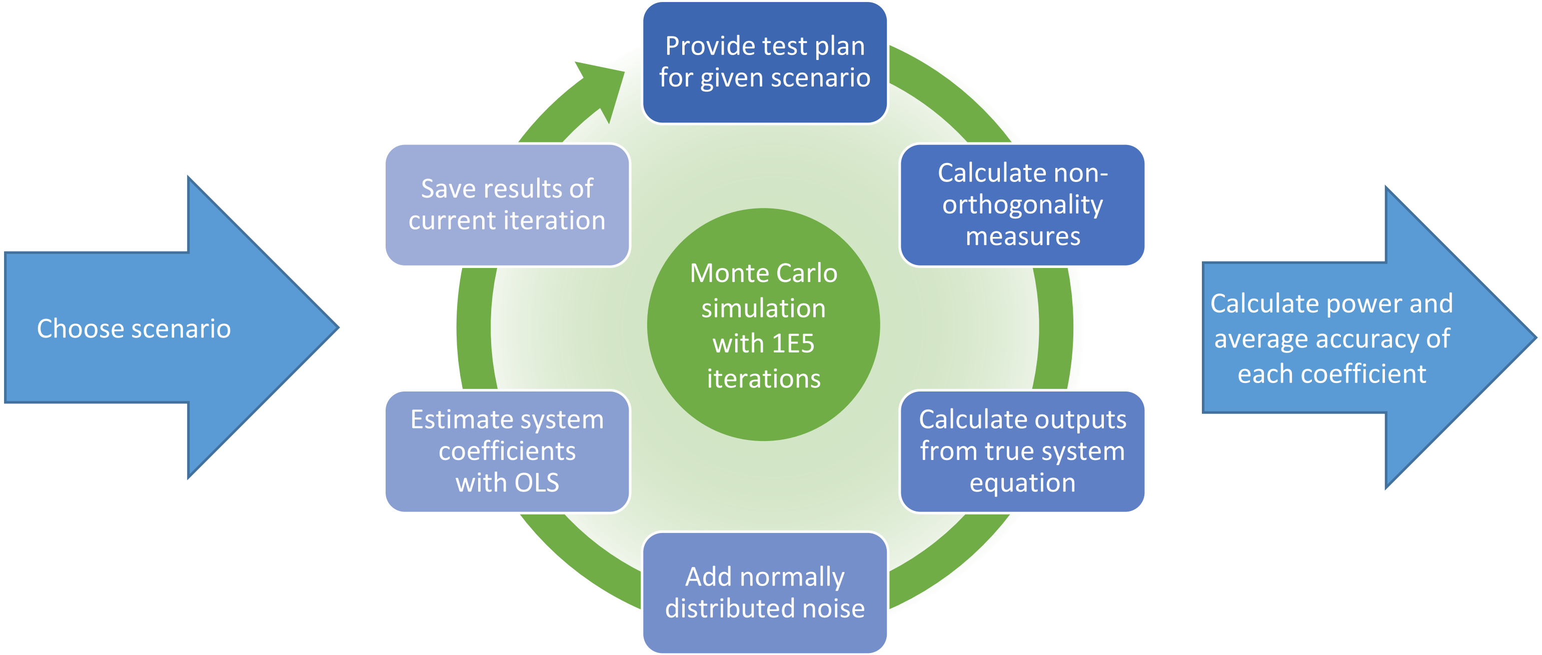
While perfect non-orthogonality is impossible in the evaluation, perfect orthogonality is impossible in the execution

| How bad is non-orthogonality? | |
|---|---|
| | Regression accuracy |
| | Test power |

| How do I even measure it, anyway? | |
|---|---|
| | Optimality criteria |
| | Other quantities |

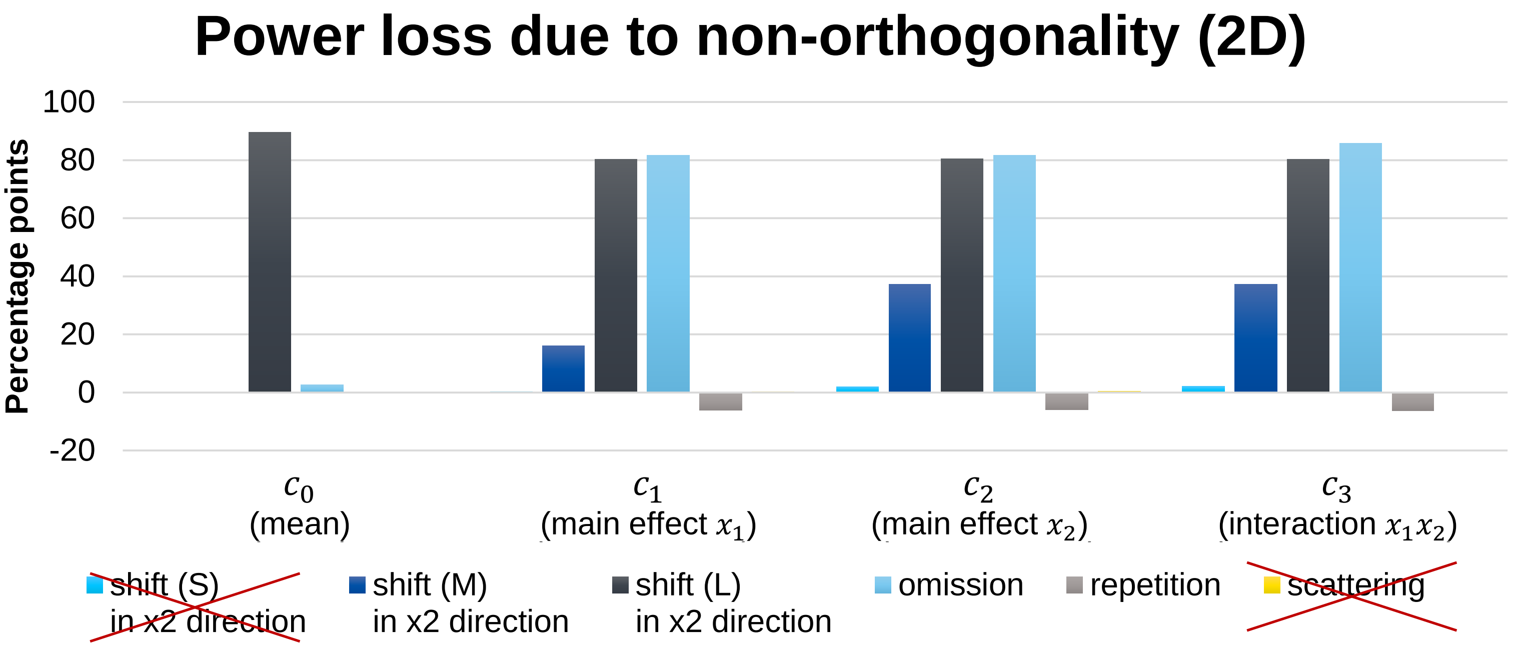
# Simulation study

## Process



# Simulation study

## Results



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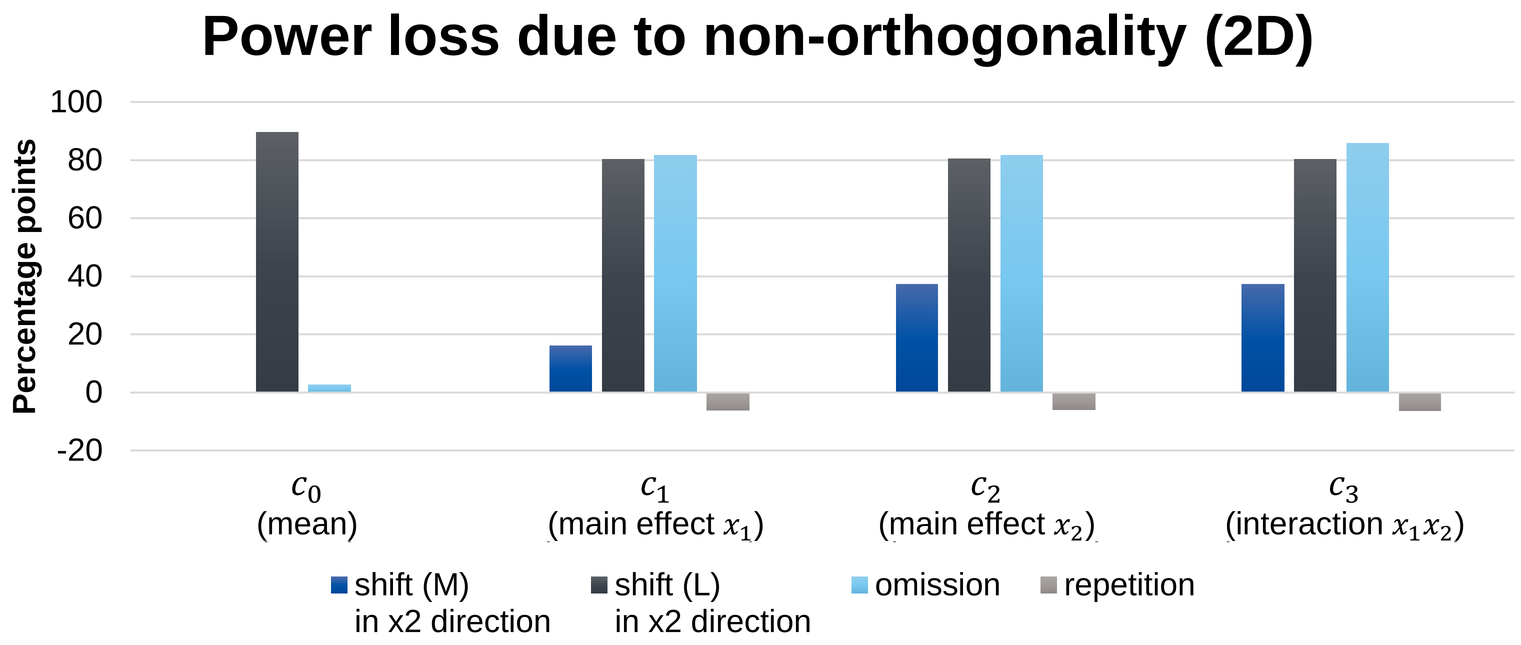
# Simulation study

Results



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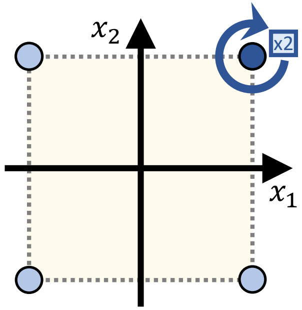
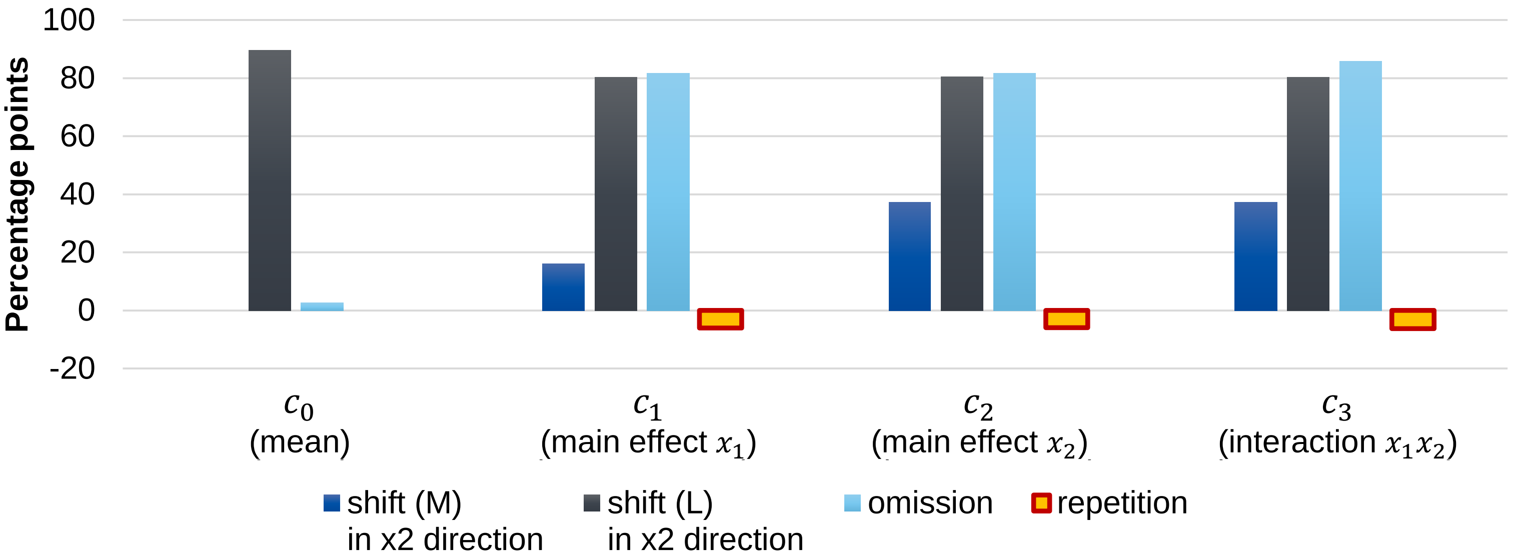
# Simulation study

## Results




**Power loss due to non-orthogonality (2D)**



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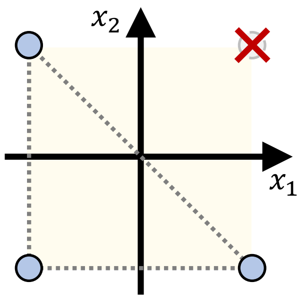
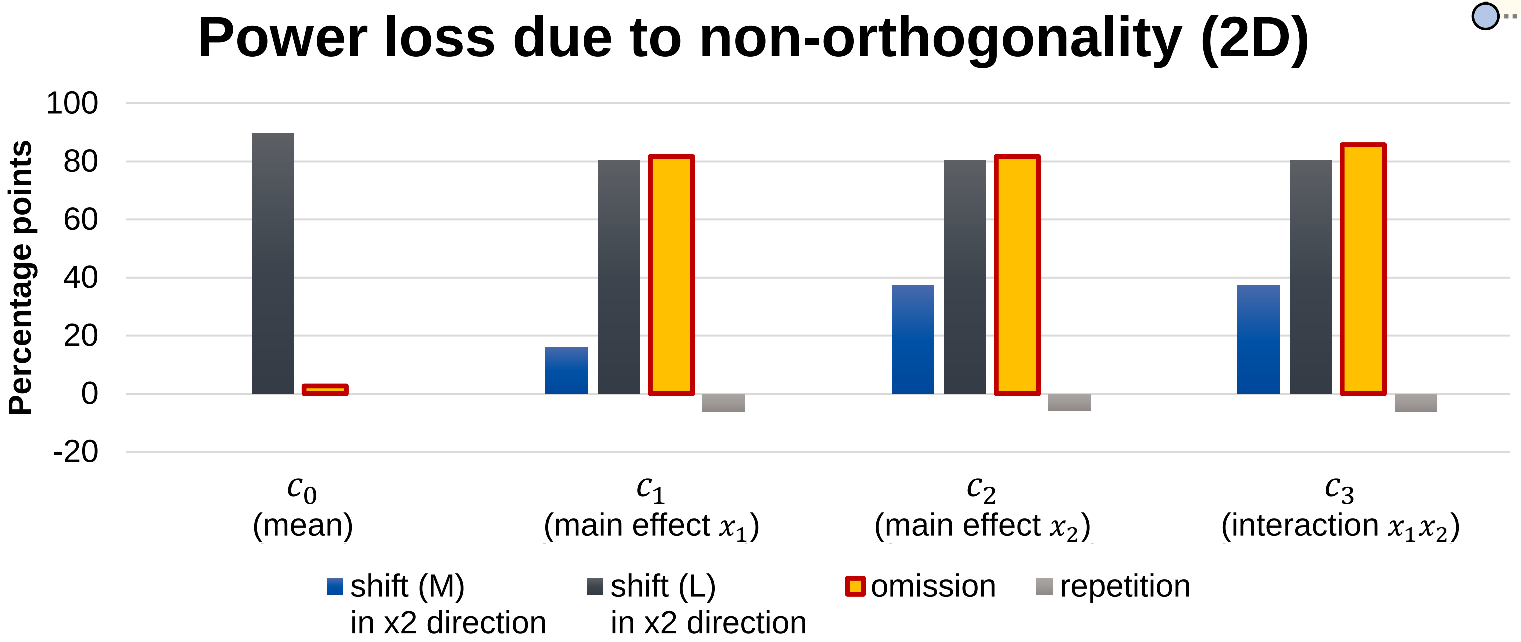
## Results



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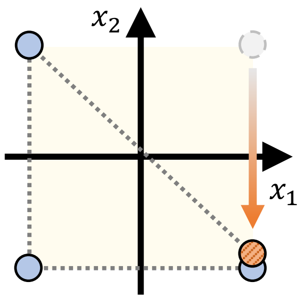


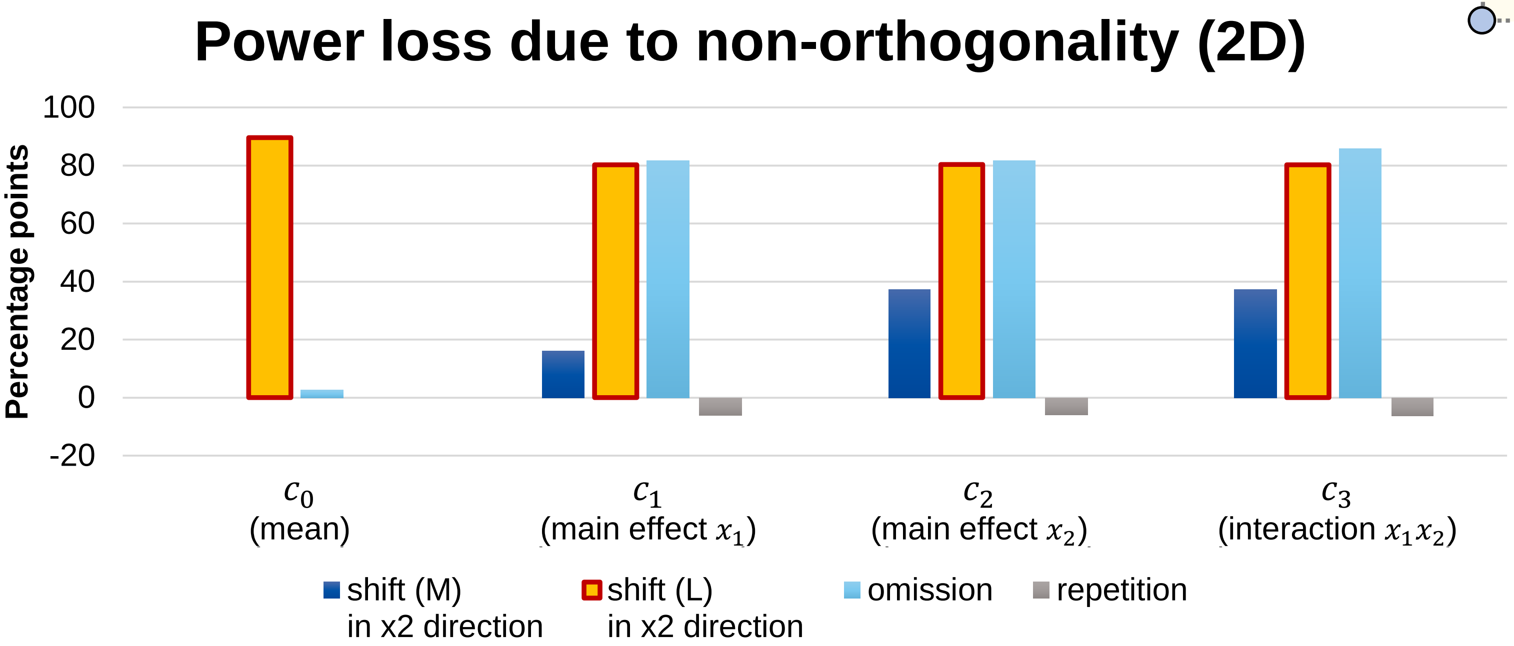

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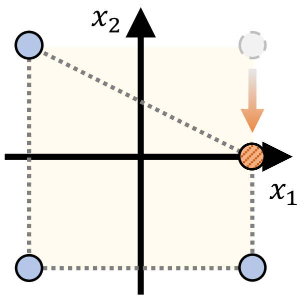
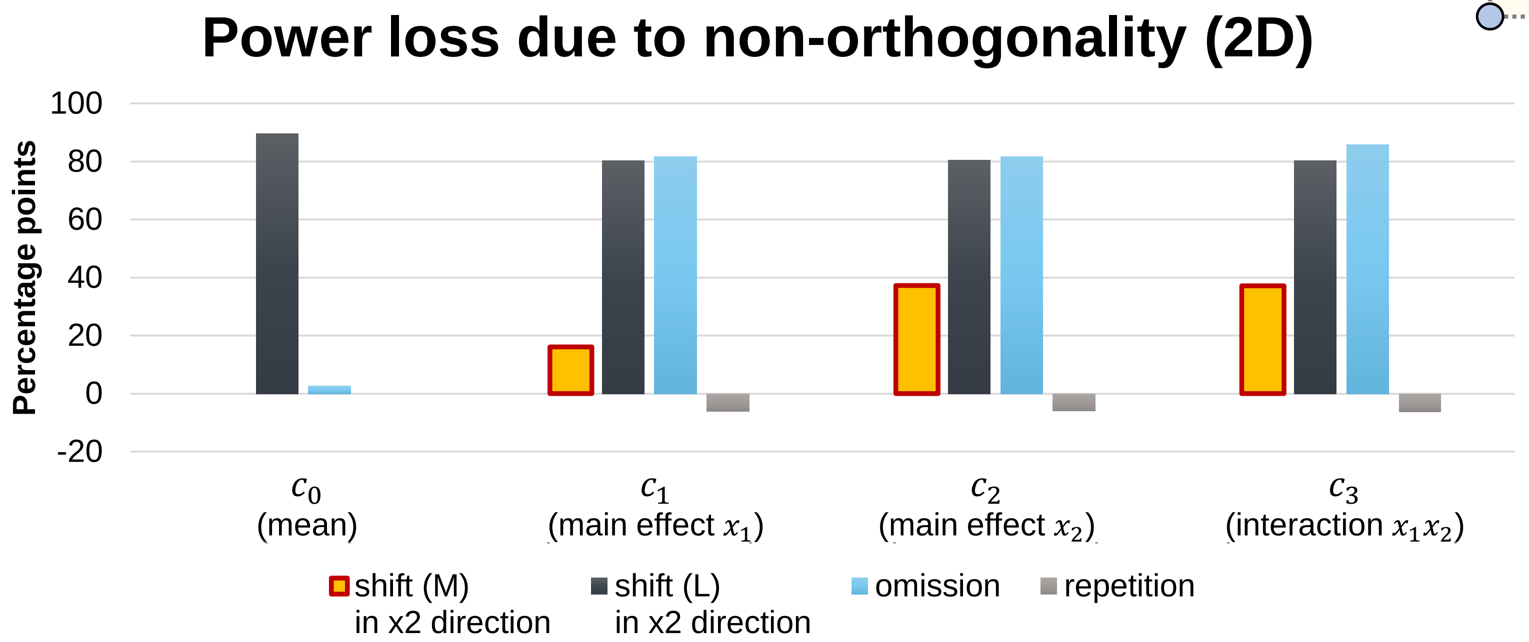
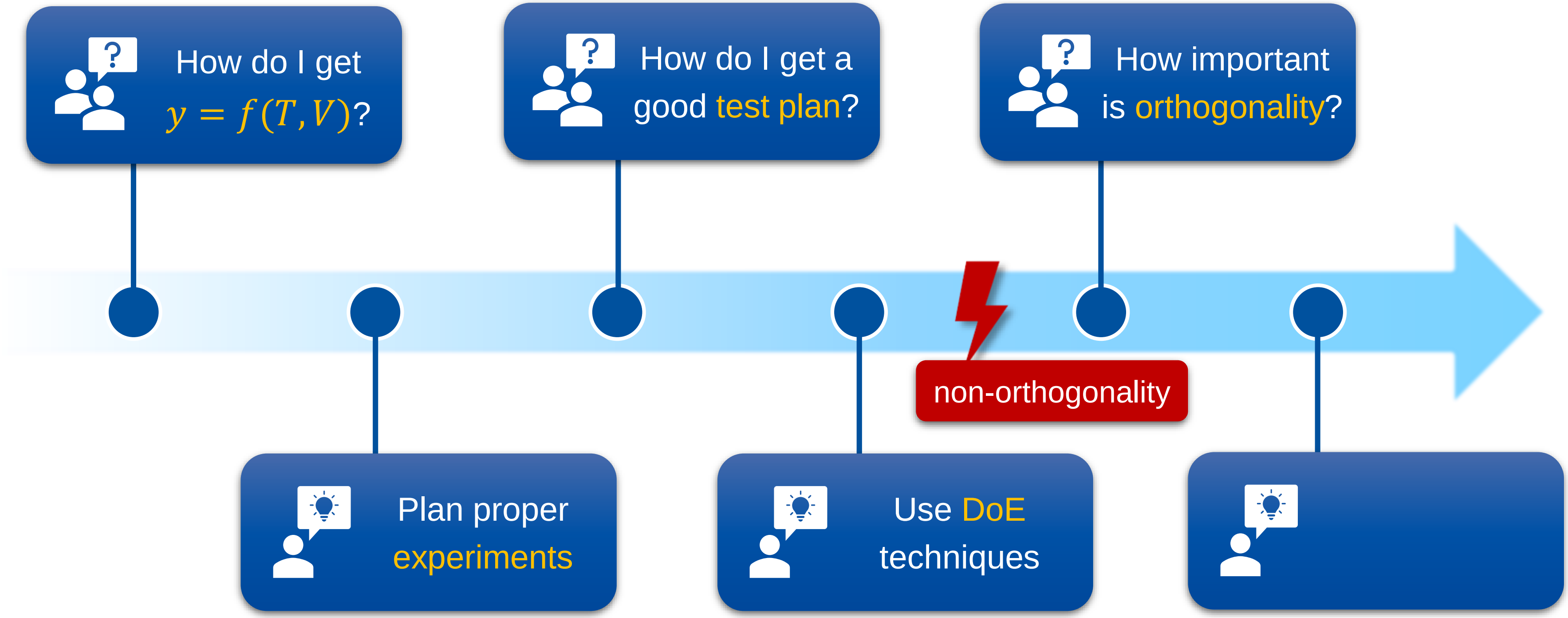


# Summary & future work

## How important is orthogonality?



# Summary & future work

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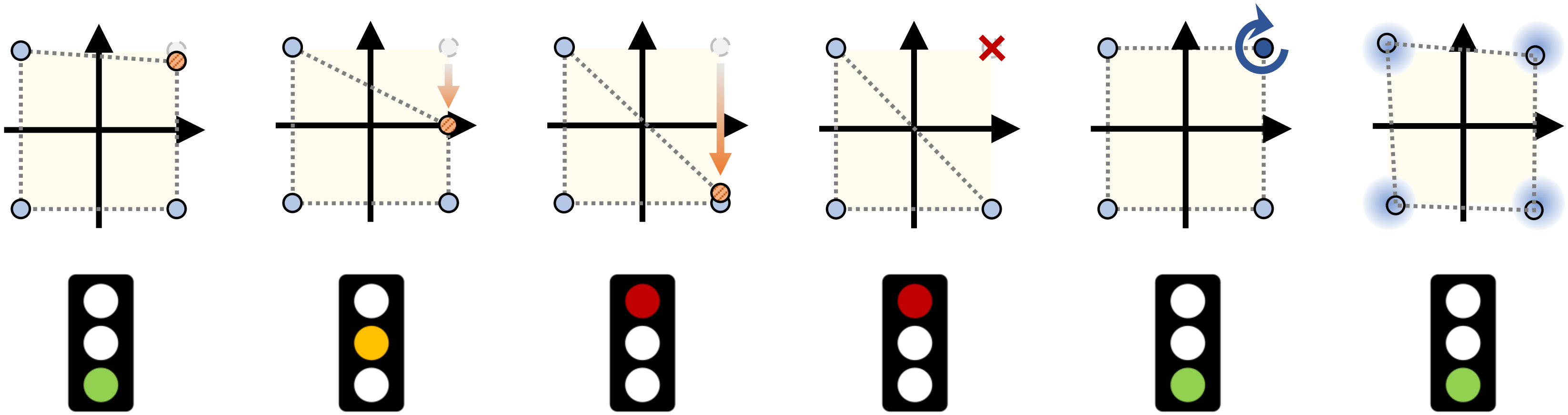

# Summary & future work

## Future work

- How do combinations of the basic non-orthogonality scenarios perform?
- How do different degrees of input quantity scattering affect the results?
- How does the behaviour change for different ratios of parameter influence?
  (main effect to main effect, main effect to interaction, interaction to interaction)
- What is the dependency on...
  - ... the number of repetitions?
  - ... the number of dimensions?
- etc...

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# Thank you!

**Philipp Mell, M.Sc.**

e-mail philipp.mell@ima.uni-stuttgart.de
phone +49 (0) 711 685-69859
www.ima.uni-stuttgart.de

University of Stuttgart
Pfaffenwaldring 9 – 70569 Stuttgart – Germany

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