University of Stuttgart
Institute of Machine Components
Reliability Department

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Generic effects of deviations from test design orthogonality on test power and regression modelling of CCDs

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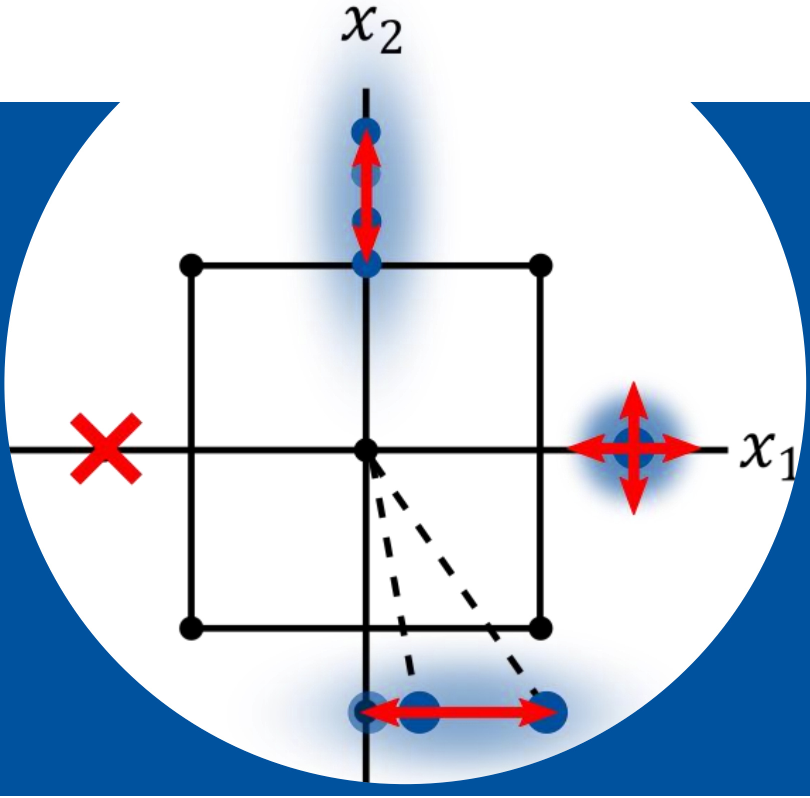

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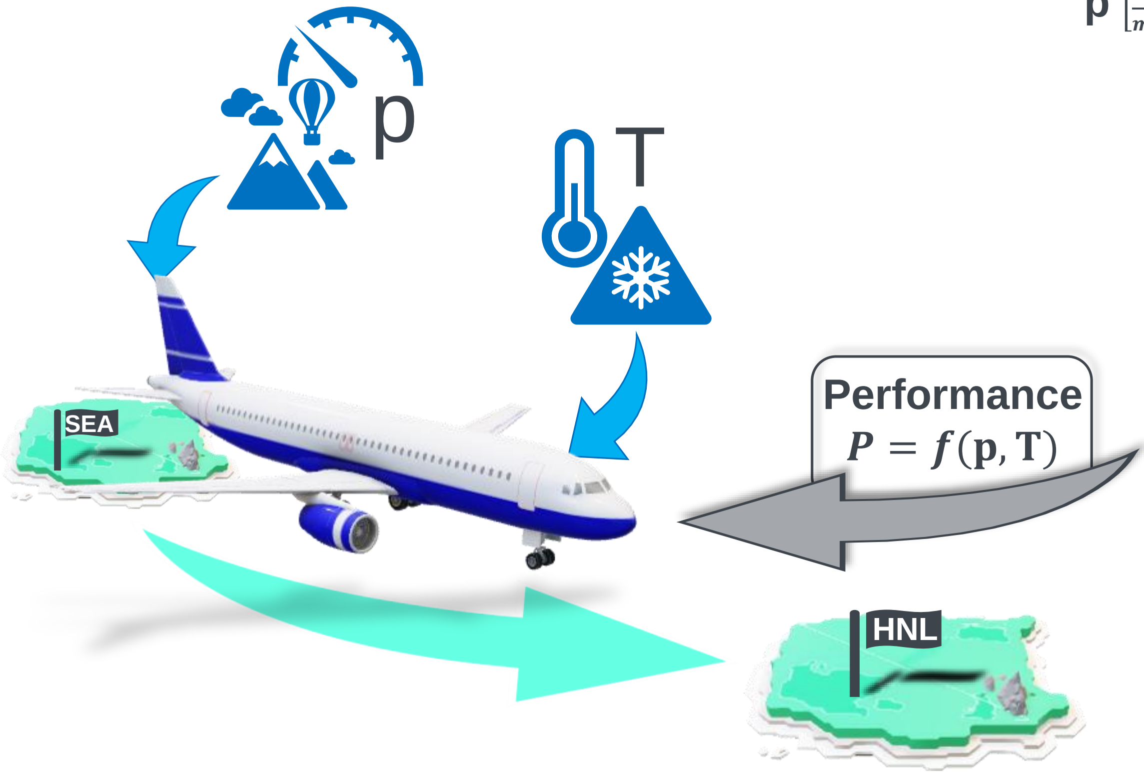
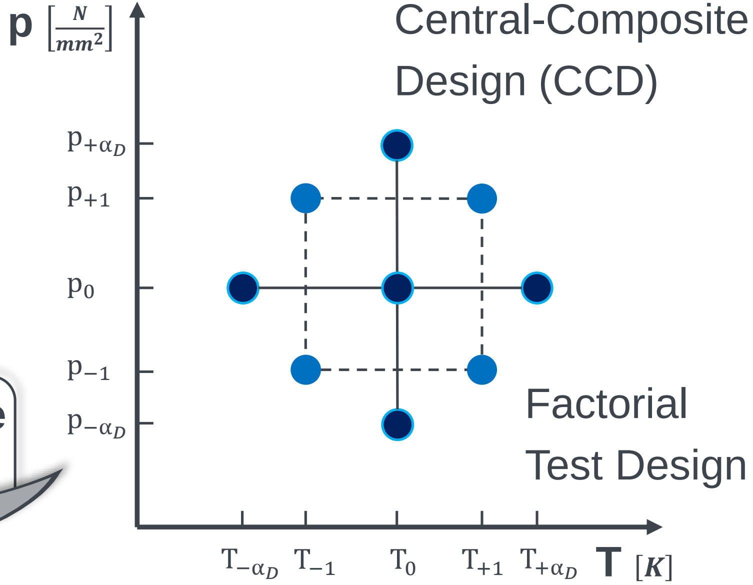
Overview and Outline

- Background and Motivation
- Orthogonality Deviations
- Study Approach
- Results
- Summary and Conclusion
- Next Steps and Future Work

Background and Motivation

Exemplary DoE Approach





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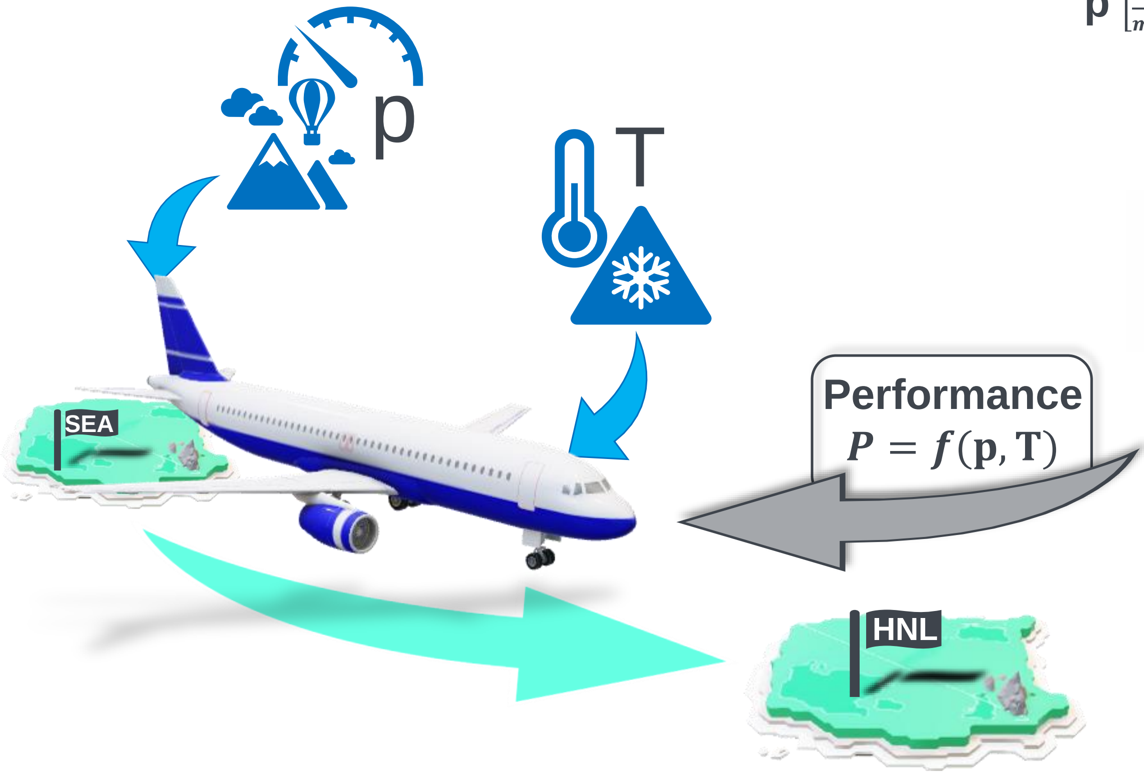
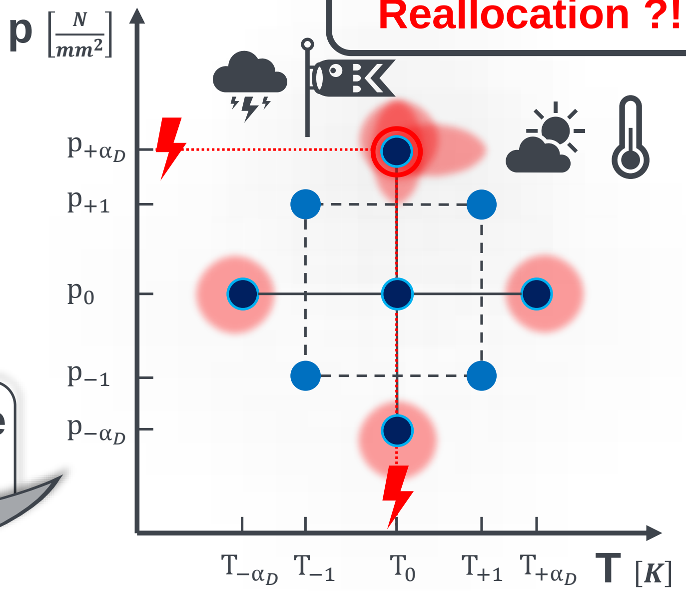
Background and Motivation

Test Run Issues causing Conflicts with Orthogonality



Parameter Space Reallocation ?!



→ What are the Effects on Model Power and Model Quality?

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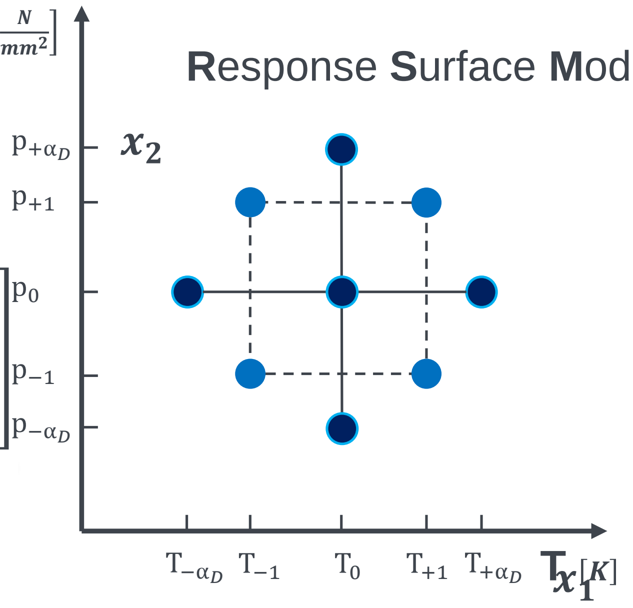
Background and Motivation

State-of-the-Art in Empirical Model Building

- Regression Model based on n Observations $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$
  - Coefficients for k Factors $\beta = \begin{bmatrix} \beta_o \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}$ and Error $\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$
- Second Order Model for $k = 2$ Factors:
  - $y = \beta_o + \beta_1 \boldsymbol{x_1} + \beta_2 \boldsymbol{x_2} + \beta_{11} \boldsymbol{x_1^2} + \beta_{22} \boldsymbol{x_2^2} + \beta_{12} \boldsymbol{x_1} \boldsymbol{x_2} + \varepsilon$

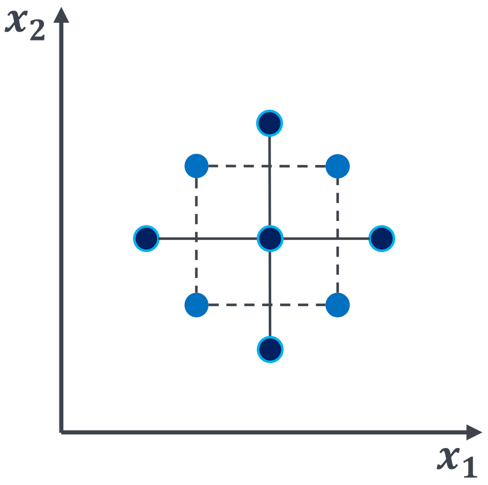
    $(x_3 = x_1^2,\ x_4 = x_2^2,\ x_5 = x_1 x_2;\ \beta_3 = \beta_{11},\ \beta_4 = \beta_{22},\ \beta_5 = \beta_{12})$



Background and Motivation

State-of-the-Art in Empirical Model Building

- Create a Function for Fitted Response Values:
  - $\hat{y} = X\hat{\beta}$, where $X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}$



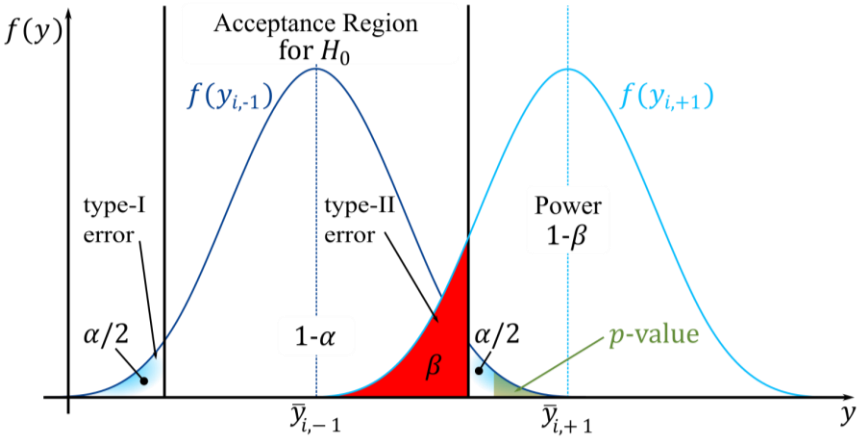
- Resulting in Least-Squares Estimator of β:
  - $\hat{\beta} = (X'X)^{-1}X'y$

Background and Motivation

Test Power

- Null hypothesis $H_0: \bar{y}_{i,-1} = \bar{y}_{i,+1}$

  Alternative hypothesis $H_1: \bar{y}_{i,-1} \neq \bar{y}_{i,+1}$



- Type-I Error:

  H_0 *is* rejected although it is *true*

  Type-II Error*:*

  H_0 is *not* rejected although it is *false*

➢ The probability to identify an existing influence on the effect correctly:

Power $= 1 - \beta$

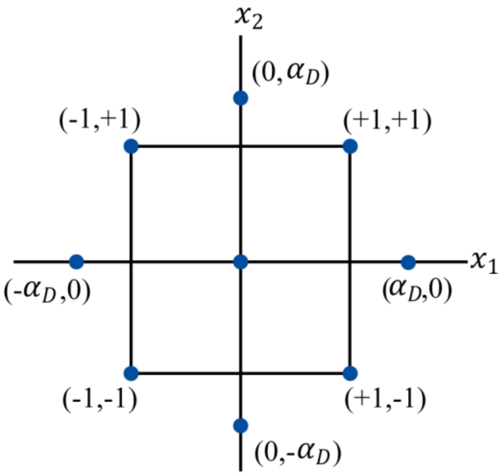
- ANOVA: $p = 1 - P(F > F_{k,n-k-1})$

Orthogonality Deviations

Orthogonality in CCDs

- Linear Independent Input Parameters → $\mathrm{M} = (\mathrm{X}'\mathrm{X})$ is a diagonal matrix
- Minimize: $Var(\hat{\beta}) = \sigma^2 \mathrm{M}^{-1}$
- Leverage value $\alpha_D = \left[\frac{\sqrt{n_F\left(n_F+2k\cdot\frac{n_S}{n_f}+\frac{n_C}{n_f}\right)}-n_F}{2\cdot\frac{n_S}{n_f}}\right]^{1/2}$

  with amounts of factorial test points (n_F), factorial runs ($n_f = r_f \cdot n_F$), central runs (n_C) and star runs ($n_s = r_s \cdot 2k$)
- Control Criteria:
  correlation matrix, A-Optimality, D-Optimality, G-Optimality, …



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Orthogonality Deviations

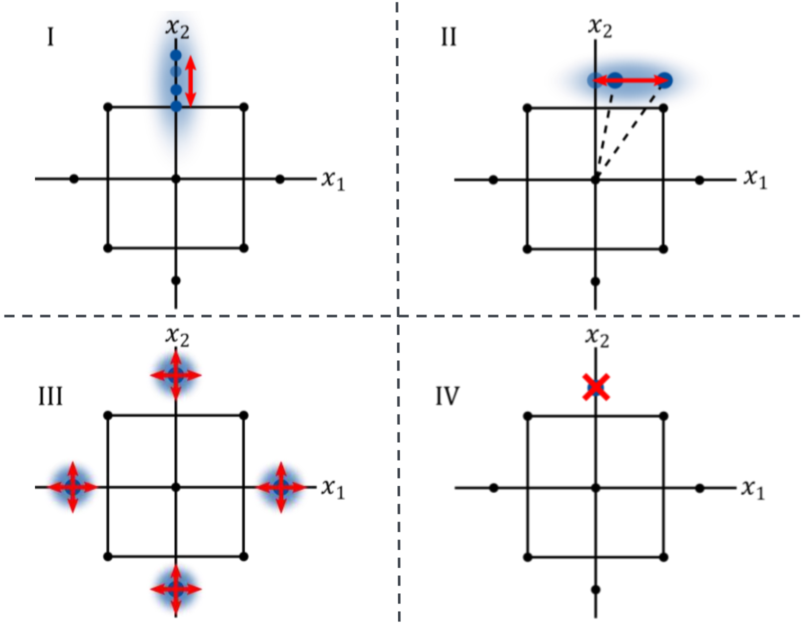
Cases for CCDs

The Star Point(s),

- is not physically or mechanically feasible, due to e.g., time and effort or ambient testing conditions;
- is just realisable at a decreased level of the original the axial run definition, therefore $\pm\alpha_D$ shrinks;
- or a factor stage combination is encountered where a pure axial run can no longer be performed

I-1: $\alpha_D = 1$
I-2: $\alpha_D = 0.8 \cdot \alpha_D$
I-3: $\alpha_D = 1.2 \cdot \alpha_D$

II-1: $(0.1, \alpha_D)$
II-2: $(1.0, \alpha_D)$



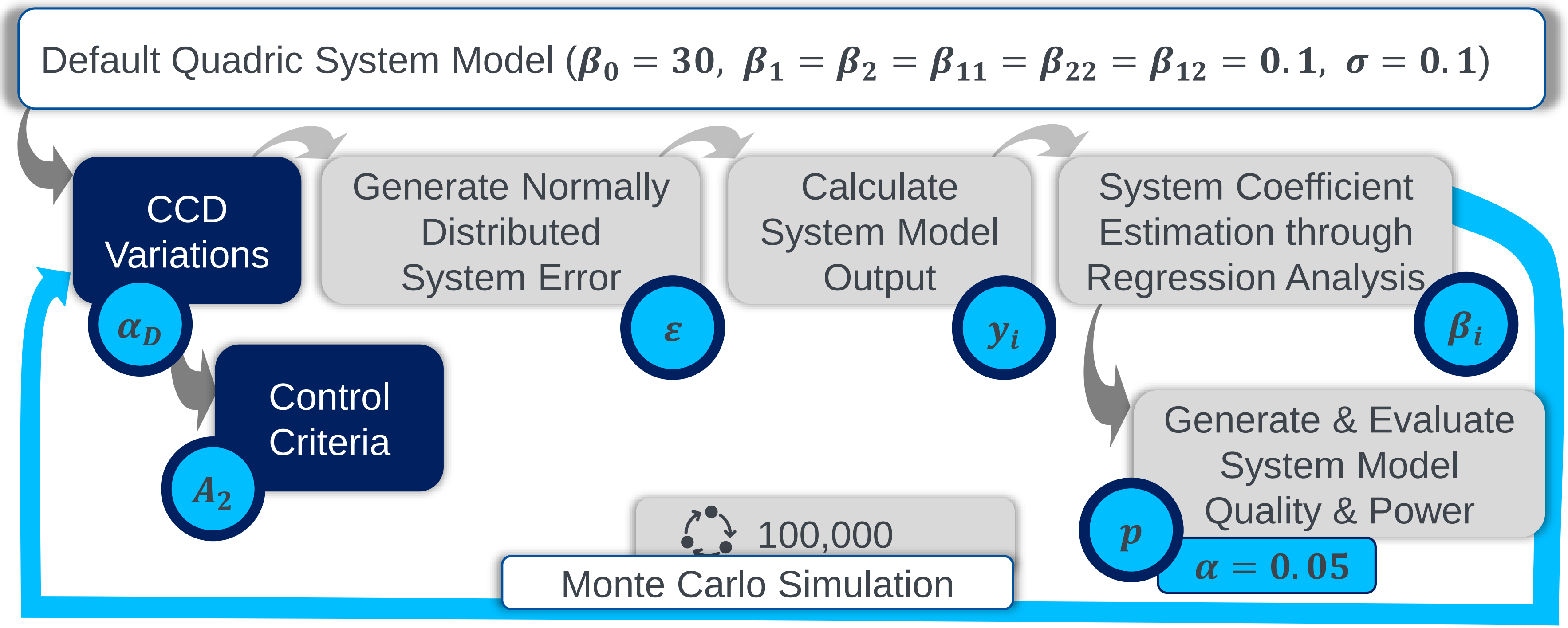
III: 10 % scatter

IV: $(0, \alpha_D)$ omitted

➢ **Sequential parameter study with discrete values**

Study Approach

Simulation Study for Generic Effects on Power and Regression



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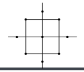
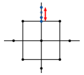
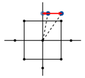
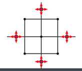
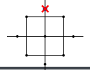
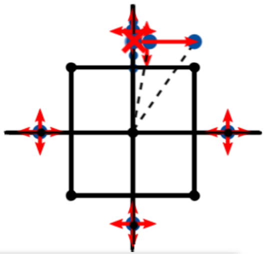
Results

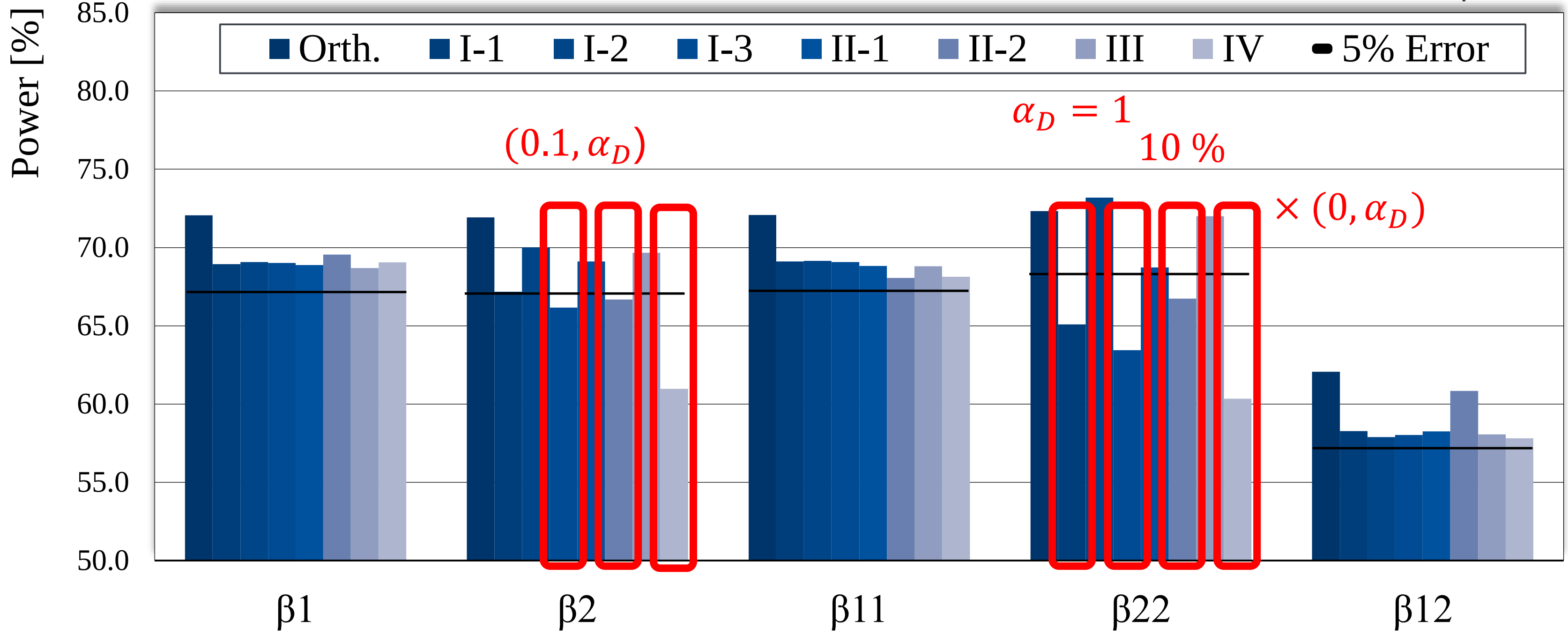
Power of Model Coefficients

Case		Star Run	A_2	Power [%]					
				β_o	β_1	β_2	β_{11}	β_{22}	β_{12}
	Orth.	$(0, \alpha_D)$	0	100.0	72.1	71.9	72.1	72.3	62.1
	I-1	$(0,1)$	0.04	100.0	68.9	67.2	69.1	65.1	58.3
	I-2	$(0, 0.8 \cdot \alpha_D)$	0.04	100.0	69.1	70.0	69.1	73.2	57.9
	I-3	$(0,1.2 \cdot \alpha_D)$	0.07	100.0	69.0	66.2	69.1	63.5	58.0
	II-1	$(0.1, \alpha_D)$	0.01	100.0	68.9	69.1	68.8	68.7	58.3
	II-2	$(1, \alpha_D)$	0.35	100.0	69.5	66.7	68.1	66.7	60.8
	III	$(0, \alpha_D) \pm 10\%$	0.01	100.0	68.7	69.7	68.8	72.0	58.1
	IV	(NaN,NaN)	NaN	100.0	69.1	61.0	68.1	60.3	57.8

Results

Power of Model Coefficients





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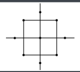
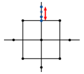
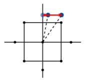
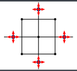
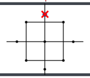
Results

Quality of Regression Coefficients

Case		Star Run	Coefficient Estimation Amount Error [%]					
			β_0	β_1	β_2	β_{11}	β_{22}	β_{12}
	Orth.	$(0, \alpha_D)$	0.00	0.74	-0.47	0.77	0.34	0.33
	I-1	$(0,1)$	-0.03	-2.88	1.55	-3.24	1.24	-8.60
	I-2	$(0, 0.8 \cdot \alpha_D)$	0.00	0.23	4.03	2.20	2.19	6.19
	I-3	$(0, 1.2 \cdot \alpha_D)$	0.01	-2.97	0.05	-3.19	4.63	4.98
	II-1	$(0.1, \alpha_D)$	0.00	-0.09	5.50	5.05	1.93	1.32
	II-2	$(1, \alpha_D)$	0.00	-1.01	0.82	-0.46	7.71	3.98
	III	$(0, \alpha_D) \pm 10\%$	0.00	-1.53	2.77	-2.95	1.61	-1.65
	IV	(NaN,NaN)	-0.01	3.30	-0.89	-8.69	-0.20	-5.75

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Results

General Findings

- Orthogonality Deviations → Generic effects on coefficient power and quality
- Factor-dependent power deviation for the coefficients shows a relation to the corresponding orthogonality deviations
- 10 % star run scattering → max. power-loss -4.0%, max. estimate error -2.95 %
- A_2-criterion → only partly useful for the investigation objective

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Summary and Conclusion

- Study Approach using Monte Carlo Simulation as presented is very well suited to investigate power and quality behavior of regression modelling with CCD orthogonality deviations
- Use of power and regression quality is recommended to indicate consequences of test design variations → also as an improvement in terms of test design efficiency
- Characteristics of the default system model determine the amount and type of consequences on model power and quality

Next Steps and Future Work

- Dependency on the given system model in the simulative environment:
  effects and system errors

- Cost model:
  trade-off of perceived opportunity costs from model degradation and cost savings

- Study approach with continuous variables and all combinations
  “using DoE to investigate DoE”

Thank you!

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