Review of Quantum(-Inspired) Optimization Methods for System Reliability Problems

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Abstract: Many industrial systems demand equipment with high levels of reliability. Companies and academia have been developing, over the years, mathematical methods and advancing engineering techniques to assist in the maintenance of active and reliable systems. There are different optimization problems in this context, highlighting (1) the redundancy allocation problem (RAP), (2) the reliability allocation problem, and (3) the reliability-redundancy allocation problem (RRAP). Many solving methods have already been applied to these problems, e.g., dynamic, linear, integer, and nonlinear programming, as well as classical metaheuristics based on evolutionary algorithms, such as the Genetic Algorithm (GA). Either way, these methods are modeled according to the specificities of the systems. However, these approaches can be very computationally expensive depending on the problem instances. Meanwhile, quantum computing has gained ground for combinatorial optimization problems. It is expected that problems with a high level of complexity can be solved more efficiently using these new methods than classical ones. Optimization methods have attempted to improve their efficiency by adding quantum concepts, as is the case of Quantum-inspired Evolutionary Algorithms (QEA). The QEA has a better diversity and convergence rate than other EAs because it uses qubit representation instead of numerical, binary, or symbolic representations. In this context, this paper aims to develop a systematic review of the literature through keyword filtering, article reading, and bibliometric analysis on applying purely quantum and quantum-inspired methods in system reliability optimization problems, specifically in RAP, the reliability allocation problem, and the RRAP. Our goal is to identify quantum-based techniques' advantages, limitations, and potential in such a context and suggest a research plan based on the observed literature gaps. We observed few studies in the context, especially in classical quantum, which indicates an open field for exploration and applications.

1. INTRODUCTION

Many industrial systems demand equipment with high levels of reliability. For this purpose, the redundancy allocation (RAP), the reliability allocation and the reliability-redundancy allocation (RRAP) problems, well known in the literature, are widely applied. According to Du and Li [1], there are applications, among others, in computer networks, electrical power systems, and telecommunications systems.

These types of problems also appear recurrently in the system development phase. Two main situations of their use can be emphasized. The first relates to reliability allocation, where, in early phases of development, where changes to components are possible, reliability allocation is important because it may guide important (re-)design decisions to meet reliability requirements at the lowest possible cost [2], [3]. Chatterjee *et al*. [3], for example, studies this problem at the development stage of a software program. Second, if the components that compose the system are existing solutions and it is not possible to intervene to improve reliability. Then, the remaining thing to enhance the system reliability is to allocate redundancies [4], [5]. This type of study has been applied, for example, in the optimization of maintenance operations [6].

Those problems are NP-hard [7]. In the case of small instances, it can be solved exactly. Otherwise, meta-heuristics have been developed and applied to deal with the problem efficiently, for example, the use of evolutionary algorithms (EA), such as Genetic Algorithms [8], Ant Colony Optimization (ACO) [9], Tabu Search [10], and Particle Swarm Optimization (PSO) [11]. Although they deal better with increasing instances, these methods are still parameter sensitive and computationally expensive.

As an advancement of these approaches, quantum-inspired models are being added to these methods for improved efficiency. A good example of them is the quantum-inspired evolutionary algorithm (QEA). A population search-based meta-heuristic that uses the concepts of quantum bits (qubits) and superposition of states to explore the solution space [12]. QEA has a better diversity and convergence rate than other EAs because it uses qubit representation instead of numerical, binary or symbolic representations [13]. This methodology has been applied by Li and Zio [7] and by Du and Li [1], who added local search techniques to this approach.

Nevertheless, quantum computing techniques based on quantum annealing have also been applied, considering the conversion of the original mathematical formulation to a quadratic unconstrained binary optimization (QUBO) of RAP [14].

To the best of our knowledge, there are few applications to this problem in this domain. So, this paper aims to conduct a literature review on these methods that have been applied in system reliability problems, specifically, RAP, RRAP and reliability-allocation problems, as well as gaps and research opportunities.

The remainder of this paper is structured as follows: Section 2 gives a short theoretical background about the system reliability problems, quantum computing and quantum-inspired methods. Section 3 addresses the research method adopted and the descriptive analyses of the sample of selected articles. Section 4 presents the results, discussions and gaps in the theme. Finally, Section 5 addresses the conclusions.

2. THEORETICAL BACKGROUND

2.1. System Reliability Optimization Problems

2.1.1. Redundancy Allocation Problem

Traditionally, RAP consists of allocating redundant components in subsystems to maximize their availability or reliability [15]. We can see several applications of this problem from a mono-objective point of view, i.e., when there is only one objective function to be optimized. The single objective function can be the above-mentioned availability under cost/budget constraints. Or the opposite: the objective is to minimize the costs of redundancy allocation under a minimum desired system reliability constraint [1], [7], [14].

Otherwise, there are also multi-objective applications [8], [9], [16], [17]. In this case, we obtain possible scenarios for conflicting objectives. For example, to increase reliability, the more redundancies, the better. On the other hand, the costs increase. So, classical applications of this problem seek to minimize costs and maximize reliability. Among those listed in an optimal Pareto front, the decision of the best solution to be implemented will be made based on the decision maker’s preferences. The latter may be maintenance managers, reliability engineers, experts, and others.

Besides the variations of the problem regarding the type of optimization (mono or multi-objective), RAPs can be classified based on the component states. There are two types, in general: (1) binary-state system (BSS) and the (2) multi-state system (MSS). In the BSS, the system and the component can only be in perfect operation or a complete failure state. The MSS, on the other hand, considers that there are different states between perfect operation and failure. Applications can also vary when it comes to system structures. For example, they can be series, parallel, series-parallel, and complex [17].

For the mathematical formulation of RAP, consider a system that needs to be designed with a specific number of subsystems in series. The latter may have different components in parallel, representing redundancies. The mono-objective problem maximizes system reliability (R), subject to a cost constraint (e.g., a limited budget c for equipment purchase and maintenance). There are different components (ct_j) available for each subsystem j (j = 1, 2, ..., s) that can perform the same function. Nevertheless, each subsystem must assume a minimum number ($n_{j,min}$) and a maximum number ($n_{j,max}$) of possible components to compose it [3, 7]. The mathematical formulation is shown in Eq. 1 through 4. Note that this is one of the possible RAP formulations. And, in particular, this case that includes multiple types of components has not yet been solved by quantum(-inspired) methods, as it will be discussed in the results section.

$$Maximize\ R(x) = \prod_{j=1}^{s}\left[1 - \prod_{k=1}^{ct_j}\left(1 - R_{jk}\right)^{x_{jk}}\right] \tag{1}$$

s.t.

$$\sum\nolimits_{j=1}^{s}\sum\nolimits_{k=1}^{ct_j} x_{jk} \cdot c_{jk} \leq c \tag{2}$$

$$n_{j,min} \leq \sum\nolimits_{k=1}^{ct_j} x_{jk} \leq n_{j,max} \tag{3}$$

$$x_{jk} \in \{0,1,\dots,n_{j,max}\} \tag{4}$$

2.1.2. Reliability Allocation Problem

RAP and the reliability allocation problem are two very different challenges. The system structure is fixed in the reliability allocation problem, while the component reliability values are continuous choice variables. There are no general constraints on the system structure for this topic. The cost of a component and other factors are mathematical functions of its reliability. Cost, weight, and other considerations, which may be incorporated as part of the restrictions or the goal function, increase when component reliability (and consequently system reliability) improves. Because the decision variables are continuous, multiple types of nonlinear programming can be employed to find the best solutions [18]. Consider a system composed of n elements. This system's purpose is to achieve a high level of reliability. The goal is to assign reliability to all or part of the system's components in order to achieve that goal at the lowest possible cost. Eq. 5-7 shows how the problem is expressed. Where, C is the total system cost, $c_i(R_i)$ is the cost of component/subsystem i, R_s is the system reliability, R_G is the system reliability goal, R_i is the reliability of component/subsystem i, $R_{i,min}$ is the minimum reliability of component/subsystem i, and, $R_{i,max}$ is the maximum achievable reliability of component/subsystem i [2], [19], [20].

$$Minimize\ C = \sum_{i=1}^{n} c_i(R_i) \tag{5}$$

s.t.

$$R_s \geq R_G \tag{6}$$

$$R_{i,min} \leq R_i \leq R_{i,max} \quad, i = 1, 2, \dots, n. \tag{7}$$

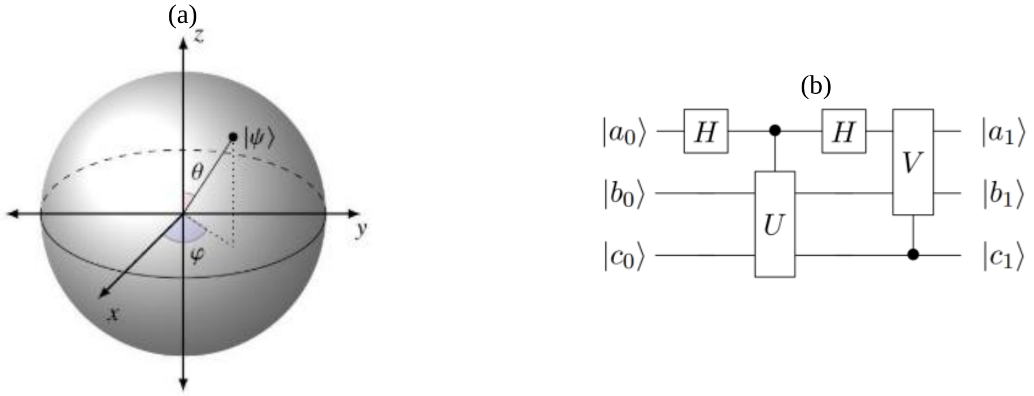
2.1.3. Reliability-redundancy Allocation Problem

The most generic problem formulation is the reliability-redundancy allocation problem. One or more "subsystems," or collections of logically connected sets of components, make up the system. Each subsystem has x_i components that are decision variables with r_i reliability. The goal is to maximize overall system reliability by allocating redundancy and reliability to the components of each subsystem in the most efficient way possible. Typically, the goal of optimization is to increase system reliability or reduce system costs. RRAP is frequently applied to series-parallel systems, but it can also be used in other systems, e.g., sequentially connected systems and sliding window systems [16], [18].

2.2. Quantum Computing

Classical computing is a system of logic gates that are based on bits 0 and 1. Meanwhile, in Quantum computing, so-called qubits can have superposition values between $|0\rangle$ and $|1\rangle$ before measuring them. A qubit can be represented by a Bloch sphere (Figure 1a), in which there are rotation functions that are carried out through its quantum gates. There are also interleaving operators that generate the connections of the qubits, which function as Not gate operators [21]. In Figure 1b, we can see a representation of a quantum circuit. The latter is a quantum computation paradigm that is comparable to classical circuits, in which a computation is made up of a series of quantum gates, measurements, and qubit initializations to known values. The circuit should be read from left to right. Initially we have $|a_0\rangle$, $|b_0\rangle$ and $|c_0\rangle$ which represent the input qubits, next there are the logic gates and the horizontal lines that represent the evolution of the qubits in time. At the end of the line, $|a_1\rangle$, $|b_1\rangle$ and $||c_1\rangle$ are the final qubits or result [22]

Figure 1: (a) Bloch Sphere as a graphical representation of a single qubit [21]**; (b) A Quantum Circuit example** [22]



If the ability to simulate classical computers were the only feature of quantum computers, there would be little point in going to all the trouble of exploiting quantum effects. The advantage of quantum computing is that much more powerful functions may be computed using qubits and quantum gates.

In recent years, many works have been studying the adoption of this whole quantum scope, previously mentioned, for optimization problems [22], [23]. The objective is to observe if these techniques bring advantages, such as increased computational speed and convergence. In this sense, quantum algorithms usually require problems to satisfy certain criteria to be applicable. For example, variational algorithms such as the Variational Quantum Eigensolver (VQE), Quantum Approximate Optimization Algorithm (QAOA) [23], Quantum annealing (QA) can only be applied to Quadratic Unconstrained Binary Optimization (QUBO) problems [14].

QUBO is an NP-hard problem and has gained prominence in recent years with the discovery that it unifies a wide variety of combinatorial optimization problems. By its association with the Ising problem in physics (Figure 2), the QUBO model has emerged as an underpinning of the quantum computing area known as quantum annealing and has become a subject of study in neuromorphic computing [14].

Figure 2: QUBO, Ising and Ising Hamiltonian models [14], [24]**.**

<u>QUBO</u>

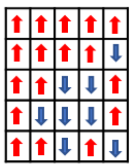
$$\min q\vec{x}^n \sum \vec{x} - \vec{x}^n \vec{\mu}$$

$$\vec{x} \in \{0,1\}^n$$

<u>Ising Model</u>

$$\min q\vec{z}^n \sum \vec{z} - \vec{z}^n \vec{\mu}$$

$$\vec{x} \in \{-1,1\}^n$$



<u>Ising Hamiltonian</u>

$$\min\langle\psi|\hat{H}|\psi\rangle$$

$$|\psi\rangle$$

$$\Downarrow$$

Ground State ≃ Optimal Solution

2.3. Quantum-inspired algorithms

Quantum classical metaheuristics consist of approximate algorithms that aim to improve the runtime and performance of the optimization process. However, they must be processed by quantum computers. In this context, quantum-inspired metaheuristics emerge so that it is possible to run the models with better performance on classical computers. Thus, quantum physics phenomena such as superposition and entanglement are simulated to explore quantum programming and predict future behavior and results [13].

In QEA the most basic element of an individual is the qubit. In contrast to the classical bit, the state of a qubit $\begin{bmatrix}\alpha\\\beta\end{bmatrix}$ could be the basics states |0> and |1> or a superposition of them. By superposition, we mean the linear combination between them. The coexistence of the states is considered mutually exclusive in the classical view [7]. The superposition can be written as:

$$|\psi >= \alpha|0 > + \beta|1 > \tag{8}$$

The probability amplitudes α and β must satisfy the following equation:

$$|\alpha^2| + |\beta^2| = 1 \tag{9}$$

In this case, $|\alpha^2|$ and $|\beta^2|$ represent, respectively, the probability of collapsing for either state '0' or state 1'. Consider q an individual consisting of a string of 'l' qubits, thus:

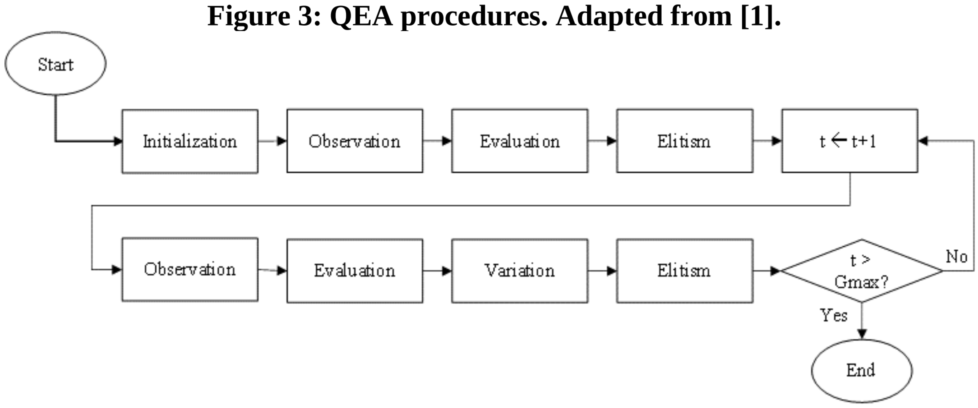
$$q = [q_1 q_2 \dots q_l] = \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_l\\ \beta_1 & \beta_2 \cdots & \beta_l\end{bmatrix} \tag{10}$$

The binary individual is needed for the fitness evaluation of the individual q. For this, each q_i is collapsed to form a binary bit $b_i \in \{0,1\}$, according to the probability $|\beta^2|$ of state "1". There is the concept of estimation family distribution of the algorithm (EDA). EDA generalizes GAs by replacing crossover and mutation operations by learning and collapsing the explicit probability models of the promising candidate solutions of each generation of the algorithm. This probability $|\beta^2|$ of each q_i define, thus a probabilistic model [1], [25]:

$$p = [|\beta_1^2|\, |\beta_2^2| \dots |\beta_l^2|] \tag{11}$$

It especially represents the probability distribution of binary solutions in search space. The general procedures of a QEA, according to Du and Li [1]follows the steps of the Figure 3.

Figure 3: QEA procedures. Adapted from [1].



3. RESEARCH MATERIALS AND METHODS

A Systematic Literature Review is a useful technique for determining the current state of knowledge in relation to a specific subject. It differs from a traditional review in that it is achieved through a repeatable, scientific, and transparent method, which eliminates the potential of bias and lack of critical examination. Furthermore, determining what might be recognized and unknowable about the specified research question necessitates a methodological approach [20]. Different approaches are used for performing systematic reviews of the literature [26]. They are organized around three primary stages: (1) planning the review, (2) gathering and selecting papers, and (3) reporting.

In the first stage, Web of Science (WoS) was the database chosen to be used in this research. It aggregates a variety of databases into a single searching tool [27]. In the second stage, one criterion used for filtering the articles was the language of publication. Only copies published in English were selected. The search interval was 1945 to 2022 (WoS default). But publications do not occur until 2004. Quantum optimization is already known to be relatively new [21]. The maximum number of articles per year is 2, in 2021. No article type filtering was used, i.e., journal and conference copies are included. The conference papers can point out this novelty since the quantum approach is new in this reliability context.

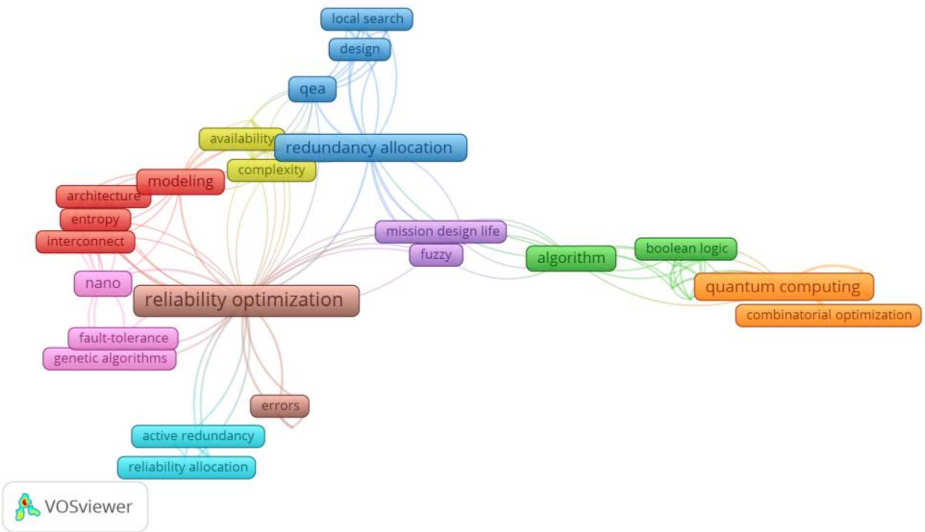
Also, in stage (2), the main keywords were also thought to conduct the study. First, keywords were combined, referring to the system reliability problems mentioned in this paper. No papers were found directly related to the keywords “Reliability allocation”, “Reliability-redundancy allocation” or “Reliability-redundancy” with “Quantum”. The only combinations that returned articles were the ones shown in Table 1. In this regard, three phases of article selection were conducted. Phase 1 consists of the general collection of articles; phase 2 deals with filtering from the reading of titles and abstracts; finally, in phase 3 the selection was made from the complete reading of the articles.

Table 1: Keywords Combination

Keywords combination	**Phase 1**	**Phase 2**	**Phase 3**
“Reliability Problem” AND “Quantum”	11	4	1
"Redundancy allocation" AND "Quantum"	4	4	4
"System reliability" AND "quantum" AND "optimization"	9	6	2
“Reliability problem” AND “Quantum” AND "Optimization"	1	0	0
Total	**25**	**14**	**7**

Another part of the descriptive analysis of the articles was carried out using the VOSviewer®, a free software tool. The keywords and significant terms used in the article portfolio can be linked using network maps [28]. As a result, a summary of the papers’ key features is obtained, see Figure 4. We can identify that reliability optimization problems, as expected, are at the center of the network most frequently. Applications are observed in problems such as “Reliability Allocation” in the light blue cluster and “Redundancy Allocation” in the dark blue cluster. “Quantum Computing” appears associated with “Combinatorial Optimization”. One can already see the introduction of quantum-inspired methods, such as the Quantum Evolutionary Algorithm, presented in the network as QEA.

Figure 4: Articles keywords co-occurrence network visualization



4. RESULTS

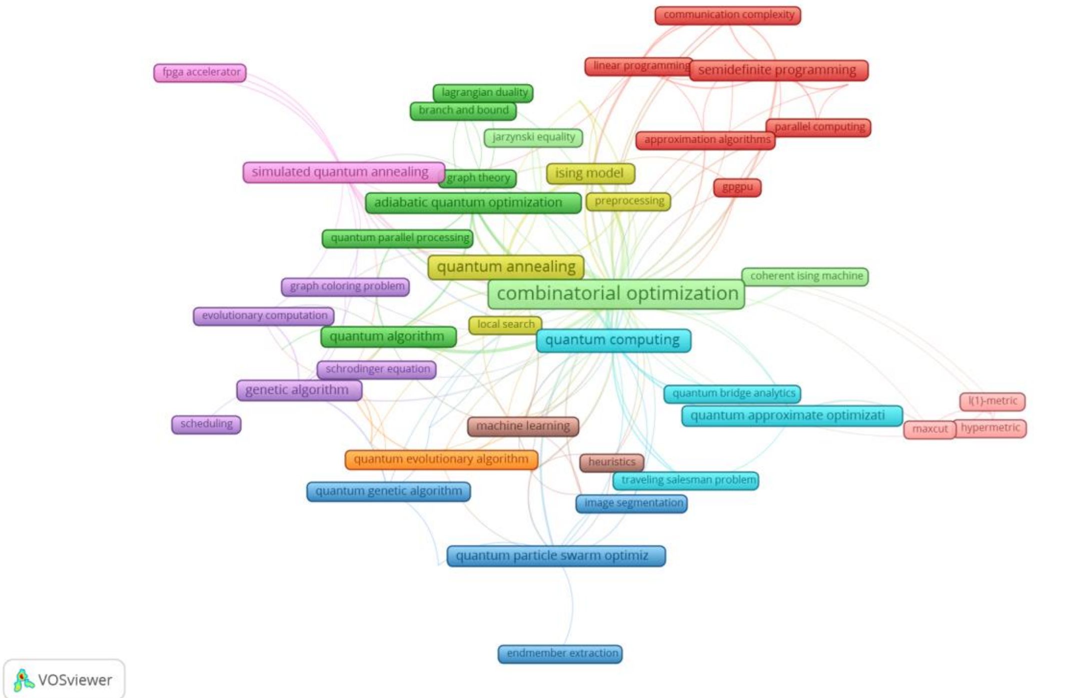
In the mid-1980s quantum computers were proposed. Quantum computing is a rapidly emerging technology that takes advantage of the laws of quantum mechanics to solve problems too complex for classical computers. In terms of optimization, it is basically divided into two camps. The first focuses on the generation of quantum algorithms using programming techniques in the quantum computers themselves. The second focuses on quantum-inspired algorithms, which use some principles of mechanics and can be applied to classical computers [12]. Next, the literature review regarding these two approaches will be presented, focusing on system reliability optimization problems.

4.1. Quantum Classical Approaches

Regarding traditional quantum computing, only one study was found related to RAP of a series-parallel system that applies a quantum algorithm for optimization [14]. The problem was modeled as QUBO. The D-Wave quantum hardware has brought focus to this method [29]. This approach uses quantum annealing for the optimization process, analogous to the classical simulated annealing, but with superior performance. From the computational speed point of view, the quantum version of the method can be eight times faster than the classical one [14].

Since the classic quantum basis is scarce in the reliability optimization context, we extended our search to identify other such methods that are applied in other combinatorial optimization problems. For analysis purposes, a Web of Science® search was performed with keywords such as "quantum algorithms" and "combinatorial optimization", excluding the words "quantum-inspired", "quantum inspired". A sample of 313 articles was found. Based on it we plotted a network of the main terms of the articles with the help of the VOSviewer® software, as shown in Figure 5. In the literature, in general, applications of different algorithms in different problems have been found. For example, quantum genetic algorithm for the MaxCut problem [30], quantum annealing [31], quantum approximative optimization - QAOA [23] quantum particle swarm optimizer applied to the backpack problem [32]. We can also detect applications in classical problems such as Travelings Salesman Problem [33]. It is worth noting that there are also multi-objective applications in the routing problem [34].

Figure 5: Articles keywords co-occurrence network visualization – quantum methods



4.2. Quantum-inspired Approaches

In the WoS base, six articles were found that relate quantum-inspired methods and systems reliability optimization problems. Li and Zio [7] apply QEA in the context of RAP. The authors bring a proposed algorithm adjusted for series-parallel multi-state systems This model assumes more than one intermediate state for the system and its components, between the two extremes of normal operation and total failure. The authors attached the method to implement a new strategy for the local search step. When performing numerical analyses, QEA showed better results regarding the number of fitness evaluations and runtime. The tests were compared with results from GA, ACO, among others.

Du and Li [1] proposed a similar approach to the previous authors. In this case, they also applied QEA with two different local search methods. One for cost reduction and the other for availability improvement. This allows a balance between exploration and exploitation. This technique is called memetic since it is an extension of the classical evolutionary algorithm. In this sense, it provides a sufficiently good solution to an optimization problem. A local search technique is used to reduce the probability of premature convergence. Additionally, the authors affirm that the results are better than those proposed by Li and Zio [7].

Bhattacharyee [35] applies an algorithm called Big-M penalty in the Weighted Quantum-behaved Particle Swarm Optimization (BWQPSO) to RAP. The author considers components with time-dependent reliability models, an objective function to maximize system reliability, and a second objective function consisting of the system "Mission Design Life (MDL)". This parameter is found by integrating the system reliability from time zero to mission time. The Big-M penalty is used to handle the constraints of the problem. The "W" of the model relates to the weight given to the particle that has a better fitness value. QPSO, on the other hand, deals with quantum inspiration aggregated with the classical PSO metaheuristic. In traditional PSO algorithms, particles are restricted to a search space, ensuring that the entire swarm does not wander and converges to a local or global optimum. In QPSO, particles can arise with a specific probability at any point in the search space, even if it is far distant from the current global optimum. In the QPSO model, the state of a particle can be defined by a wave function $\Psi(X,t)$, rather than its position and velocity. Thus, the probability that a given particle appears at position X of the probability density function $|\Psi(X,\ t)|$, depends on the potential field in which the particle is located. The operating procedure of the QPSO algorithm is similar to the PSO, but has different evolution equations since there is no velocity vector for each particle [35].

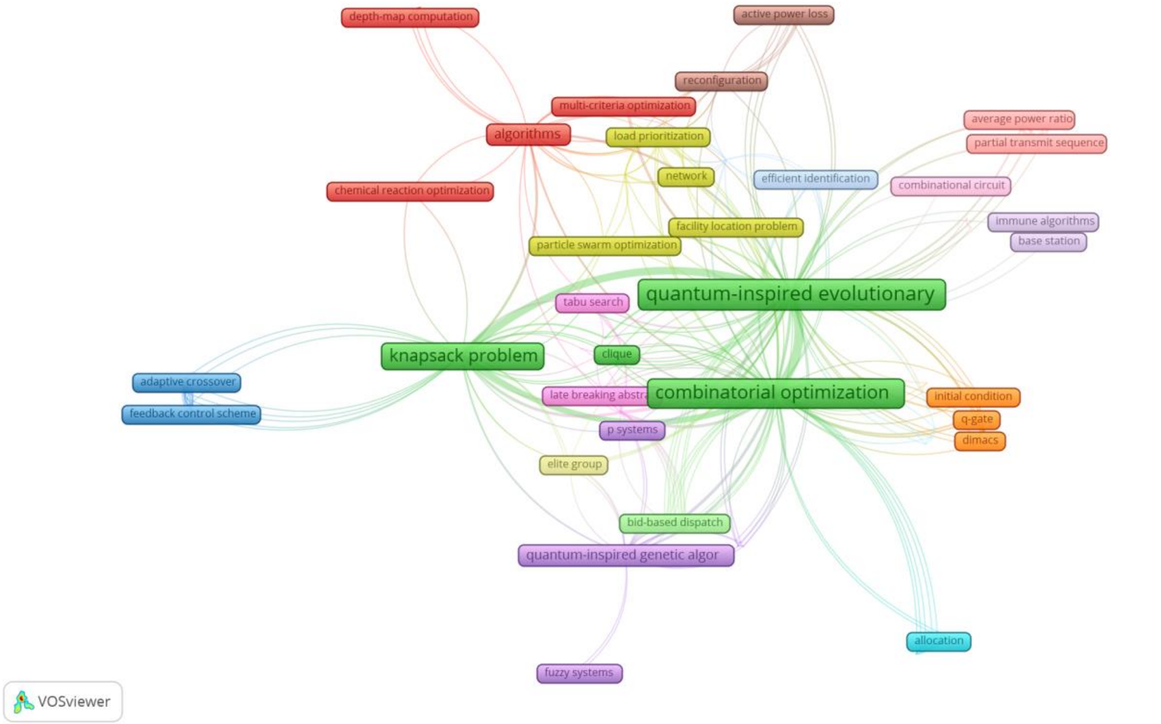
Despite not being one of the three combinatorial optimization problems (RAP, RRAP, Reliability-redundancy allocation), the next references that will be described are related to the context of reliability optimization using quantum-inspired methods. Zhang, Zhou and Yang [36] solved the System Reliability-aware Energy Optimization Problem. In this case, the objective is to minimize the energy consumption of a piece of equipment, subject to the reliability constraint. The algorithm proposed for solving part of this problem was also QPSO. They used the multi-neighborhood simulated annealing (MNSA) technique to find the best particles after manipulating QPSO.

Alvarez-Alvarado and Jayaweera [37] propose a technique for estimating the operational states of components connected to a grid using Sequential Monte Carlo Simulation (SMCS). The program then identifies the optimal size and location of Static Var Compensators (SVCs) using an Accelerated Quantum Particle Swarm Optimization (AQPSO). The method maximizes the smart grid's level of reliability, which is sensitive to voltage control.

Dueñas-Osorio, Vardi and Rojo [38] discuss the quantification of the reliability of networked systems. The authors emphasize that there are already different numerical and analytical methods that use strategies such as Monte Carlo, Boolean functions, and statistical learning for reliability estimation. Both have their efficient estimation with some guarantee that there are errors. On the other hand, the research provides a quantum-inspired computing perspective for system reliability bounds as a precursor to more complex applications, including overall infrastructure reliability and resilience. They linked concepts of Boolean logic, quantum tensor networks (TNs), and network reliability. A procedure to evaluate the number of settings contributing to the entire network-terminal reliability was outlined to assess the reliability problem as a Boolean formula in conjunctive normal form (CNF). It is usually done for logic-based satisfiability (SAT) problems. This structure, represented as a network, embodies a SATNET or graph-based satisfiability problem. It can be endowed with qubits, so TN contractions unravel the number of satisfiable settings encoded in a superposed Boolean quantum state.

Similar to what was done with quantum algorithms, we expanded our search to a general context of combinatorial optimization problems. In the WoS database, 42 studies addressed the theme "Quantum-Inspired Algorithms" and "Combinatorial Optimization". With the help of the VOSviewer® software, we created a network that relates the main terms of these articles (Figure 6). We observe a strong relationship between combinatorial optimization, QEA, and the knapsack problem. The literature has shown developments in this methodology for different variations of the problem [20, 21]. We can also find other methods for the same problem, such as quantum-inspired differential evolution and quantum-inspired tabu search [25]. In other problems, we can see quantum-inspired ant colony [41] and quantum-inspired genetic algorithm [42].

Figure 6: Keywords quantum-inspired publications and combinatorial optimization



4.3. Discussions and Gaps

Based on the survey of what has been studied in the literature regarding quantum applied to system reliability optimization problems, some interesting aspects can be highlighted. First, it is an area with little exploration so far. Only seven studies address these types of approaches to handle those problems to the best of our knowledge. One is for quantum algorithms run on quantum computers, and six are related to quantum-inspired analysis. Table 2 below summarizes the methods encountered.

Table 2: Quantum(-inspired) methods in the literature review

Source	Method	Method Category
Leon and Cascal [14]	Simulated Annealing	Classic Quantum
Li and Zio [7]	QEA	Quantum-inspired
Du and Li [1]	QEA	Quantum-inspired
Bhattacharyee [35]	BWQPSO	Quantum-inspired

Zhang, Zhou and Yang [36]	QPSO	Quantum-inspired
Alvarez-Alvarado and Jayaweera [37]	AQPSO	Quantum-inspired
Dueñas-Osorio, Vardi and Rojo [38]	Quantum-inspired Boolean states	Quantum-inspired

Besides being few, the applications are made only in the mono-objective context. With reliability optimization and cost constraint, or vice-versa. Thus, there is a gap concerning multi-objective applications. Furthermore, all analyses concerns only series-parallel systems.

Broadening the horizon of applications, we observe that there are different possible methodologies to be used in the context, besides the ones that have been applied specifically for the RAP. For example, quantum-inspired, as seen, can be used in different metaheuristics, such as ACO, GA, and tabu search. Nevertheless, among the methodologies that were applied in the RAP, in QEA, different local search techniques can be inserted to refine the solution, signaling that this method can also be further explored.

5. CONCLUSION

This paper aimed to perform a literature review to identify the applications of quantum and quantum-inspired optimization in systems reliability optimization problems. Given the discussions and gaps found, it is possible to identify a research agenda to be carried out in a practical way to expand the knowledge of the quantum-based methods on the specific problems. Some of the possibilities of studies seen are the applications of quantum(-inspired) algorithms:

- for experimentation, evaluation, and creation of a benchmarking. Given the difficulty of obtaining quantum computers, there are libraries in Python, such as Qiskit, that can run the models for a limited number of qubits;
- on RRAP problem;
- in multi-objective contexts;
- in systems with different reliability models beyond series-parallel,
- in BSS and MSS;
- with other local search engines, like the Hill Climbing [43] and the Iterative Local Search (ILS) [44] techniques.

In summary, there are many possibilities for research. Notably, expanding this research field will enable managers, reliability engineers, maintenance professionals, and other decision-makers to use more computationally efficient methods to solve these problems. This is especially true for complex systems involving many subsystems and multiple redundancy possibilities. Applying fundamental models to support the decision-making process in more classical methodologies becomes onerous.

As a limitation of this study, we emphasize that not all methods found in the literature were detailed. So, to understand more about them, it is necessary to access the references. Furthermore, it is crucial to expand the search to other databases, such as Scopus. Despite these points mentioned above, this study can provide relevant information about the problem, generate insights for new reviews and empirical studies, and fill the previously identified gaps.

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