

Balanced Fault Tree Modelling of Alternating Operation Systems in Probabilistic Safety Assessment

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Abstract: Nuclear power plants (NPPs) have alternating operation systems, such as component cooling water system (CCWS), essential service water system (ESWS), essential chilled water system (ECWS), and chemical and volume control system (CVCS). Single-unit probabilistic safety assessment (SUPSA) models for NPPs have many failures of alternating operation systems. Furthermore, since NPPs undergo alternating operations between full power and low power and shutdown (LPSD), multi-unit PSA (MUPSA) models have failures of NPPs that undergo alternating operations between full power and LPSD. Failures for alternating operation systems are modelled using fraction or partitioning events in seismic SUPSA and MUPSA fault trees. Since partitioning events for one system are mutually exclusive, their combinations should be excluded in exact solutions. However, it is difficult to eliminate the combinations of mutually exclusive events without modifying PSA tools for generating MCSs from a fault tree. If the combinations of mutually exclusive events are not deleted, core damage frequency (CDF) is underestimated. To avoid CDF underestimation in SUPSAs and MUPSAs, this paper introduces a process of converting partitioning events into conditional events, and conditional events are then inserted explicitly in a fault tree. With this conversion, accurate CDF can be calculated without modifying PSA tools. It is strongly recommended that the method suggested in this paper be employed for avoiding CDF underestimation in seismic SUPSAs and MUPSAs.

1. INTRODUCTION

1.1. SUPSA and MUPSA

Probabilistic safety assessments (PSAs) that calculate core damage frequency (CDF) are divided into single-unit PSAs (SUPSAs) and multi-unit PSAs (MUPSAs).

Many SUPSAs for nuclear power plants (NPPs) have been performed since the first PSA of WASH-1400 [1]. The initial MUPSA studies [2,3] were performed due to the gradually increasing concern regarding multi-unit nuclear accidents. After the Fukushima accident in 2011, many studies and reports on MUPSA [4–16] were published. In 2019, the International Atomic Energy Agency (IAEA) published a technical report [17] as part of a safety report series that provides comprehensive guidance for performing MUPSA. One method proposed for seismic MUPSA was converting correlated seismic failures into seismic common cause failures (CCFs) [18].

In MUPSA [17], multi-unit core damage frequency (MUCDF), site core damage frequency (SCDF), and single-unit core damage frequency (SUCDF) are defined as accident frequencies in which at least two NPPs, at least one NPP, and only one NPP are in a core damage state following an initiating event, respectively.

1.2. Alternating Operation Systems

A Boolean equation for the failures in alternating operation systems can be expressed as Equation (1) [19]. The Boolean equation can be a fault tree or minimal cut sets (MCSs) that are calculated from the fault tree. In this paper, the operation fractions of X_1 and X_2 are defined as partitioning events. If one NPP has S alternating systems and each system has T partitioning events, the Boolean AND

combination number of partitioning events in MCSs might be up to S^T . The other combinations, such as X_1X_2 , are not allowed in MCSs since they are mutually exclusive.

$$\begin{aligned}
 f(\mathbf{X}, \mathbf{B}) &= X_1f_1(\mathbf{B}) + X_2f_2(\mathbf{B}) \\
 X_1 &= \text{Fraction of train 1 operation} \\
 X_2 &= \text{Fraction of train 2 operation} \\
 \{f_1(\mathbf{B}), f_2(\mathbf{B})\} &= \text{Operation failures during } \{X_1, X_2\}
 \end{aligned} \tag{1}$$

Seismic SUPSA models have random failures of alternating operation systems that are combined with many seismic failures of components and structures. Furthermore, seismic MUPSA models have failures of NPPs that undergo alternating operations between full power and low power and shutdown (LPSD).

An NPP is in full-power operation for 1 or 2 years and in LPSD operation for 1 or 2 months to replace or reload nuclear fuels. That is, NPPs are a kind of alternating operation system. A Boolean equation to calculate the failures in alternating operation NPPs can be expressed as Equation (2). In this paper, the operation fractions of X_1 , X_2 , and X_3 are also defined as partitioning events.

$$\begin{aligned}
 f(\mathbf{X}, \mathbf{B}) &= X_1f_1(\mathbf{B}) + X_2f_2(\mathbf{B}) + X_3f_3(\mathbf{B}) \\
 X_1 &= \text{Fraction of full-power operation} \\
 X_2 &= \text{Fraction of LPSD operation with nuclear fuel} \\
 X_3 &= \text{Fraction of LPSD operation without nuclear fuel} \\
 \{f_1(\mathbf{B}), f_2(\mathbf{B}), f_3(\mathbf{B})\} &= \text{Operation failures during } \{X_1, X_2, X_3\}
 \end{aligned} \tag{2}$$

Thus, multiple NPPs in a single nuclear site are considered a group of alternating operation NPPs. Since the Kori nuclear site in Korea has nine NPPs and each LPSD PSA has 15 plant operating states (POSSs), there might be 16^9 combinations of plant-level partitioning events in the MCSs of MUPSA.

2. Exclusive Modelling of One Group of partitioning Events

If a system has n trains that are alternatively operated one by one or an NPP undergoes n full-power and LPSD operations periodically, a Boolean equation to calculate system failure can be expressed as Equation (3) [19].

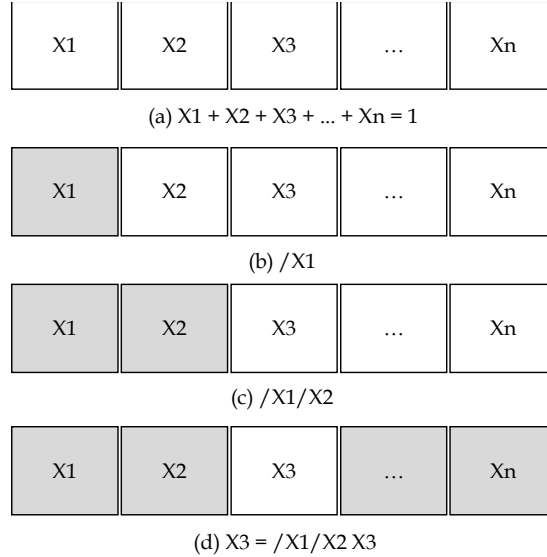
$$f(\mathbf{X}, \mathbf{B}) = X_1f_1(\mathbf{B}) + X_2f_2(\mathbf{B}) + \dots + X_nf_n(\mathbf{B}) \tag{3}$$

where \mathbf{X} has n partitioning events and $f_i(\mathbf{B})$ can be a complex Boolean equation that consists of random failure events \mathbf{B} . The fault tree in Equation (3) is a typical case. Usual fault trees can have Boolean AND combinations of X_i 's, and these Boolean AND combinations should be deleted in MCSs since they are mutually exclusive.

The partitioning events are mutually exclusive events that satisfy the following equations in Equation (4). They can be depicted by the Venn diagram in Fig. 1 that has no intersections of mutually exclusive partitioning events.

$$\begin{aligned}
 X_iX_j &= 0, i \neq j \\
 \sum_{i=1}^n X_i &= 1
 \end{aligned} \tag{4}$$

Here, 0 and 1 denote empty and union sets, respectively.



$$p(X_2 | /X_1) = p(X_2) / (p(X_2) + \dots + p(X_n))$$

$$p(X_3 | /X_1/X_2) = p(X_3) / (p(X_3) + \dots + p(X_n))$$

Fig. 1. Venn diagram for partitioning events (no intersections).

If members of \mathbf{X} are not mutually exclusive, $p(f(\mathbf{X}, \mathbf{B}))$ can be calculated by the inclusion–exclusion equation [20] in Equation (5).

$$\begin{aligned}
 p(f(\mathbf{X}, \mathbf{B})) &= p(X_1 f_1(\mathbf{B})) + p(X_2 f_2(\mathbf{B})) + \dots \\
 &+ (-1)^1 [p(X_1 X_2 f_1(\mathbf{B}) f_2(\mathbf{B})) + p(X_1 X_3 f_1(\mathbf{B}) f_3(\mathbf{B})) + \dots] \\
 &+ (-1)^2 [p(X_1 X_2 X_3 f_1(\mathbf{B}) f_2(\mathbf{B}) f_3(\mathbf{B})) + p(X_1 X_2 X_4 f_1(\mathbf{B}) f_2(\mathbf{B}) f_4(\mathbf{B})) + \dots] \\
 &+ \dots \\
 &+ (-1)^{n-1} p(X_1 X_2 X_3 \dots X_n f_1(\mathbf{B}) f_2(\mathbf{B}) f_3(\mathbf{B}) \dots f_n(\mathbf{B}))
 \end{aligned} \tag{5}$$

However, since X_1, X_2, \dots are mutually exclusive events, Equation (5) should be

$$p(f(\mathbf{X}, \mathbf{B})) = p(X_1)p(f_1(\mathbf{B})) + p(X_2)p(f_2(\mathbf{B})) + \dots + p(X_n)p(f_n(\mathbf{B})). \tag{6}$$

The probability in Equation (5) is much smaller than that in Equation (6). That is, if Boolean AND combinations of mutually exclusive partitioning events in a single group are not eliminated in the inclusion–exclusion equation or in the exact solutions, such as a binary decision diagram (BDD) [21, 22], the system failure probability or CDF would be underestimated. However, it is impossible to eliminate mutually exclusive event combinations without modifying calculation tools. Therefore, there is a great need to explicitly model the partitioning events in fault trees instead of revising such tools. This is an objective of this paper.

To accomplish this objective, partitioning events are expressed as shown in Equation (7). They can be confirmed by reflecting the terms on the right-hand side in Equation (7) into the Venn diagram in Fig. 1.

$$\begin{aligned}
 X_2 &= /X_1 X_2 \\
 X_3 &= /X_1 /X_2 X_3 \\
 X_n &= /X_1 /X_2 \dots /X_{n-1} X_n
 \end{aligned} \tag{7}$$

Here, the probabilities of Equation (7) can be expressed by employing the conditional probabilities as

$$p(X_2) = p(/X_1 X_2) = p(/X_1)p(X_2 | /X_1) \tag{8}$$

$$p(X_3) = p(X_1/X_2X_3) = p(X_1)p(X_2/X_1)p(X_3/X_1/X_2)$$

$$p(X_n) = p(X_1/X_2 \dots /X_{n-1}X_n) = p(X_1)p(X_2/X_1) \dots p(X_n/X_1/X_2 \dots /X_{n-1}).$$

Equation (8) shows that partitioning events are not independent events, since

$$p(X_2) \neq p(X_2/X_1)$$

$$p(X_3) \neq p(X_3/X_1/X_2) \quad (9)$$

$$p(X_n) \neq p(X_n/X_1/X_2 \dots /X_{n-1}).$$

In this paper, conditional events are intentionally defined as in Equation (10) to explicitly model partitioning events with explicit events \mathbf{X}^c inside a fault tree.

$$X_1^c \equiv X_1$$

$$X_2^c \equiv X_2/X_1$$

$$X_3^c \equiv X_3/X_1/X_2 \quad (10)$$

$$X_n^c \equiv X_n/X_1/X_2 \dots /X_{n-1}$$

Then, probabilities of conditional events can be easily derived from the Venn diagram in Fig. 1.

$$p(X_1^c) = p(X_1)$$

$$p(X_2^c) = \frac{p(X_2)}{\sum_{i=2}^n p(X_i)}, p(/X_2^c) = 1 - \frac{p(X_2)}{\sum_{i=2}^n p(X_i)} \quad (11)$$

$$p(X_3^c) = \frac{p(X_3)}{\sum_{i=3}^n p(X_i)}, p(/X_3^c) = 1 - \frac{p(X_3)}{\sum_{i=3}^n p(X_i)}$$

$$p(X_n^c) = 1$$

Using the conditional events in Equations (10) and (11), partitioning events and their probabilities can be expressed as in Equations (12) and (13).

$$X_2 = /X_1^c X_2^c$$

$$X_3 = /X_1^c /X_2^c X_3^c \quad (12)$$

$$X_n = /X_1^c /X_2^c \dots /X_{n-1}^c X_n^c$$

$$p(X_2) = p(/X_1^c)p(X_2^c)$$

$$p(X_3) = p(/X_1^c)p(/X_2^c)p(X_3^c) \quad (13)$$

$$p(X_n) = p(/X_1^c)p(/X_2^c) \dots p(/X_{n-1}^c)p(X_n^c)$$

Finally, $f(\mathbf{X}, \mathbf{B})$ in Equation (3) can be converted to Equation (14). Please note that the terms on the right-hand side in Equation (14) are explicitly mutually exclusive since $X_i^c/X_i^c = 0$.

When converting MCSs of $f(\mathbf{X}, \mathbf{B})$ into exact solutions, any combination of the terms on the right-hand side in Equation (14) becomes an empty set since $X_i^c/X_i^c = 0$. This is a strength of the method proposed in this paper.

$$f(\mathbf{X}, \mathbf{B}) = X_1^c f_1(\mathbf{B}) + /X_1^c X_2^c f_2(\mathbf{B}) + \dots + /X_1^c /X_2^c \dots /X_{n-1}^c X_n^c f_n(\mathbf{B}) \quad (14)$$

The Boolean equations in Equations (3) and (14) are identical. It should be noted that the partitioning events in Equation (3) can be modelled in the fault tree using conditional events as in Equation (14). If a fault tree has Boolean AND combinations of partitioning events and they are converted into conditional events, the MCS generation tool from the fault tree automatically deletes these AND combinations (e.g., $X_2X_3 = (/X_1^c X_2^c)(/X_1^c /X_2^c X_3^c) = 0$). Furthermore, the MCS conversion tool to exact solutions automatically deletes similar combinations of conditional events. With this modelling, the underestimation of CDF can be avoided.

3. EXCLUSIVE MODELLING OF MULTIPLE GROUP PARTITIONING EVENTS

A system or NPP can have multiple groups of partitioning events as in Equation (15) [19]. Here, \mathbf{X} , \mathbf{Y} , and \mathbf{Z} are the first, second, and third groups of partitioning events, respectively. \mathbf{B} has regular basic events.

$$\begin{aligned} f(\mathbf{X}, \mathbf{B}) &= X_1 f_1(\mathbf{B}) + X_2 f_2(\mathbf{B}) + \dots \\ g(\mathbf{Y}, \mathbf{B}) &= Y_1 g_1(\mathbf{B}) + Y_2 g_2(\mathbf{B}) + \dots \\ h(\mathbf{Z}, \mathbf{B}) &= Z_1 h_1(\mathbf{B}) + Z_2 h_2(\mathbf{B}) + \dots \end{aligned} \quad (15)$$

The partitioning events satisfy the following equations. This can be shown by using the Venn diagram in Fig. 1.

$$\begin{aligned} X_i X_j &= Y_i Y_j = Z_i Z_j = 0, i \neq j \\ \sum_i X_i &= \sum_j Y_j = \sum_k Z_k = 1 \end{aligned} \quad (16)$$

Similar to the conversion in Section 3, all terms on the right-hand in Equation (15) can be exclusively converted to Equation (17). Please note that the terms on the right-hand side in Equation (17) are explicitly mutually exclusive. When calculating probabilities of $f(\mathbf{X}, \mathbf{B})$, $g(\mathbf{Y}, \mathbf{B})$, $h(\mathbf{Z}, \mathbf{B})$, $f(\mathbf{X}, \mathbf{B})g(\mathbf{Y}, \mathbf{B})h(\mathbf{Z}, \mathbf{B})$, and $(\mathbf{X}, \mathbf{B}) + g(\mathbf{Y}, \mathbf{B}) + h(\mathbf{Z}, \mathbf{B})$, combinations that have X_i^c/X_i^c , Y_j^c/Y_j^c , or Z_k^c/Z_k^c are automatically deleted since they have explicitly mutually exclusive events combinations. This is a strength of the new method presented in this paper.

$$\begin{aligned} f(\mathbf{X}, \mathbf{B}) &= X_1^c f_1(\mathbf{B}) + X_2^c f_2(\mathbf{B}) + \dots \\ g(\mathbf{Y}, \mathbf{B}) &= Y_1^c g_1(\mathbf{B}) + Y_2^c g_2(\mathbf{B}) + \dots \\ h(\mathbf{Z}, \mathbf{B}) &= Z_1^c h_1(\mathbf{B}) + Z_2^c h_2(\mathbf{B}) + \dots \end{aligned} \quad (17)$$

4. APPLICATION TO A SIMPLE SYSTEM

The new method was applied to the simple Boolean equation in Equation (18) [19]. Probabilities of partitioning events and regular basic events are shown in Equation (19).

$$f(\mathbf{X}, \mathbf{B}) = X_1 B_1 + X_2 B_2 + X_3 B_3 \quad (18)$$

$$p(X_1) = 0.5, p(X_2) = 0.3, p(X_3) = 0.2 \quad (19)$$

$$p(B_1) = p(B_2) = p(B_3) = 0.9$$

The probability of a Boolean equation $f(\mathbf{X}, \mathbf{B})$ in Equation (18) can be calculated by the inclusion–exclusion equation [20] as in Equation (20). To avoid the underestimated $p(f(\mathbf{X}, \mathbf{B}))$ in Equation (20), the fourth to seventh terms on the right-hand side in Equation (20) should be deleted since they have partitioning event combinations. If an NPP has many alternating operation systems, the fault tree for this NPP would have multiple group partitioning events. In this case, it is difficult to find and delete complex combinations of partitioning events. Furthermore, there is no dedicated tool to delete these complex combinations of partitioning events.

$$\begin{aligned} p(f(\mathbf{X}, \mathbf{B})) &= p(X_1 B_1) + p(X_2 B_2) + p(X_3 B_3) \\ &- p(X_1 X_2 B_1 B_2) - p(X_1 X_3 B_1 B_3) - p(X_2 X_3 B_2 B_3) \\ &+ p(X_1 X_2 X_3 B_1 B_2 B_3) = 0.67077 \end{aligned} \quad (20)$$

To explicitly avoid the underestimation of the probability of $f(\mathbf{X}, \mathbf{B})$, Equation (18) can be converted into Equation (21), which is similar to Equation (14). Here, the conditional events and their probabilities are shown via Equations (22) and (23).

$$f(\mathbf{X}, \mathbf{B}) = X_1^c B_1 + /X_1^c X_2^c B_2 + /X_1^c /X_2^c X_3^c B_3 \quad (21)$$

where the conditional events are defined as

$$\begin{aligned} X_1^c &\equiv X_1 \\ X_2^c &\equiv X_2 | /X_1 \\ X_3^c &\equiv X_3 | /X_1 /X_2 \end{aligned} \quad (22)$$

and their probabilities are

$$\begin{aligned} p(X_1^c) &= p(X_1) = 0.5 \\ p(X_2^c) &= \frac{p(X_2)}{p(X_2) + p(X_3)} = \frac{0.3}{0.3 + 0.2} = 0.6 \\ p(X_3^c) &= 1 \end{aligned} \quad (23)$$

The accurate probability of $f(\mathbf{X}, \mathbf{B})$ can be calculated by Equation (24) without employing any other techniques or dedicated PSA tools. This is a great strength of the method proposed in this paper.

$$\begin{aligned} p(f(\mathbf{X}, \mathbf{B})) &= p(X_1^c B_1) + p(/X_1^c X_2^c B_2) + p(/X_1^c /X_2^c X_3^c B_3) \\ &= 0.5 \times 0.9 + 0.5 \times 0.6 \times 0.9 + 0.5 \times 0.4 \times 1.0 \times 0.9 = 0.9 \end{aligned} \quad (24)$$

5. APPLICATION TO CCW SYSTEM

A configuration of component cooling water system (CCWS) with four trains is depicted in Fig. 2. This system has (1) two trains ① and ② for CCW load A, (2) two trains ③ and ④ for CCW load B, and (3) four trains ①, ②, ③, and ④ for CCW common load. Each train has a designated pump and heat exchanger. The train failures and load failures are defined in Table 1. There are three possible fault tree modelling methods for the failures of CCW load A, load B, and common load in Table 2.

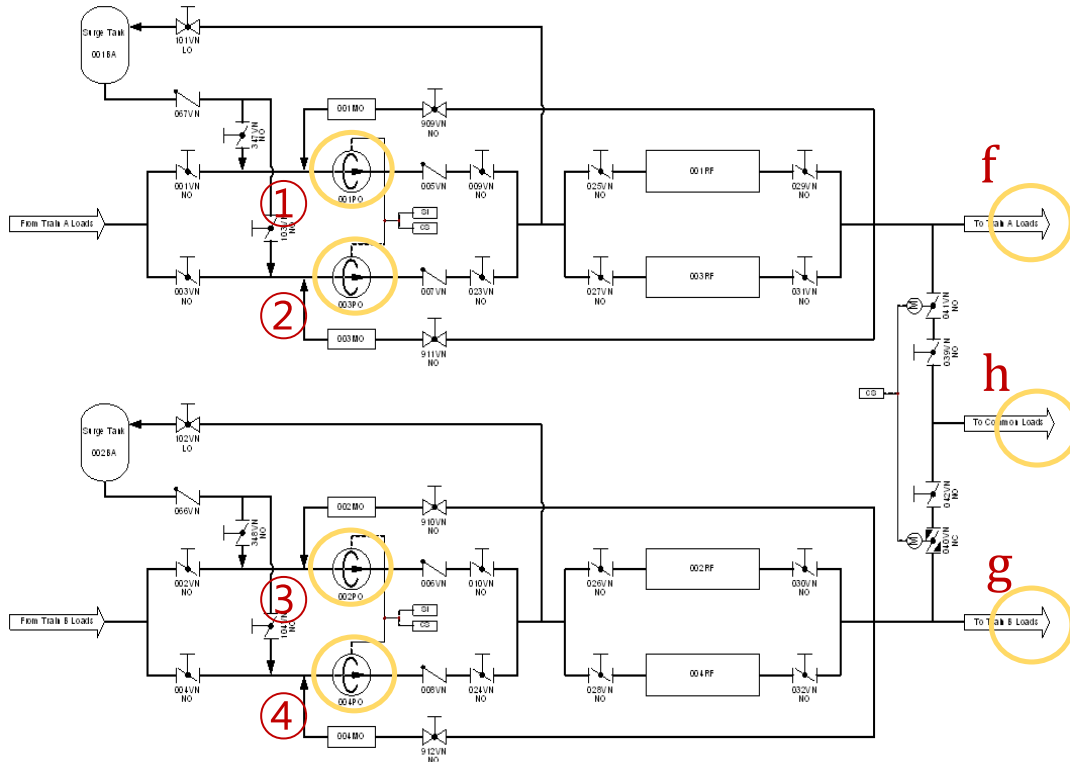


Fig. 2. Simplified P&ID of CCWS

Table 1. Failure definitions

<p>f = failure of CCW load A = failure of trains ① and ② g = failure of CCW load B = failure of trains ③ and ④ h = failure of CCW comon load = failure of trains ①, ②, ③ and ④ f₁ = train ① failure f₂ = train ② failure g₁ = train ③ failure g₂ = train ④ failure</p>
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Table 2. Fault tree modelling methods

Method 1	Usual	Fault tree is modelled for one selected system alignment. That is, it has no partitioning or conditioning events.
Method 2	Partitioning events	Fault tree is modelled for N system alignments. Partitioning events are used for system alignment modelling.
Method 3	Conditional events	Fault tree is modelled for N system alignments. Conditional events are used for system alignment modelling.

For a normal CCWS operation, one train out of ① and ②, and another train out of ③ and ④ are in operation. Therefore, two trains out of four combinations {①③, ①④, ②③, ②④} can be in operation, and the other two trains are in a standby state. If the operation of one train fails, standby train should be started. For example, if the train ① fails, the standby train ② should be started.

Load A and B failures for Method 2 can be formulated with partitioning events as Eqs. (25) and (26). Here, ①_s and ①_r denotes start and running failure of train ①. As listed in Table 3, load A and B failures for Method 3 are generated by converting partitioning events into conditional events.

$$\begin{aligned}
 f &= X_1 f_1 + X_2 f_2 \\
 g &= Y_1 g_1 + Y_2 g_2
 \end{aligned}
 \tag{25}$$

where

$$\begin{aligned}
 X_1 + X_2 &= 1 \text{ and } X_1 X_2 = 0 \\
 Y_1 + Y_2 &= 1 \text{ and } Y_1 Y_2 = 0 \\
 f_1 &= \textcircled{1}_r \times (\textcircled{2}_s + \textcircled{2}_r) \\
 f_2 &= (\textcircled{1}_s + \textcircled{1}_r) \times \textcircled{2}_r \\
 g_1 &= \textcircled{3}_r \times (\textcircled{4}_s + \textcircled{4}_r) \\
 g_2 &= (\textcircled{3}_s + \textcircled{3}_r) \times \textcircled{4}_r
 \end{aligned}
 \tag{26}$$

Table 3. Fault modelling methods for CCW load A and B failures

Method 1	f = f ₁	p(X ₁) = 1.0, p(X ₂) = 0.0
	g = g ₁	p(Y ₁) = 1.0, p(Y ₂) = 0.0
Method 2	f = X ₁ f ₁ + X ₂ f ₂	p(X ₁) = p(X ₂) = 0.5
	g = Y ₁ g ₁ + Y ₂ g ₂	p(Y ₁) = p(Y ₂) = 0.5
Method 3	f = /X ₁ ^c f ₁ + /X ₁ ^c X ₂ ^c f ₂	p(X ₁ ^c) = p(X ₁) = 0.5 p(X ₂ ^c) = 0.5/0.5 = 1.0
	g = /Y ₁ ^c g ₁ + /Y ₁ ^c Y ₂ ^c g ₂	p(Y ₁ ^c) = p(Y ₁) = 0.5 p(Y ₂ ^c) = 0.5/0.5 = 1.0

The fault tree for the common load failure for Method 2 can be formulated with four partitioning events as Eqs. (27) and (28). As listed in Table 4, common load failure for Method 3 can be generated by converting partitioning events into conditional events.

$$h = X_1h_1 + X_2h_2 + X_3h_3 + X_4h_4 \quad (27)$$

where $X_1 + X_2 + X_3 + X_4 = 1$ and $X_iX_j = 0, i \neq j$

$$\begin{aligned} h_1 &= \textcircled{1}_r \times (\textcircled{2}_s + \textcircled{2}_r) \times \textcircled{3}_r \times (\textcircled{4}_s + \textcircled{4}_r) \text{ when train } \textcircled{1} \text{ and } \textcircled{3} \text{ are running} \\ h_2 &= \textcircled{1}_r \times (\textcircled{2}_s + \textcircled{2}_r) \times (\textcircled{3}_s + \textcircled{3}_r) \times \textcircled{4}_r \text{ when train } \textcircled{1} \text{ and } \textcircled{4} \text{ are running} \\ h_3 &= (\textcircled{1}_s + \textcircled{1}_r) \times \textcircled{2}_r \times \textcircled{3}_r \times (\textcircled{4}_s + \textcircled{4}_r) \text{ when train } \textcircled{2} \text{ and } \textcircled{3} \text{ are running} \\ h_4 &= (\textcircled{1}_s + \textcircled{1}_r) \times \textcircled{2}_r \times (\textcircled{3}_s + \textcircled{3}_r) \times \textcircled{4}_r \text{ when train } \textcircled{2} \text{ and } \textcircled{4} \text{ are running} \end{aligned} \quad (28)$$

Table 4. Fault tree modelling methods for CCW common load failure

Method 1	$h = h_1$	$p(X_1) = 1.0, p(X_2) = p(X_3) = p(X_4) = 0.0$
Method 2	$h = X_1h_1 + X_2h_2 + X_3h_3 + X_4h_4$	$p(X_1) = p(X_2) = p(X_3) = p(X_4) = \frac{1}{4}$
Method 3	$h = X_1^c h_1 + X_1^c X_2^c h_2 + X_1^c / X_2^c X_3^c h_3 + X_1^c / X_2^c X_3^c X_4^c h_4$	$p(X_1^c) = p(X_1) = \frac{1}{4}$ $p(X_2^c) = \frac{p(X_2)}{p(X_2) + p(X_3) + p(X_4)} = \frac{0.25}{0.75} = \frac{1}{3}$ $p(X_3^c) = \frac{p(X_3)}{p(X_3) + p(X_4)} = \frac{0.25}{0.50} = \frac{1}{2}$ $p(X_4^c) = \frac{0.25}{0.25} = 1$

In this application, CDF was defined in the fault tree as the product of h (common load failure) in Table 1 and an initiator (internal event initiator or seismic initiator) as Eq. (29).

$$\begin{aligned} \text{CDF}_{\text{internal}} &= \%I_{\text{internal}} \times h \\ \text{CDF}_{\text{seismic}} &= \%I_{\text{seismic}} \times h \end{aligned} \quad (29)$$

Fault trees for seismic event were developed by replacing internal event initiator with seismic initiator and by adding seismic induced failure events. The added seismic induced failure events are SIF-CC-HXS and SIF-CC-PPS as shown in Appendix A. All the basic events data of CCWS fault trees for internal and seismic events for Method 1, 2, and 3 are listed in Table A.1. The CCWS fault tree for seismic event with Method 3 is listed in Table A.2. System alignment 1 is for h_1 , 2 for h_2 , 3 for h_3 , and 4 for h_4 .

For internal and seismic CDF calculation, fault trees and minimal cut sets (MCSs) were generated from the fault tree modelling methods in Table 4. Internal and seismic CDFs were calculated with MCSs using three different methods of rare event approximation (REA), min cut upper bound (MCUB), and binary decision diagram (BDD) [21, 22]. The internal and seismic CDFs are listed in Tables 5 and 6. As shown in Table 5, there are no significant CDF differences in the various CDF calculation methods in Table 4. It is because all internal events are rare events. As listed in Table 6, seismic BDD-based CDF of Method 2 was drastically underestimated. Thus, these results show that Method 3 that is proposed in this study is recommended to correctly model the fault trees of alternating operation systems and correctly calculate importance measures of all components.

Table 5. Internal event CDFs

Methods	No of MCSs	CDF by REA	CDF by MCUB	CDF by BDD
1	246	4.751E-10	4.751E-10	4.751E-10
2	981	4.751E-10	4.751E-10	4.751E-10
3	981	4.751E-10	4.751E-10	4.751E-10

Table 6. Seismic event CDFs

Methods	No of MCSs	CDF by REA	CDF by MCUB	CDF by BDD
1	248	5.830E-5	5.830E-5	4.998E-5
2	986	5.830E-5	5.830E-5	4.207E-5
3	986	5.830E-5	5.830E-5	4.998E-5

6. CONCLUSIONS

There are several systems undergoing alternating operations in NPPs, and each NPP alternates between full power and LPSD. Therefore, complex Boolean AND combinations of mutually exclusive partitioning events should be eliminated when generating MCSs from a fault tree and converting MCSs into exact solutions.

For the correct probability calculation of a fault tree that has partitioning events, a proper modelling method of these events was proposed in Section 3, and the strength and simplicity of this modelling method were demonstrated by the applications in Sections 4 and 5. If MCSs for seismic SUPSA and MUPSA are generated and converted into exact solutions without deleting combinations of mutually exclusive partitioning events, final CDFs (SUCDF, MUCDF, and SCDF) can be underestimated. Unfortunately, it is impossible to eliminate mutually exclusive event combinations without modifying PSA tools for generating MCSs from a fault tree and converting MCSs into a BDD.

Therefore, there has been a great need to explicitly model the partitioning events in fault trees instead of revising PSA tools. This paper provides a solution to avoid CDF underestimation. If the partitioning events are modelled with conditional events in the seismic SUPSA and MUPSA fault trees with the method suggested in this paper, accurate CDF calculation is possible using the existing PSA tools. This is the strength of the proposed method. The use of the method suggested in this paper is strongly recommended for avoiding CDF underestimation in seismic SUPSA and MUPSA.

The failures of alternating operation systems are frequently modelled in internal, flooding, and fire event SUPSAs. Therefore, for calculating accurate CDF, it is also recommended that the modelling method of partitioning events suggested in this paper be applied to any SUPSAs where the failures of alternating operation systems are modelled.

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Appendix A.

Table A.1. Basic events and initiators of CCWS

Basic Event Name	Data	Basic Event Description	Remark
%IE-INT	1.00E-02	Internal event initiator	For internal event
%IE-SEIS	1.00E-04	Seismic event initiator	For seismic event
AO-CASE-01	2.50E-01	Partitioning event for system alignment 01	For Method 2, partitioning event
AO-CASE-01-CE	2.50E-01	Conditional event for system alignment 01	For Method 3, conditional event
AO-CASE-02	2.50E-01	Partitioning event for system alignment 02	For Method 2, partitioning event
AO-CASE-02-CE	3.33E-01	Conditional event for system alignment 02	For Method 3, conditional event
AO-CASE-03	2.50E-01	Partitioning event for system alignment 03	For Method 2, partitioning event
AO-CASE-03-CE	5.00E-01	Conditional event for system alignment 03	For Method 3, conditional event
AO-CASE-04	2.50E-01	Partitioning event for system alignment 04	For Method 2, partitioning event
AO-CASE-04-CE	1.00E+00	Conditional event for system alignment 04	For Method 3, conditional event
CCHXY-001RF	7.29E-06	CCW HX 001RF fails to run	For internal and seismic event
CCHXY-002RF	7.29E-06	CCW HX 002RF fails to run	For internal and seismic event
CCHXY-003RF	7.29E-06	CCW HX 003RF fails to run	For internal and seismic event
CCHXY-004RF	7.29E-06	CCW HX 004RF fails to run	For internal and seismic event
CCMPKQ2-PP01/02PO	1.14E-07	2/4 CCF OF CCW PP 01/02PO fail to run	For internal and seismic event
CCMPKQ2-PP01/03PO	1.14E-07	2/4 CCF OF CCW PP 01/03PO fail to run	For internal and seismic event
CCMPKQ2-PP01/04PO	1.14E-07	2/4 CCF OF CCW PP 01/04PO fail to run	For internal and seismic event
CCMPKQ2-PP02/03PO	1.14E-07	2/4 CCF OF CCW PP 02/03PO fail to run	For internal and seismic event
CCMPKQ2-PP02/04PO	1.14E-07	2/4 CCF OF CCW PP 02/04PO fail to run	For internal and seismic event
CCMPKQ2-PP03/04PO	1.14E-07	2/4 CCF OF CCW PP 03/04PO fail to run	For internal and seismic event
CCMPKQ3-PP01/02/03PO	3.17E-08	3/4 CCF OF CCW PP 01/02/03PO fail to run	For internal and seismic event
CCMPKQ3-PP01/02/04PO	3.17E-08	3/4 CCF OF CCW PP 01/02/04PO fail to run	For internal and seismic event
CCMPKQ3-PP01/03/04PO	3.17E-08	3/4 CCF OF CCW PP 01/03/04PO fail to run	For internal and seismic event
CCMPKQ3-PP02/03/04PO	3.17E-08	3/4 CCF OF CCW PP 02/03/04PO fail to run	For internal and seismic event
CCMPKQ4-PP01/02/03/04PO	4.74E-08	4/4 CCF OF CCW PP 01/02/03/04PO fail to run	For internal and seismic event
CCMPR-PP01PO	2.59E-05	CCW PP 01PO fails to run	For internal and seismic event
CCMPR-PP02PO	2.59E-05	CCW PP 02PO fails to run	For internal and seismic event
CCMPR-PP03PO	2.59E-05	CCW PP 03PO fails to run	For internal and seismic event
CCMPR-PP04PO	2.59E-05	CCW PP 04PO fails to run	For internal and seismic event
CCMPS-PP01PO	1.88E-03	CCW PP 01PO fails to start	For internal and seismic event
CCMPS-PP02PO	1.88E-03	CCW PP 02PO fails to start	For internal and seismic event
CCMPS-PP03PO	1.88E-03	CCW PP 03PO fails to start	For internal and seismic event
CCMPS-PP04PO	1.88E-03	CCW PP 04PO fails to start	For internal and seismic event
CCMPWQ2-PP01/02PO	5.46E-06	2/4 CCF OF CCW PP 01/02PO	For internal and seismic event
CCMPWQ2-PP01/03PO	5.46E-06	2/4 CCF OF CCW PP 01/03PO	For internal and seismic event
CCMPWQ2-PP01/04PO	5.46E-06	2/4 CCF OF CCW PP 01/04PO	For internal and seismic event
CCMPWQ2-PP02/03PO	5.46E-06	2/4 CCF OF CCW PP 02/03PO	For internal and seismic event
CCMPWQ2-PP02/04PO	5.46E-06	2/4 CCF OF CCW PP 02/04PO	For internal and seismic event
CCMPWQ2-PP03/04PO	5.46E-06	2/4 CCF OF CCW PP 03/04PO	For internal and seismic event
CCMPWQ3-PP01/02/03PO	1.80E-06	3/4 CCF OF CCW PP 01/02/03PO	For internal and seismic event
CCMPWQ3-PP01/02/04PO	1.80E-06	3/4 CCF OF CCW PP 01/02/04PO	For internal and seismic event
CCMPWQ3-PP01/03/04PO	1.80E-06	3/4 CCF OF CCW PP 01/03/04PO	For internal and seismic event
CCMPWQ3-PP02/03/04PO	1.80E-06	3/4 CCF OF CCW PP 02/03/04PO	For internal and seismic event
CCMPWQ4-PP01/02/03/04PO	2.98E-06	4/4 CCF OF CCW PP 01/02/03/04PO	For internal and seismic event
SIF-CC-HXS	2.50E-01	Seismic induced failure of CCW heat exchangers	For seismic event
SIF-CC-PPS	3.33E-01	Seismic induced failure of CCW pumps	For seismic event

Table A.2. Fault tree for calculating seismic CDF with Method 3

CDF-SEIS-CE * %IE-SEIS GCC-TRAIN-AB-SEIS	
GCC-TRAIN-AB-SEIS * GCC-TRAIN-AB-SYS	
G-AO-CASE-01 + AO-CASE-01-CE	
G-AO-CASE-02 * AO-CASE-02-CE -AO-CASE-01-CE	
G-AO-CASE-03 * AO-CASE-03-CE G-NOT-CASE-01 G-NOT-CASE-02	
G-NOT-CASE-01 + -AO-CASE-01-CE	
G-NOT-CASE-02 + -AO-CASE-02-CE	
G-AO-CASE-04 * AO-CASE-04-CE G-NOT-CASE-01 G-NOT-CASE-02 G-NOT-CASE-03	
G-NOT-CASE-03 + -AO-CASE-03-CE	
G-AO-FTR + G-AO-CASE-01 G-AO-CASE-02 G-AO-CASE-03 G-AO-CASE-04	
G-AO-PP01PO-FTS + G-AO-CASE-03 G-AO-CASE-04	
G-AO-PP02PO-FTS + G-AO-CASE-02 G-AO-CASE-04	
G-AO-PP03PO-FTS + G-AO-CASE-01 G-AO-CASE-02	
G-AO-PP04PO-FTS + G-AO-CASE-01 G-AO-CASE-03	
GCC-HX-A * CCHXY-001RF CCHXY-003RF	
GCC-HX-A-SEIS + GCC-HX-A SIF-CC-HXS	
GCC-HX-B * CCHXY-002RF CCHXY-004RF	
GCC-HX-B-SEIS + GCC-HX-B SIF-CC-HXS	
GCC-PP-A * GCC-PP01PO GCC-PP03PO	
GCC-PP01PO + GCC-PP01PO-FTS-PE GCC-PP01PO-FTR-PE	
GCC-PP03PO + GCC-PP03PO-FTS-PE GCC-PP03PO-FTR-PE	
GCC-PP-B * GCC-PP02PO GCC-PP04PO	
GCC-PP02PO + GCC-PP02PO-FTS-PE GCC-PP02PO-FTR-PE	
GCC-PP04PO + GCC-PP04PO-FTS-PE GCC-PP04PO-FTR-PE	
GCC-PP01PO-FTS-PE * GCC-PP01PO-FTS G-AO-PP01PO-FTS	
GCC-PP01PO-FTR-PE * GCC-PP01PO-FTR G-AO-FTR	
GCC-PP01PO-CCF-D + CCMPWQ4-PP01/02/03/04PO GCC-PP01PO-CCFQ2-D GCC-PP01PO-CCFQ3-D	
GCC-PP01PO-CCFQ2-D + CCMPWQ2-PP01/03PO CCMPWQ2-PP01/02PO CCMPWQ2-PP01/04PO	
GCC-PP01PO-CCFQ3-D + CCMPWQ3-PP01/02/03PO CCMPWQ3-PP01/03/04PO CCMPWQ3-PP01/02/04PO	
GCC-PP01PO-CCF-R + CCMPKQ4-PP01/02/03/04PO GCC-PP01PO-CCFQ2-R GCC-PP01PO-CCFQ3-R	
GCC-PP01PO-CCFQ2-R + CCMPKQ2-PP01/02PO CCMPKQ2-PP01/03PO CCMPKQ2-PP01/04PO	
GCC-PP01PO-CCFQ3-R + CCMPKQ3-PP01/02/03PO CCMPKQ3-PP01/02/04PO CCMPKQ3-PP01/03/04PO	
GCC-PP01PO-FTR + CCMPR-PP01PO GCC-PP01PO-CCF-R SIF-CC-PPS	
GCC-PP01PO-FTS + CCMPs-PP01PO GCC-PP01PO-CCF-D	
GCC-PP02PO-FTS-PE * GCC-PP02PO-FTS G-AO-PP02PO-FTS	
GCC-PP02PO-FTR-PE * GCC-PP02PO-FTR G-AO-FTR	
GCC-PP02PO-CCF-D + CCMPWQ4-PP01/02/03/04PO GCC-PP02PO-CCFQ2-S GCC-PP02PO-CCFQ3-S	
GCC-PP02PO-CCFQ2-S + CCMPWQ2-PP01/02PO CCMPWQ2-PP02/03PO CCMPWQ2-PP02/04PO	
GCC-PP02PO-CCFQ3-S + CCMPWQ3-PP01/02/03PO CCMPWQ3-PP01/02/04PO CCMPWQ3-PP02/03/04PO	
GCC-PP02PO-CCF-R + CCMPKQ4-PP01/02/03/04PO GCC-PP02PO-CCFQ2-R GCC-PP02PO-CCFQ3-R	
GCC-PP02PO-CCFQ2-R + CCMPKQ2-PP01/02PO CCMPKQ2-PP02/03PO CCMPKQ2-PP02/04PO	
GCC-PP02PO-CCFQ3-R + CCMPKQ3-PP01/02/03PO CCMPKQ3-PP01/02/04PO CCMPKQ3-PP02/03/04PO	
GCC-PP02PO-FTR + CCMPR-PP02PO SIF-CC-PPS GCC-PP02PO-CCF-R	
GCC-PP02PO-FTS + CCMPs-PP02PO GCC-PP02PO-CCF-D	
GCC-PP03PO-FTS-PE * GCC-PP03PO-FTS G-AO-PP03PO-FTS	
GCC-PP03PO-FTR-PE * GCC-PP03PO-FTR G-AO-FTR	
GCC-PP03PO-CCF-D + CCMPWQ4-PP01/02/03/04PO GCC-PP03PO-CCFQ2-D GCC-PP03PO-CCFQ3-D	
GCC-PP03PO-CCFQ2-D + CCMPWQ2-PP01/03PO CCMPWQ2-PP02/03PO CCMPWQ2-PP03/04PO	
GCC-PP03PO-CCFQ3-D + CCMPWQ3-PP01/02/03PO CCMPWQ3-PP01/03/04PO CCMPWQ3-PP02/03/04PO	
GCC-PP03PO-CCF-R + CCMPKQ4-PP01/02/03/04PO GCC-PP03PO-CCFQ2-R GCC-PP03PO-CCFQ3-R	
GCC-PP03PO-CCFQ2-R + CCMPKQ2-PP01/03PO CCMPKQ2-PP02/03PO CCMPKQ2-PP03/04PO	
GCC-PP03PO-CCFQ3-R + CCMPKQ3-PP01/02/03PO CCMPKQ3-PP01/03/04PO CCMPKQ3-PP02/03/04PO	
GCC-PP03PO-FTR + CCMPR-PP03PO SIF-CC-PPS GCC-PP03PO-CCF-R	
GCC-PP03PO-FTS + CCMPs-PP03PO GCC-PP03PO-CCF-D	
GCC-PP04PO-FTS-PE * GCC-PP04PO-FTS G-AO-PP04PO-FTS	
GCC-PP04PO-FTR-PE * GCC-PP04PO-FTR G-AO-FTR	
GCC-PP04PO-CCF-R + CCMPKQ4-PP01/02/03/04PO GCC-PP04PO-CCFQ2-R GCC-PP04PO-CCFQ3-R	
GCC-PP04PO-CCFQ2-R + CCMPKQ2-PP01/04PO CCMPKQ2-PP02/04PO CCMPKQ2-PP03/04PO	
GCC-PP04PO-CCFQ3-R + CCMPKQ3-PP01/02/04PO CCMPKQ3-PP01/03/04PO CCMPKQ3-PP02/03/04PO	
GCC-PP04PO-CCF-S + CCMPWQ4-PP01/02/03/04PO GCC-PP04PO-CCFQ2-S GCC-PP04PO-CCFQ3-S	
GCC-PP04PO-CCFQ2-S + CCMPWQ2-PP01/04PO CCMPWQ2-PP02/04PO CCMPWQ2-PP03/04PO	
GCC-PP04PO-CCFQ3-S + CCMPWQ3-PP01/02/04PO CCMPWQ3-PP01/03/04PO CCMPWQ3-PP02/03/04PO	
GCC-PP04PO-FTR + CCMPR-PP04PO SIF-CC-PPS GCC-PP04PO-CCF-R	
GCC-PP04PO-FTS + CCMPs-PP04PO GCC-PP04PO-CCF-S	
GCC-TRAIN-A + GCC-PP-A GCC-HX-A-SEIS	
GCC-TRAIN-AB-SYS * GCC-TRAIN-A GCC-TRAIN-B	
GCC-TRAIN-B + GCC-PP-B GCC-HX-B-SEIS	
ENDTREE	
PROCESS	CDF-SEIS-CE