

# HIERARCHICAL BAYESIAN ESTIMATION OF FIRE GROWTH RATE FOR VARIOUS BUILDING USAGES BASED ON THE FIRE INCIDENTS REPORT

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**Abstract:** In this study, distributions of fire growth rate for rooms of various uses, including those with scarce data availability, have been estimated by using the Hierarchical Bayesian Estimation. And the levels of fire growth rate of *the Verification method for evacuation safety* have been examined by comparing them with the estimated distributions of fire growth rate.

The following results were obtained:

- The regression by the Hierarchical Bayesian Estimation was robust.
- In most uses, assuming the interior materials were quasi-non-combustible materials fire growth rate of the method lay between 75 and 95 percentile of estimated distributions of fire growth rate.

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## 1. INTRODUCTION

In order to design evacuation safety for fires, it is important to set the fire source so that is appropriately corresponds with the amount, quality, arrangement, etc. of combustible materials in target rooms. Generally, in the fields of evacuation safety design, there are many cases where the  $\alpha^2$  fire source is used where the heat release rate  $Q$ , which is a design fire source of the fire, grows with the square of time. In other words, the setting of the fire growth rate  $\alpha$  is very important. It should be noted that in *the Japanese Verification method for evacuation safety*<sup>[1]</sup>, (hereafter, *the verification method*), the fire growth rate  $\alpha$  is given as the sum of the fire growth rate of the loaded combustibles  $\alpha_f$  and the fire growth rate of the interior materials  $\alpha_m$ . Among these, the fire growth rate of the loaded combustibles  $\alpha_f$  is calculated using the heat of combustion per unit floor area  $q_l$  that is set for each room use. The greater the amount of loaded combustibles, the larger the fire growth rate in the design. In addition, the fire growth rate of the interior materials  $\alpha_m$  is given as a value that corresponds to the combustibility of the finish, such as 0.0035 for non-combustible materials, 0.014 for quasi-non-combustible materials, and 0.056 for flame-retardant materials. However, the level of standards that will be required in *the verification method* for fire growth rate  $\alpha$  for actual fires is not clear. We can examine the level of standards that will be required in *the verification method* for fire growth rate  $\alpha$  for actual fires.

In previous studies, the purpose was to estimate the distribution of the fire growth rate  $\alpha$  of typical room uses and to use the distribution for risk-based evacuation safety design. Using fire statistics from a 14-year period, estimates were made of the distribution of the fire growth rate of typical uses, such as rooms in office, residence, restaurant and merchandise store. As a result, it was reported that the distribution of the fire growth rate  $\alpha$  could be explained with the log-normal distribution. However, because of insufficient data, for uses outside of the aforementioned 4 uses, the distribution of the fire growth rate  $\alpha$  could not be estimated.

Therefore, in the present study, a follow-up to the previous study was made to estimate the distribution of the fire growth rate using fire statistics for a wide range of uses. Furthermore, by comparing the estimated fire growth rate  $\alpha$  with the fire growth rate  $\alpha$  of *the verification method*, we can examine the level of standards that will be required in *the verification method* for fire growth rate

for actual fires. At the present time, even for uses for which the distribution could not be estimated due to insufficient data in the previous study<sup>[2]</sup>, we made estimates using Hierarchical Bayesian Estimation that could probabilistically supplement deficiencies in the data.

## 2. ESTIMATING FIRE GROWTH RATE USING FIRE STATISTICS

### 2.1. Overview of fire reports

In the present study, we used fire reports<sup>note1),[3]</sup> as fire statistical data. Here, we set screening conditions<sup>note2)</sup> of fire reports following previous study<sup>[2]</sup>. It should be noted that in the present study, we gave priority to using as much data as possible, so the condition that data are with burnt floor area of 90% or less (excluding the upper 10% of large burnt floor areas) was excluded from the screening conditions, which was set in the previous study in order to remove diverged values.

- Time period: FY1995-2018 (24 years)
- Fire classification: Building fires
- Explosions: No explosive fires
- Number of buildings lost to fire: 1
- Burnt floor area: At least 1 m<sup>2</sup>
- First-aid firefighting: None
- Time from start of water spraying to extinguishment: 60 minutes or less
- Time from fire occurrence to start of water spraying: 20 minutes or less
- Locations of fire occurrence: 41 uses
- Data with estimated fire growth rate of 1 kW/s<sup>2</sup> or over were excluded.

From all data of the 1,262,676 incidents that occurred over the 24-year period, 28,702 incidents were extracted based on the above screening. It should be noted that, the same types of analyses were conducted on the office, residence, restaurant and merchandise store for which sufficient data were available in the previous study<sup>[2]</sup>. However, as will be discussed later, in the present study we were able to examine data for which scarce data exist by introducing the Hierarchical Bayesian Estimation.

### 2.2. Method for handling fire reports

As with the previous study<sup>[2]</sup>, the present study dealt with fire report data. Figure 1 is a conceptual diagram of the method for handling data for calculating the fire growth rate from fire reports. The broken lines are images of floor area lost in actual fires, while the solid lines are an approximate growth curve of burnt floor area that were used to make estimates. In addition, the ★ symbol in the diagram represents data that have been listed in fire reports.

The time frame of fire occurrence (C), which is estimated by fire departments, appears to have just a slight margin of error with the true value (A), so it can be considered to be more or less correct. The time frame of start of water spraying time (B) is a fairly accurate value, but at this time the burnt floor area (D) could not be ascertained from the fire report data, so the data that were obtained were the final burnt floor area (F) after the fire had been completely extinguished. However, because fire in the early stage was a target of the present study, the scale of fires at the start of water spraying time was not so large, so we assumed that there was not a noticeable spread of the fire after the start of spraying (that is, by having the time from start of spraying to extinguishment limited to 60 minutes or less), and the burnt floor area (E) of the time frame of water spraying (B) was approximated with the final burnt floor area (F).



Table1 Correspondence table about use of the Verification method for evacuation safety and fire statistics

id	VERIFICATION METHOD FOR EVACUATION SAFETY			FIRE STATISTIC		Number of Data
	Type of room	$q_f$ (MJ/m <sup>2</sup> )	$q''$ (kW/m <sup>2</sup> )	Fire room usage		
1	Habitable room in a dwelling	720	249.7	Habitable room(1010)	18,002	
2	Bedroom in a building other than a dwelling	240	173.1	Dressing room(1160), Guest room(1760), Patient bedroom(1860), Night-duty room(2120), Resting room(2020)	686	
3	Office or room similar thereto	560	229.6	Private room(1910), Office(1930), Security/ Building Manager room(2130)	1,010	
4	Conference room or room similar thereto	160	151.2	Reception room(1920), Meeting/Conference room(1940)	91	
5	Classroom	400	205.3	Laboratory(1950), Classroom(1980)	96	
6	Arena in a gymnasium or other room similar thereto	80	120.0	Gym(2010)	38	
7	Exhibition room in a museum or art gallery or other room similar thereto	240	173.1	Exhibition room(1960)	6	
8	Department store, store engaged in commodity sales or other store similar thereto	960	274.8	Amusement room(1740), Library(1970)	70	
9	Other parts	480	218.1	Commodity sales store(1710), Service store(1720)	621	
10	Dining facility or other room used for dining	240	173.1	Dining room(1030)	363	
11	Other dining room	480	218.1	Restaurant(1730)	899	
12	Room in a theater, movie theater, entertainment hall, grand-stand, public hall, assembly room, or room provided for other use similar thereto	400	205.3	Auditorium(1750)	3	
13	Seating space	480	218.1			
14	Stage	240	173.1	Stage(1330), Trap room(1350)	4	
15	Parking garage or automobile repair shop	240	173.1	Parking garage(1610), Indoor parking(1620)	707	
16	Corridor, stairway, or other passageway	32	88.4	Corridor(1130), Stairway(1150)	715	
17	Vestibule, lobby, or other space similar thereto	160	151.2	Vestibule(1110), Entrance hall(1120), Reception(2110)	606	
18	Machine room for elevator or other building equipment	80	120.0	Hot-water service room(1230), Electric room(1430), Machine room(1460), Elevator(1470), Escalator(1480), Projection booth(1440), Communication equipment room(1450)	196	
19	Rooftop plaza or balcony	80	120.0	Rooftop(2250), Veranda/Balcony(2260)	274	
20	Storage or other room used to store goods	2,000	351.0	Storage(1650)	4,315	

### 3. ESTIMATION OF THE EXPANSION RATE OF BURNT FLOOR AREA USING HIERARCHICAL BAYESIAN ESTIMATION

#### 3.1. Estimation method

Bayesian estimation<sup>note3)</sup> is a method for probabilistically estimating parameters of probability distribution from the data<sup>e.g.[4]</sup>. Here, Bayesian estimation is used to estimate the probability distribution of the expansion rate of burnt floor area  $A_f/t^2$  from fire report data. First, in order to decide on a probability distribution model of the expansion rate of burnt floor area  $A_f/t^2$ , we applied typical probability distributions such as gamma distribution, Rayleigh distribution, lognormal distribution, etc., and confirmed the degree of conformity with the least squares method. The data were divided by use and year, then compared. As a result, here we could see that the gamma distribution had the best degree of conformity. Therefore, in the present study, the gamma distribution was used as a model of the probability distribution of the expansion rate of burnt floor area  $A_f/t^2$ , as shown in Eq. (3). It should be noted that there were two parameters for gamma distribution: shape parameter  $a$ , and scale parameter  $b$ .

$$\text{Gamma}(y|a, b) = \frac{b^a}{\Gamma(a)} y^{a-1} \exp(-by) \quad (3)$$

Figure 2 shows a graphical model of the distribution of the expansion rate of burnt floor area, with the statistical data classified for each use when the parameters are subject to Bayesian estimation (hereafter, Bayesian estimation for each use). In other words, the parameters  $a_i, b_i$  of the gamma distribution of uses  $i$  are estimated using the statistical data  $d_{i1}-d_{iN}$  of uses. In this case, it appears that in uses for which there are obviously scarce statistical data, the estimation accuracy decreases, and there are some cases where estimates cannot be made. On the other hand, Figure 3 shows a graphical model of the Hierarchical Bayesian model. As shown in Eq. (4), this model assumes that the parameters  $a_i, b_i$  of use  $i$  are the parameters  $a, b$  of all data plus the group differences  $a_{id}, b_{id}$ . It should be noted that group differences  $a_{id}, b_{id}$  assume an average of 0 and a normal distribution of the standard deviation  $s_a, s_b$ <sup>note4)</sup>. In this way, even for uses with scarce data, stable estimates can be made using overall trends. It should be noted that in the present study, PyStan was used in the calculation algorithm of the Bayesian model. There were 5,000 trials of MCMC, of which 500 trials were discarded as burn-in. There were 3 repetitions. For the prior distribution, the non-informative prior distribution was used. In addition, as will be discussed later, in all of the Bayesian estimates that were carried out in the present study, Rhat, which was the indicator for MCMC convergence to a steady state, was 1.0 (generally, anything less than 1.1 is considered to be convergent).

$$\begin{cases} a_i = a + a_{id} \\ b_i = b + b_{id} \end{cases} \begin{cases} a_{id} \sim N(0, s_a) \\ b_{id} \sim N(0, s_b) \end{cases} \quad (4)$$

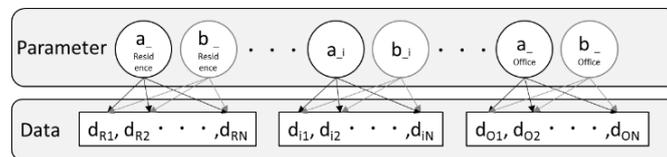


Fig.2: Graphical model of the Bayesian model

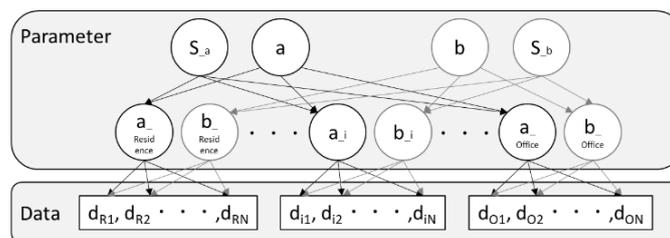


Fig.3: Graphical model of the Hierarchical Bayesian model

### 3.2. Results of the estimation of the distribution of the expansion rate of burnt floor area and discussion

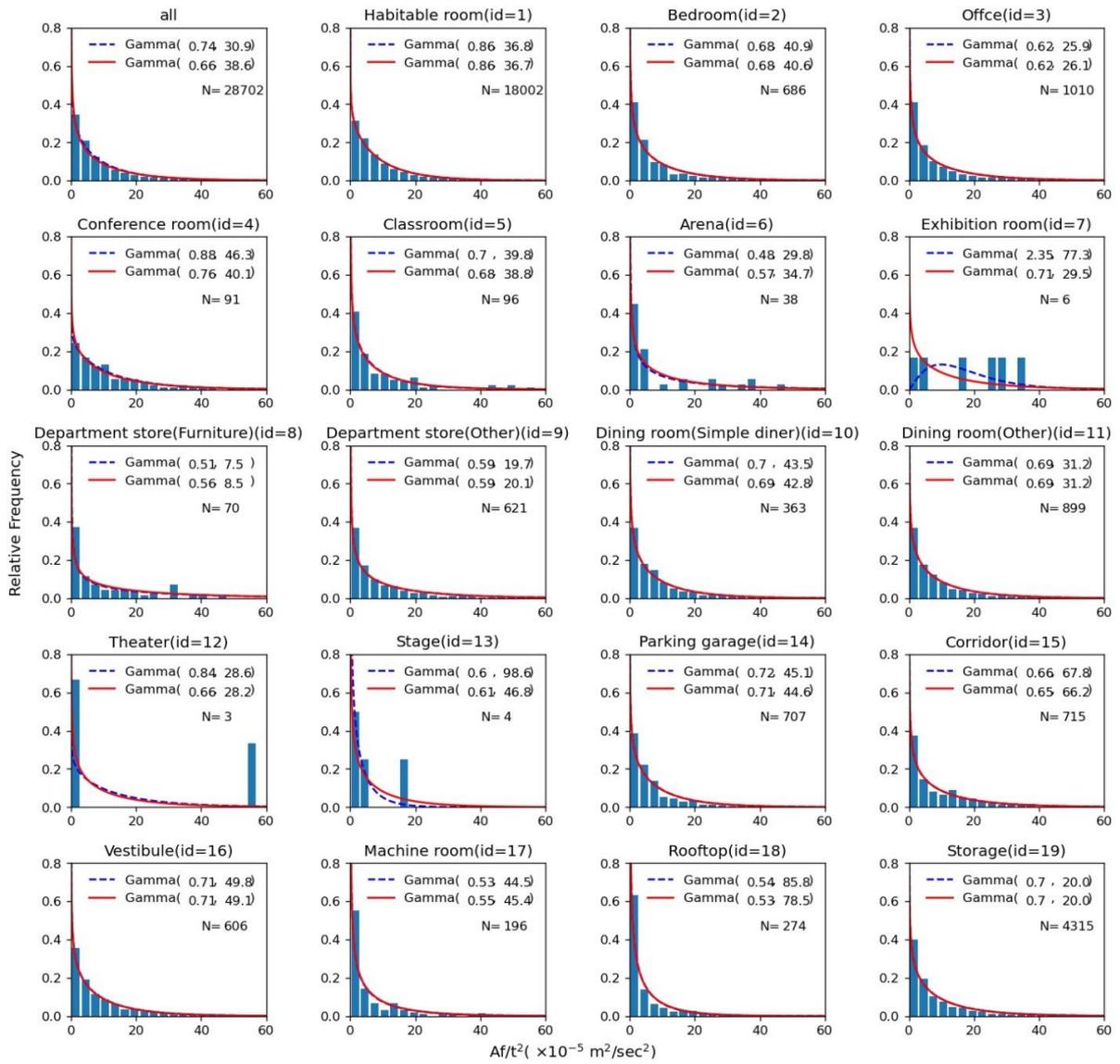
Figure 4 shows the estimated results of the distribution of expansion rate of burnt floor area  $A_i/t^2$  (parameters  $a$  and  $b$  are average of estimated values) that were obtained from the Bayesian and Hierarchical Bayesian models for each use. The bars in the figure are histograms, the dotted lines are the results of Bayesian estimation for each use, and the solid lines are the results of the Hierarchical Bayesian model.

For example, even if there are around 100 bits of data, as is the case with “Conference room” (id=4), the shape of the histogram can be reproduced well based on estimations for each use, and there is only a small difference with the Bayesian model. On the other hand, if there is an extremely small amount of data, less than 10 bits as is the case with “Exhibition room” (id=7), and the use shows special characteristics in the distribution of its data, we can see that the estimated distribution pattern differs from those for other uses and is difficult to predict. In such a case, in the Hierarchical Bayesian model the overall trend can be used to stabilize the fire growth rate distribution. At such a time, it can be confirmed that the shape parameter  $a$  of the gamma distribution goes from 2.35 to 0.71, and the scale parameter  $b$  goes from 77.3 to 29.5, which are both close to the respective values of the general parameters, 0.66 and 38.6.

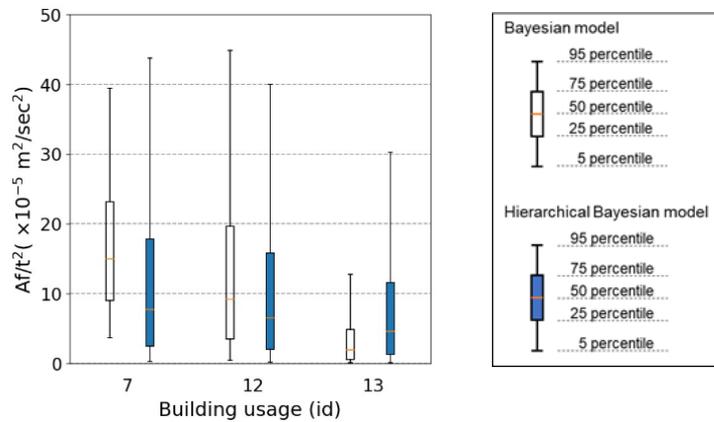
Next, let’s look at effectiveness of the Hierarchical Bayesian model for three uses where data consists of one digit and the conditions of variance differ: “Exhibition room” (id=7), “Room in a theater (Seating space (Fixed))” (id=12) and “Stage” (id=13). Figure 5 is a box plot of the Bayesian model and Hierarchical Bayesian model for each of these three uses. It should be noted that the box plots in the present study differ from regular box plots, as the percentile values are 5, 25, 50, 75 and 95, in order from the tip of the lowest whisker. We can confirm this from Figure 4 where, for example, the data ( $N = 4$ ) for “Stage” (id=13) is collected at 0.10, 0.16, 3.52 and  $16.28 \times 10^{-5} \text{ m}^2/\text{s}^2$  or less. We can also confirm this trend for the Bayesian models of each use in Figure 5. However, in the Hierarchical Bayesian model, even when there is such skewness in data, we can use the overall averages well to confirm distributions over a wide range.

Next, Figure 6 shows the estimated results of the distribution of expansion rate of burnt floor area  $A_i/t^2$  of all uses in the Hierarchical Bayesian model as the heat of combustion per unit floor area  $q_1$  on the x-axis. The values shown above the upper whiskers in the figure indicate the use id. It should be noted that the heat release per unit floor area of “Storage” (id=19) is 2,000 MJ/m<sup>2</sup>, but for convenience it is plotted at the position of 960 MJ/m<sup>2</sup> in the figure.

If we exclude “Department store (Furniture)” (id=8), then the correlation between expansion rate of burnt floor area and the heat release per unit floor area is small in Fig. 6, and we can say that the use (number of loaded combustible units) has little effect on the expansion rate of burnt floor area  $A_i/t^2$ . Although the reason for that could not be ascertained, we can probably say that, unlike *the verification method* which assumes an ideal space where fire growth continues in concentric circles, in actual fires there is the occurrence of phenomena in which fire growth stops or gently expands. We hope that the cause of this will be a topic of future research.



**Fig.4: Distribution of Fire Growth Rate estimated by the Bayesian Model and Hierarchical Bayesian Model**



**Fig.5: Effect of the Hierarchical Bayesian Model**

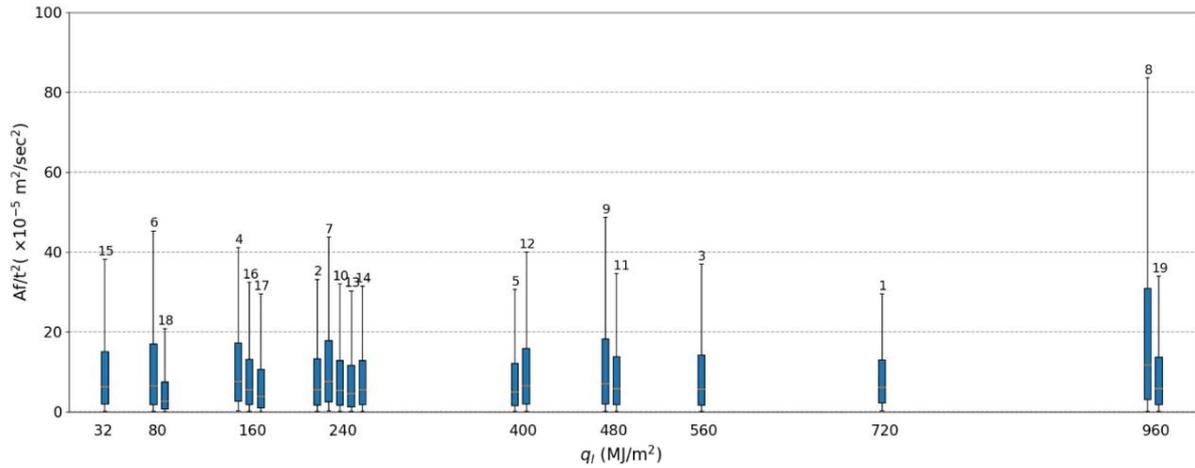


Fig.6: Heat of combustion per unit floor area vs expansion rate of burnt floor area

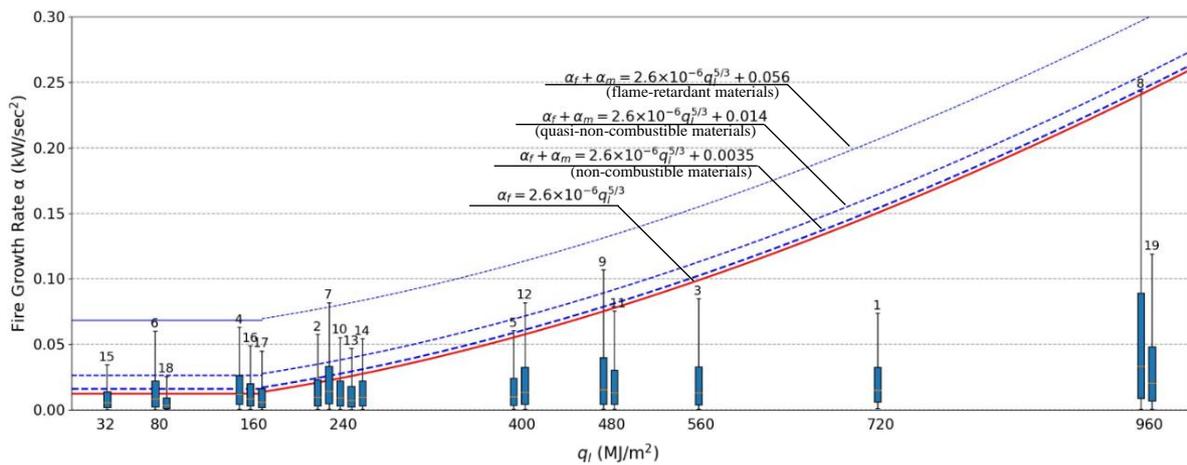


Fig.7: Heat of combustion per unit floor area vs Fire Growth Rate

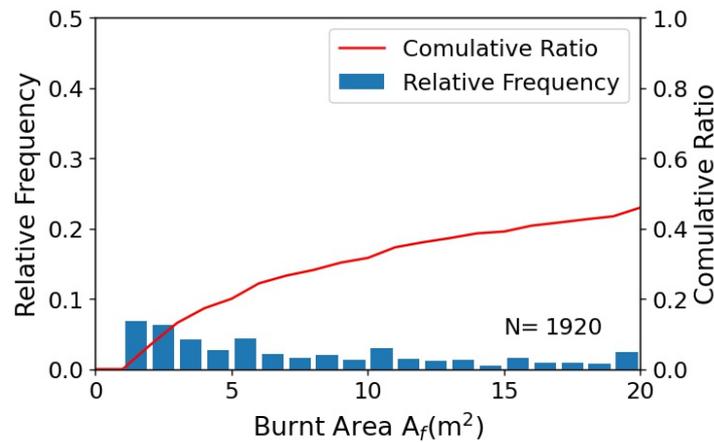
### 3.3. Comparison with the fire growth rate of the verification method

Because the expansion rate of burnt floor area  $A_f/t^2$  is not handled in *the verification method*, it is difficult to make a comparison between actual fires and the set values of *the verification method* without modifications. Therefore, using Eqs. (1) and (2) shown earlier, we converted the expansion rate of burnt floor area  $A_f/t^2$  into fire growth rate. Here, the formula for calculating the heat release rate per unit floor area  $q''$  in Eq. (2) is an equation that was formulated based on studies of exposed surface area of combustibles that were in the rooms of designated uses. In addition, the coefficient used in the equation is a mixture of wooden objects and wooden and plastic combustibles which are assumed to be of about the same amount. In the present study, we applied this to all uses, but its validity has not been investigated yet. The previous study<sup>[2]</sup> as well noted that “when estimating the fire growth rate, it is extremely important to set the heat release rate per unit floor area”, and it is necessary to pay attention to how the calculated fire growth rate is handled.

Similar to Fig. 6, Fig. 7 shows the estimated results of the distribution of the fire growth rate of all uses in the Hierarchical Bayesian model as the heat release per unit floor area on the x-axis. It should be noted that the solid line in the figure represents the fire growth rate of loaded combustibles  $\alpha_f$  of *the verification method* that is shown in Eq. (5). To this are added, as the interior fire growth rate  $\alpha_m$ , the values of non-combustible materials (0.0035), quasi-non-combustible materials (0.014), and flame-retardant materials (0.056) that were stipulated by *the verification method* and are shown by the dotted lines.

$$\alpha_f = \begin{cases} 0.0125 & (q_l \leq 170) \\ 2.6 \times 10^{-6} q_l^{5/3} & (q_l > 170) \end{cases} \quad (5)$$

Because we cannot specify the interior finish of fire rooms from the fire statistics from Fig. 7, here we will consider an example of assuming that quasi-non-combustible materials are used as interior materials. The fire growth rate of *the verification method* for all uses had a larger value than the estimated 75 percentile value. However, in many uses it was smaller than the estimated 95 percentile value. This was especially the case with combustible amount per unit floor area which, at 240 MJ/m<sup>2</sup> or less, showed a value smaller than the estimated 95 percentile value for fire growth rate of *the verification method*, for all uses. Now we will consider the reason for this. According to the literature<sup>[2]</sup> and Eq.(5), “when the heat of combustion per unit floor area  $q_l$  is 170 or less, and considering that only a single combustible unit burns,  $\alpha_f$  is set at a uniform 0.0125. However, when it becomes higher than 170, it is considered that multiple stored combustibles burn simultaneously, and the fire growth rate is changed in response to the heat of combustion per unit floor area  $q_l$ .” On the other hand, when we examined the distribution of burnt floor area of the fire statistics as a target of uses where the combustible amount per unit floor area was 170 MJ/m<sup>2</sup> or less, then, as shown in Fig. 8, the proportion of burnt floor area of 1 m<sup>2</sup> or less was 6.8%, for 3 m<sup>2</sup> or less it was 17.3% and for 5 m<sup>2</sup> or less it was 24.4%. It should be noted that in uses where the heat of combustion per unit floor area  $q_l$  exceeded 170 MJ/m<sup>2</sup>, the proportion of burnt floor area of 1 m<sup>2</sup> or less was 3.2%, for 3 m<sup>2</sup> or less it was 8.5% and for 5 m<sup>2</sup> or less it was 11.9%. Here, when considering fires where the burnt floor area was more than 3 m<sup>2</sup> and the fires had not stopped at burning a single unit combustible even when the heat of combustion per unit floor area  $q_l$  was 170 MJ/m<sup>2</sup> or less, simultaneous combustion occurred in more than 80% of fires. This was especially the case for uses where the heat of combustion per unit floor area  $q_l$  was 240 MJ/m<sup>2</sup> or less, and this is considered to be one of the reasons why the fire growth rate of *the verification method* was less than the estimated value.



**Fig.8: Relative frequency of burnt floor area ( $q_l \leq 170$ )**

#### 4. CONCLUSION

Using fire statistics from a 24-year period, we estimated the distribution of the fire growth rate using Bayesian estimation. This time, by carrying out Hierarchical Bayesian estimation, we could estimate distribution patterns even for uses where there were few data and there were special characteristics in the distribution of the data. In addition, by comparing the estimated distribution of the fire growth rate with the fire growth rate of *the verification method*, the fire growth rate of *the verification method* corroborated what levels of standards are required for actual fires. As a result, when using examples of cases where interior materials were quasi-non-combustible materials, the fire growth rate of *the verification method* was set at a value that was larger than the value of the estimated 75 percentile, but in some of the uses we could see that it was less than the value of the 95 percentile. This trend was especially noticeable for uses where the heat release per unit floor area was less than 240 MJ/m<sup>2</sup>. One

of the reasons for this was that in cases where the heat release per unit floor area was less than 170 MJ/m<sup>2</sup>, in *the verification method* we assume a single unit combustible burns, but in actual fires, more than 80% involved the burning of multiple combustibles, rather than assuming the burning of single combustible unit.

## References

- [1] The Ministry of Land, Infrastructure, Transport and Tourism, Housing Bureau, Building Guidance Division et.al “*Verification Methods for Determining Safe Evacuation of Floor and Building*”, Kaibundo, 2001, in Japanese.
- [2] Deguchi, Y., Notake, H., Yamaguchi, J. and Tanaka, T. “*Statistical Estimation of the Distribution of Fire Growth Factor -Study on Risk Based Evacuation Safety Design Method-*”, Fire Safety Science, Vol.10, No.2, pp. 1087-1100, (2011).
- [3] Disaster Prevention Administration Study Group “*Handbook of Fire Statistics*”, Tokyo Hourei, 2021, in Japanese.
- [4] Matsuura, K. “*Bayesian Statistical Modelling Using Stan and R*”, Kyoritsu Shuppan, 2016, in Japanese.

## Notes

### Note 1)

All fires reported by fire departments were converted into a database. Various types of information were recorded, such as buildings, materials contained therein, temporal information such as time frames of fires and awareness, whether or not first-aid firefighting was carried out, the cause of the fire, state of damage, weather information, monetary amount of damages, number of human casualties, etc.

### Note 2)

The purpose of the present study was to estimate the fire growth rate, so we set screening conditions so that we could exclude, as much as possible, small fires that naturally extinguished by themselves, and fully-developed fires. In order to exclude small fires, we only targeted fires that had a burnt floor area of at least 1 m<sup>2</sup>. In addition, by setting an upper limit on the time from fire occurrence to start of water spraying, we excluded fires that could lead to fully-developed fires. It should be noted that fires that could lead to fully-developed fires were apparently included in the 20 minutes data (time from fire occurrence to start of water spraying) that was used in the analysis, so there is room to debate whether or not 20 minutes was an appropriate standard for making judgments.

### Note 3)

For example, in the Maximum Likelihood estimation, one value of parameters was derived as the maximum likelihood, but in the Bayesian estimation, the distribution of parameters could be obtained directly by MCMC. It should be noted that here, the mean values of parameters were used in the analyses.

### Note 4)

Even though the number of combustible units, arrangement, materials, etc., depend on the use, the fire phenomenon itself, in which fires spread as combustibles burn, does not change. Therefore, in the present study, we assumed that the use parameters could be estimated by adding group differences to the overall mean values.