Probabilistic Methods for Cyclical and Coupled Systems with Changing Failure Rates

Courtney Otani[a], Robby Christian[a], Arjun Earthperson[b], Steven Prescott[a], and Mihai A. Diaconeasa[b]
[a] Idaho National Laboratory, Idaho Falls, ID, USA
[b] North Carolina State University, Raleigh, NC, USA

Abstract: Advancements in nuclear system designs with automated control features provide many benefits, but can lead to complex coupled systems and dynamic failure scenarios. This is especially true for microreactor designs where components are not expected to be replaced during the reactor's lifetime. Hence, the life of the system, in addition to the safety, needs to be evaluated. Modeling these sequences of time-dependent events requires addressing cyclical processes and changing failure rates in ways that represent the actual system dynamics in contrast to a single sampling for a component's time to failure. This research presents two distinct analytical methods for several failure distributions that evaluate a final time to failure used for different scenarios where the time to failure must be sampled multiple times. The first method is used when evaluating a component whose failure rate increases due to an outside event after the initial sampling but before the initially sampled time to failure. The second method is used when evaluating multiple identical components or a component that has been replaced with a new identical version before the second sampling. The two methods were implemented in a few representative case studies developed in the dynamic probabilistic risk assessment tool Event Modeling Risk Assessment using Linked Diagrams. Overall, this paper provides guidelines on how these approaches give a more realistic and accurate dynamic probabilistic risk assessment of complex systems.

1. INTRODUCTION

Dynamic probabilistic risk assessment (PRA) provides a range of analysis capabilities that allow for more realistic behavior modeling of nuclear facilities. As these features are used in new scenarios, care taken while modeling and during simulation will ensure appropriate probabilistic failure modeling is achieved. A recent research project at Idaho National Laboratory evaluated combining multiple methods into the dynamic PRA tool Event Modeling Risk Assessment using Linked Diagrams (EMRALD) [1]. One postulated scenario involved a change in the component failure rate depending on if it is in a given condition and how to determine a new time to failure in the simulation. This paper outlines some plausible modeling scenarios and how to both correctly model and quantify the failure probabilities in the simulation.

The ability to change or reset the failure rate or inputs to different distributions for PRA is an important feature needed in many scenarios:

- Aerospace modeling uses different mission phases and changing component or system failure rates for different phases of the mission, such as launch, orbit, and reentry. [2]
- Degraded components or other conditions can induce other failure modes, causing a change in the failure rate.
- Seasonal environmental conditions at nuclear installations can cause events, such as algae blooms, that could cause heat sink reduction, reduce output capabilities, or affect support systems. [3]
- Risk modeling considering degradation and preventative maintenance is used for predictive monitoring of nuclear plant components. [4]

2. BACKGROUND

EMRALD is based on a three-phase discrete event simulation that uses Monte Carlo simulation techniques to sample when random events occur over time. No fixed duration time steps are used in the systems modeled. The simulation jumps to the next event's time because nothing that affects the model will happen until the next event. When the simulation runs out of events or hits a terminal event, it is complete.

This event-driven methodology is different from the static PRA using discrete event trees, where each path in the event tree is explicitly explored. Each method has its advantages and disadvantages; one advantage of the discrete event is that it makes it easy for an analyst to model looping behaviors or dynamically adjust time-dependent failure rates.

Dynamic PRA modeling promises new ways to more accurately represent facility behavior; however, it is also easy for an analyst to unknowingly cause unintended behavior in the model. This also means that the dynamic PRA tool must provide the features necessary to generate the desired behaviors. For example, this paper goes over why the dynamic PRA software must not ignore or only resample for a new failure time if failure parameters change, as this could have dramatically different outcomes. This is easily seen when parameters change inside a cyclic process. An example of this would be systems designed to load-follow that hypothetically have a component with higher failure contributions when at the maximum output. The analysis assumed the component can be modeled with a constant failure rate, lambda, of 0.01 per 24 hours and the system enters peak production an average of once every 24 hours. To illustrate the error in resampling for this example, the same lambda of 0.01 per 24 hours is used for the peak production but still causes a resampling event. As shown in the table below, resampling causes the mean time to failure (MTTF) to drastically reduce compared to no resampling. If the goal is to determine the probability that the system will fail before the desired life span, given a frequency of use at max operating level, an adjustment, not resampling of the lambda, is needed.

Table 1: Fixed versus resampled results

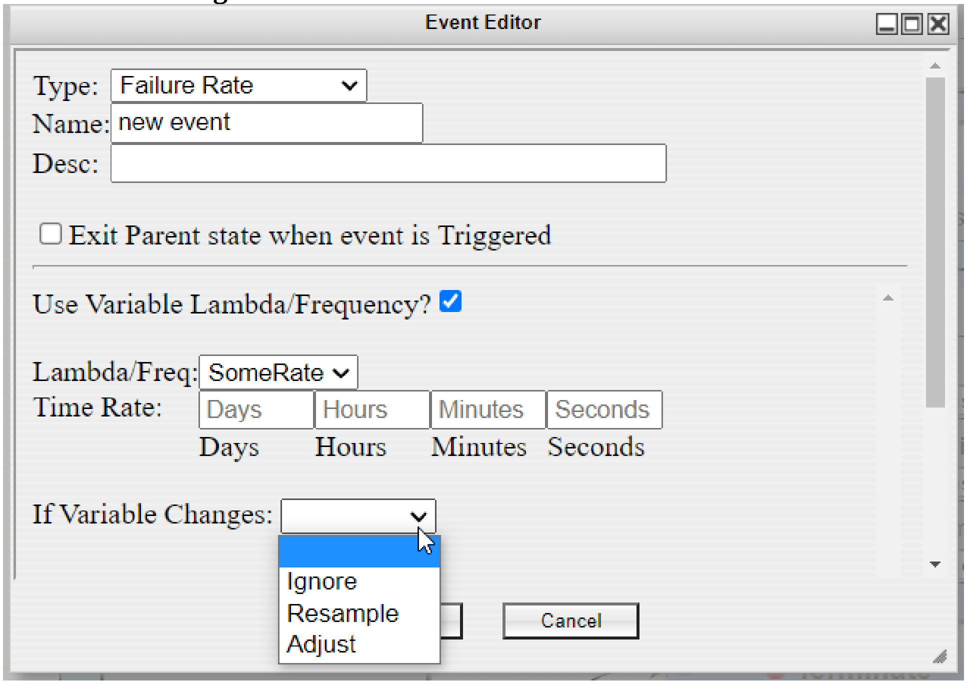
Method using 1,000 Runs	MTTF (Days.Hours)
Fixed Lambda (0.01 per 24 hours)	90.23
Resample Lambda (0.01 per 24 hours)	11.14

An accurate analysis would need both sets of failure data. However, if there is no failure data for altered component running conditions, estimated data could be used to determine if additional testing data, and at what level, would be significant in the overall system results.

If the failure rate or distribution input value can change, the analyst must decide if they want to ignore any rate changes; resample the failure time, if a component is being replaced; or adjust the failure time, if it is running in an abnormal condition. In EMRALD, this is done in the user interface by selecting the appropriate dropdown item if a variable is used in the parameter of the sampling method. This interface is shown in Figure 1.

If the user wants to "Resample," that is a simple calculation for the simulation, which repeats the calculation but with the new frequency/rate. To "Adjust" the event time, the simulation determines a new time using a calculation that takes into account that the event has not occurred up until the current time due to the previous failure rate or distribution values. These methods depend on the distribution type being sampled. The following section describes the methods that determine the new event times.

Figure 1: EMRALD failure rate event interface



3. METHODOLOGY

3.1. Scenario 1—Adjust

3.1.1. Description

The first method is used to determine the time of failure when evaluating a component whose failure rate changes after an initial sampling but before the initially sampled failure time. This updated time of failure will be referred to as *newOccurTime*. Other relative points in simulation time, *t*, and the differences in time, Δt, are outlined in Table 2 and Table 3, respectively. They are shown graphically in Figure 2. This figure describes an increase in failure rate, but this could also be applied to a decrease in failure rate.

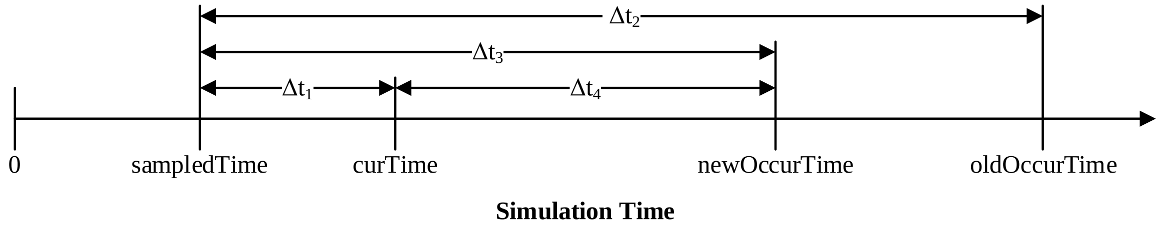
Table 2: Points in time, t, for Scenario 1

Reference Name	Known?	Description
sampledTime	Known	Original time sampled- datum
curTime	Known	Current simulation time, time of change in failure rate
oldOccurTime	Unknown	Sampled time of failure given just the first failure rate
newOccurTime	Unknown	Sampled time of conditional failure given the new failure rate following the elapsed time with the initial failure rate

Table 3: Sampled lengths of time, Δt, for Scenario 1

Lengths of Time	In Terms of t	Description
Δt_1	curTime – sampledTime	Sampled time to transition back to the state that samples failure
Δt_2	oldOccurTime – sampledTime	Sampled time to failure using the first failure rate from time first sampled
Δt_3	newOccurTime – sampledTime	Conditional sampled time to failure given new failure rate following the elapsed time with the initial failure rate relative to sampledTime
Δt_4	newOccurTime - curTime	Conditional sampled time to failure with new failure rate following the elapsed time with the initial failure rate minus Δt1 elapsed

Figure 2: Simulation times of interest for Scenario 1



Using the outlined points in time, the scenario can be described as:

$$\text{curTime} < \text{oldOccurTime} \quad (1)$$

For the following methodology to be applicable, the following equivalent inequality must be true:

$$\text{newOccurTime} > \text{curTime} \quad (2)$$

or

$$\Delta t_3 > \Delta t_1 \quad (3)$$

Inequality (2) and (3) have a difference of *sampledTime*.

If the result of *newOccurTime* falls outside of that range, *newOccurTime* should be considered as follows:

$$\lambda_C = \text{curTime}, \qquad \text{newOccurTime} \leq \text{curTime} \quad (4)$$

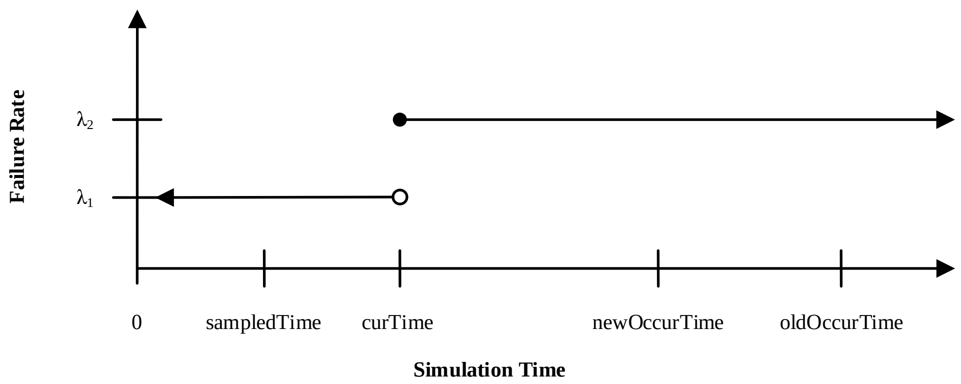
Throughout this description, we have used the term "failure rate." *t* will be described in terms of λ of a Poisson distribution but can be replaced with whatever distribution parameters are needed for the type of failure distribution that best describes the component's failure behavior. This is mathematically described with Equation (5), given Table 4, and graphically with Figure 3. This figure describes an increase in failure rate, but this could also be applied to a decrease in failure rate. It also shows two constant failure rates because of the nature of a Poisson distribution, but if another distribution was used, the distribution parameters could be functions of time as well as external event occurrence.

$$\lambda_C = \begin{cases} \lambda_1, t < \text{curTime} \\ \lambda_2, t \geq \text{curTime} \end{cases} \quad (5)$$

Table 4: Failure rates, λ, for Scenario 1

Failure Rate	Description
λ_C	Failure rate of the single specific component being analyzed
λ_1	First failure rate, before being affected
λ_2	Second failure rate, at and after being affected

Figure 3: Failure rate as a function of simulation time for Scenario 1



3.1.2. Methodology

Using basic rules and probability methods, a relationship of failure probabilities in terms of *Δt* can be written to solve for *newOccurTime*. The general steps are as follows and can be applied to any failure distribution, *F(Δt)*.

1. Use probability integral transformation on the failure distribution with the initial failure distribution parameters to sample for *oldOccurTime*.

2. Given the following definitions of conditional probability and reliability and failure, develop an equation relating the probability of failure due to the change in failure rate and parameters given some time has already elapsed ($F(\Delta t_4|\Delta t_1)$) to other probabilities.
   a. Conditional probability [1]:
   $$\mathrm{R}(\Delta \mathrm{t}_4|\Delta \mathrm{t}_1) = \frac{\mathrm{R}(\Delta \mathrm{t}_3)}{\mathrm{R}(\Delta \mathrm{t}_1)} \leftrightarrow \mathrm{R}(\Delta \mathrm{t}_3) = \mathrm{R}(\Delta \mathrm{t}_4|\Delta \mathrm{t}_1) * \mathrm{R}(\Delta \mathrm{t}_1) \tag{6}$$

   b. Reliability and failure:
   $$\mathrm{R}(\Delta \mathrm{t}) = 1 - \mathrm{F}(\Delta \mathrm{t}) \leftrightarrow \mathrm{F}(\Delta \mathrm{t}) = 1 - \mathrm{R}(\Delta \mathrm{t}) \tag{7}$$

   where *R(Δt)* is the reliability, *F(Δt)* is the failure probability, and *Δt* is the time to transition to an event.

   c. Combine to create an equation in terms of failure probability since the failure distribution is known.
   $$\mathrm{F}(\Delta \mathrm{t}_3) = 1 - [1 - \mathrm{F}(\Delta \mathrm{t}_1)] * [1 - \mathrm{F}(\Delta \mathrm{t}_4|\Delta \mathrm{t}_1)] \tag{8}$$

3. Substitute in the failure distributions on the right-hand side.

4. Use probability integral transformation on the resulting equation for failure probability to sample for *newOccurTime*.

5. Repeat Steps 2–4 as often as failure parameters change.

If it is assumed that the failure probability can be described using a Poisson distribution, the following description of failure probability is true:
$$F(T < t) = 1 - e^{-\lambda t} \tag{9}$$

where λ is as described in Equation (5) and *t* is time to transition to failure. An analytical result can be achieved following the general steps outlined above.

1. Use the initial value for the failure rate to sample for *oldOccurTime*.
   Substitute the appropriate variable values into Equation (9):
   $$F(\Delta t_2) = 1 - e^{-\lambda_1 \Delta t_2} \tag{10}$$

   Invert the failure probability to get an equation for t_2:
   $$\Delta t_2 = \frac{ln[1 - F(\Delta t_2)]}{-\lambda_1} \tag{11}$$

   By the probability integral transformation theorem, the failure probability is a uniform random variable with domain [0,1] [6]:
   $$1 - F(\Delta t_2) = U \tag{12}$$

Substitute Equation (13) into Equation (12) understanding that Equation (14) is also true because *U* is on the domain [0,1]:

$$\Delta t_2 = \frac{ln[U]}{-\lambda_1} \tag{13}$$

$$U = 1 - U \tag{14}$$

Sample *U* to yield a value for t_2, then use that sampled value and definition of t_2 (Row 2 of Table 3) to solve for *oldOccurTime*:

$$oldOccurTime = sampledTime + \Delta t_2 \tag{15}$$

2. Utilize Equation (8).

3. Substitute the appropriate variable values into Equation (9):

$$F(\Delta t_1) = 1 - e^{-\lambda_1 \Delta t_1} \tag{16}$$

$$F(\Delta t_4 | \Delta t_1) = 1 - e^{-\lambda_2 \Delta t_4} \tag{17}$$

Then substitute those equations into Equation (8).

$$F(\Delta t_3) = 1 - \left[1 - \left(1 - e^{-\lambda_1 \Delta t_1}\right)\right] * \left[1 - \left(1 - e^{-\lambda_2 \Delta t_4}\right)\right] \tag{18}$$

4. Invert Equation (18) to solve for t_4 and use the probability integral transformation theorem as detailed in Step 1 when solving for *oldOccurTime*:

$$\Delta t_4 = \frac{ln[U] + \lambda_1 \Delta t_1}{-\lambda_2} \tag{19}$$

Sample U to yield a value for t_4, then use that sampled value and definition of t_4 (row 4 of Table 3) to solve for *newOccurTime*:

$$newOccurTime = curTime + \Delta t_4 \tag{20}$$

5. Repeat as necessary.

3.1.3. Example Model Scenario

In this example, the operation and failure of the Intelligent Automation module of a fission battery plant is modeled as shown in Figure 3. Since the fission battery will be operated in remote areas with minimum human intervention, intelligent automation and decision-making capabilities are vital to its operation [7]. Here, the intelligent automation module is housed in a dedicated computer, which dissipates considerable heat due to the real-time monitoring, processing, and remote communication of the battery's sensor data. For that reason, the computer's failure rate depends on the room's cooling as regulated by the heating, ventilation, and air conditioning (HVAC) system. The HVAC has a fixed failure rate, and its failure increases the computer's failure rate from 1E-3/hr to 5E-3/hr, as shown in the "Update_ComputerFailureRate" action in Figure 4. The change in failure rate is modeled in the "Computer_Failure_Rate" event by using a variable, "ComputerFailureRate," and the "Adjust" option when the value changes, as seen in Figure 5. This option incorporates the dynamic failure, as modeled in Equation (18) – (20).

Figure 3. EMRALD diagram of fission battery intelligent automation module

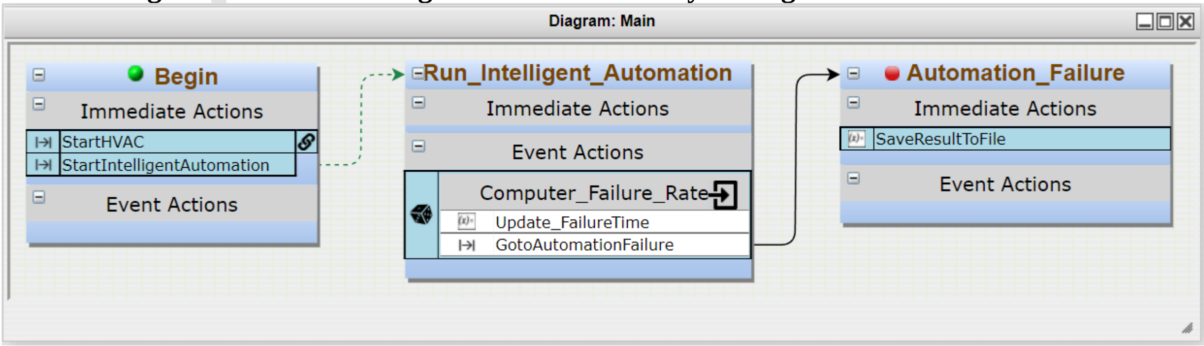


Figure 4. EMRALD model of HVAC system

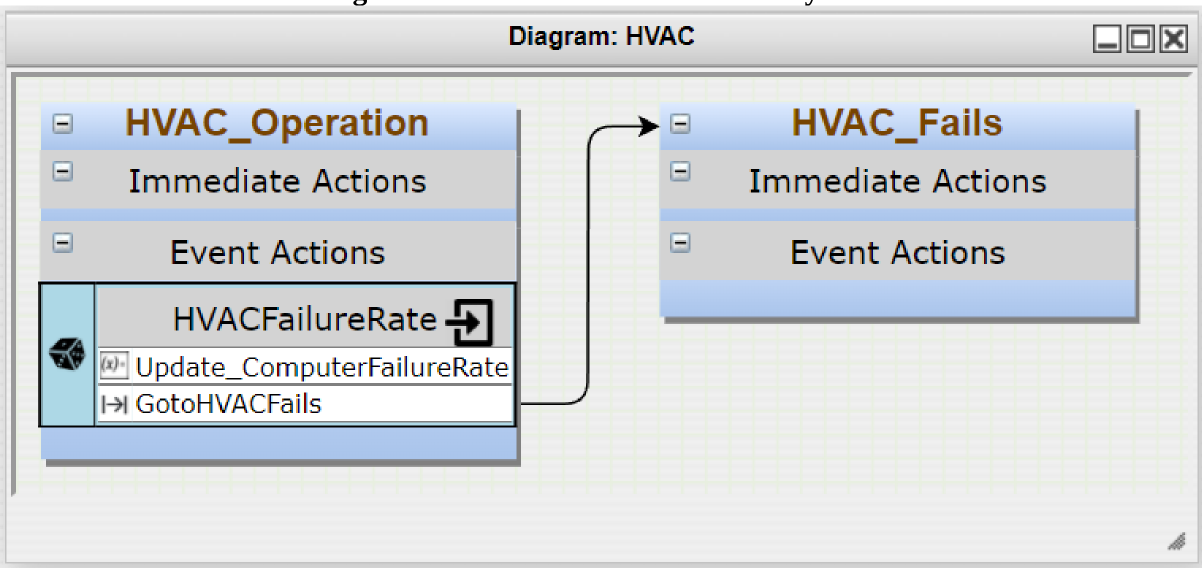
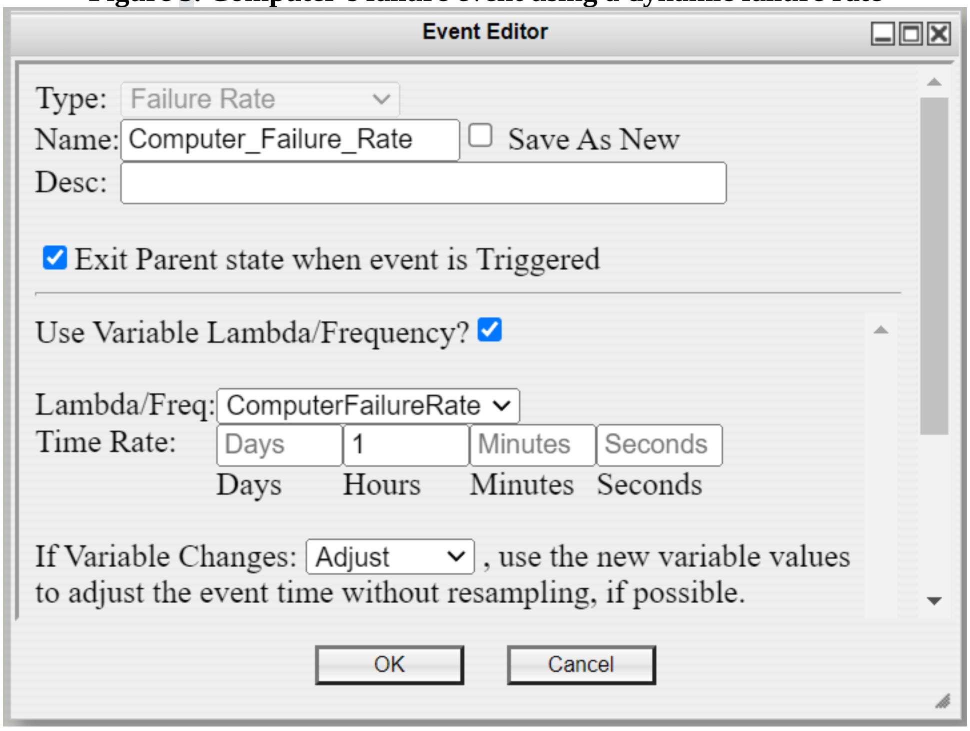


Figure 5. Computer's failure event using a dynamic failure rate



3.2. Scenario 2—Resample

3.2.1. Description

The second method determines the time to failure of distinct instances of a component with identical behavior. This would include sampling multiple similar components at different times or sampling a component that was recovered or had a similar replacement for a component that previously failed. This newly sampled time to failure will also be referred to as *newOccurTime*. The names of all other points in time will be consistent with Table 2 but have slightly different descriptions, as shown in Table 5.

Table 5: Points in time, t, for Scenario 2

Reference Name	Known?	Description
sampledTime	Known	Original time sampled- datum
curTime	Known	Current simulation time, time of second sampling
oldOccurTime	Unknown	Sampled time of failure of the first sampled component
newOccurTime	Unknown	Sampled time of failure of the second sampled component

Unlike in the first scenario, *newOccurTime* does not necessarily have to occur before *oldOccurTime*. For the replacement scenario, *oldOccurTime* will occur before *newOccurTime* by nature of the scenario. The first and second sampling to obtain *oldOccurTime* and *newOccurTime*, respectively, are now independent of each other, see the change in definition of the failure rate shown in Table 6. Now that they are independent of each other, their failure rate values are also independent of each other. They could be identical if the component being sampled has identical failure rates, or they could be slightly different from each other either from variance in component manufacturing or other external factors.

Table 6: Failure rates, λ, for Scenario 2

Failure Rate	Description
λ_1	First failure rate of first sampled component
λ_2	Second failure rate of second sampled component

3.2.2. Methodology

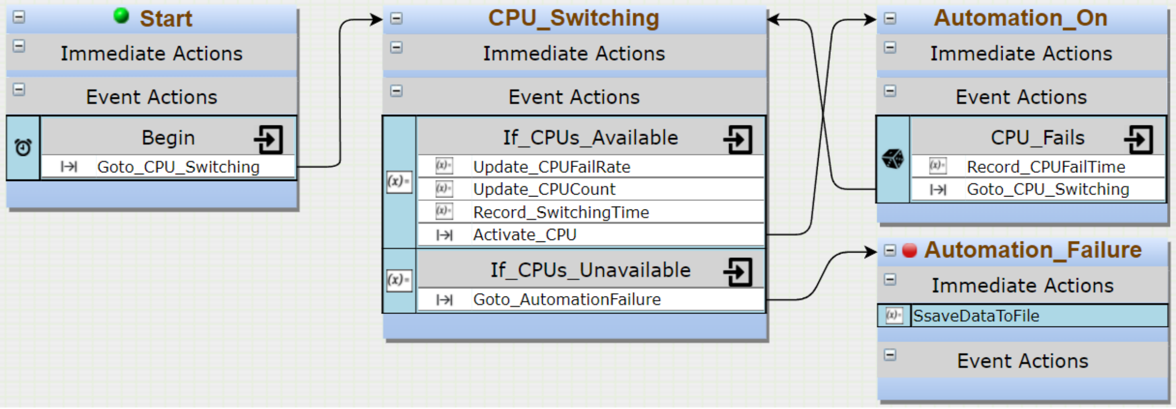
The methodology for this is to simply sample each instance independently using the probability integral transformation on the failure distributions with the appropriate failure distribution parameters to solve for *oldOccurTime* and *newOccurTime*.

If the failure probability of each component can be described using a Poisson distribution, Equation (9), using the values defined in Table 6 and applying the probability integral transformation, can yield an analytical result. This is done using Equations (10) – (15) with the appropriate substitutions.

3.2.3. Example Model Scenario

In this example, the intelligent automation of a fission battery uses redundant CPUs, as shown in Figure 5. When the main CPU fails, the system switches to the backup CPU as seen in the interaction between the "If_CPUs_Available" event in the "CPU_Switching" state and the "CPU_Fails" event in the "Automation_On" state. For this illustration, the main CPU has 1E-3 failures/hour while the backup unit has 5E-3 failures/hour. These values are arbitrarily selected to highlight the difference in failure timings due to different failure rates. Switching from the main to backup CPU requires a resampling of failure time. The simulation ends when the backup CPU eventually fails, causing the loss of the intelligent automation capability as seen in the state transition under the "If_CPUs_Unavailable" state.

Figure 5. EMRALD diagram of automation failure



4. RESULTS AND DISCUSSION

A series of 10,000 EMRALD runs was simulated for both models. A summary of simulation conditions is shown in Table 7 and the failure time probability density results are shown in Figure 6. The figure shows the bins when the fission battery lost its intelligent automation capability, assuming the computer unit had a static failure rate λ_1 of 1E-3, λ_2 of 5E-3 or a dynamic failure rate. The results of the static failure rate λ_1 and λ_2 are identical between models. The results of the dynamic failure rate are necessarily only a result of the model described in Section 3.1.3 that uses the "Adjust" methodology.

Table 7: EMRALD model inputs

Parameter	Value
Number of Runs	10,000
λ_1	1E-3
λ_2	5E-3

As seen in Figure 6, more early failures are observed in the backup CPU with the λ_2 failure rate than the more reliable main CPU. Meanwhile, the dynamic failure methodology shows estimates between the two failure timings. The cumulative probability of failure time from 10,000 EMRALD runs is presented in Figure 7 and shows a similar trend. The figure conveys a similar message that the proposed methodology in this work can estimate the change in failure time due to a change in failure rate, which may be caused by environmental conditions.

Another important observation from Figure 6 and Figure 7 is that the component initially behaved following the initial failure rate λ_1, with a lower failure density at the start of operation. However, this behavior gradually changed as the HVAC fails, causing an undesirable operating condition, and the computer's failure density started to reflect that of the degraded computer with the λ_2 failure rate. This explains why the initial sections of the dynamic lambda cumulative failure probability curve and the failure with λ_1 coincide with each other.

The three sets of data shown in Figure 6 and Figure 7 reveal the practical application of the conditional probability formulation presented in this paper and the significance of differentiating the methodologies. The distributions are all significantly different, thus representing significantly different estimated times to failure depending on the way the changed failure rate is considered. Since the result of the dynamic lambda has influence from λ_1 and λ_2, it is reasonable that it has a distribution that lies between the static λ_1 and λ_2. When the influence of the changed failure rate was not considered, the result was just the data sets of the static λ_1 and λ_2, which over and underestimate the time to failure, respectively.

Figure 6. Probability density of failure time from 10,000 runs

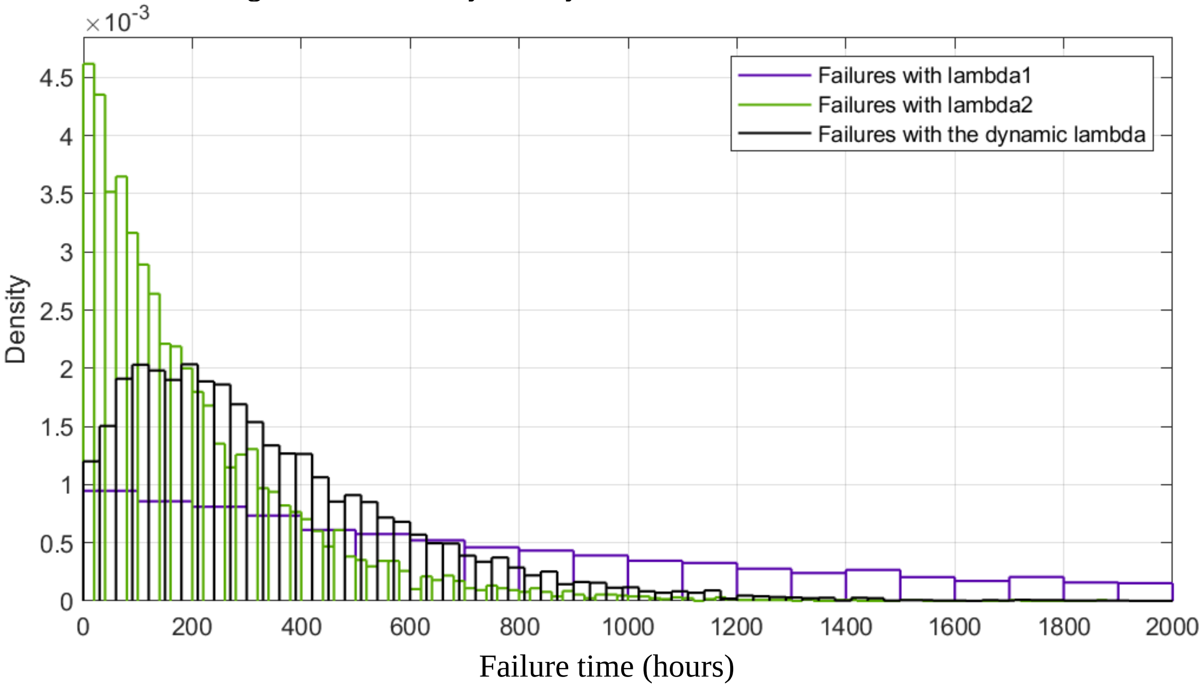
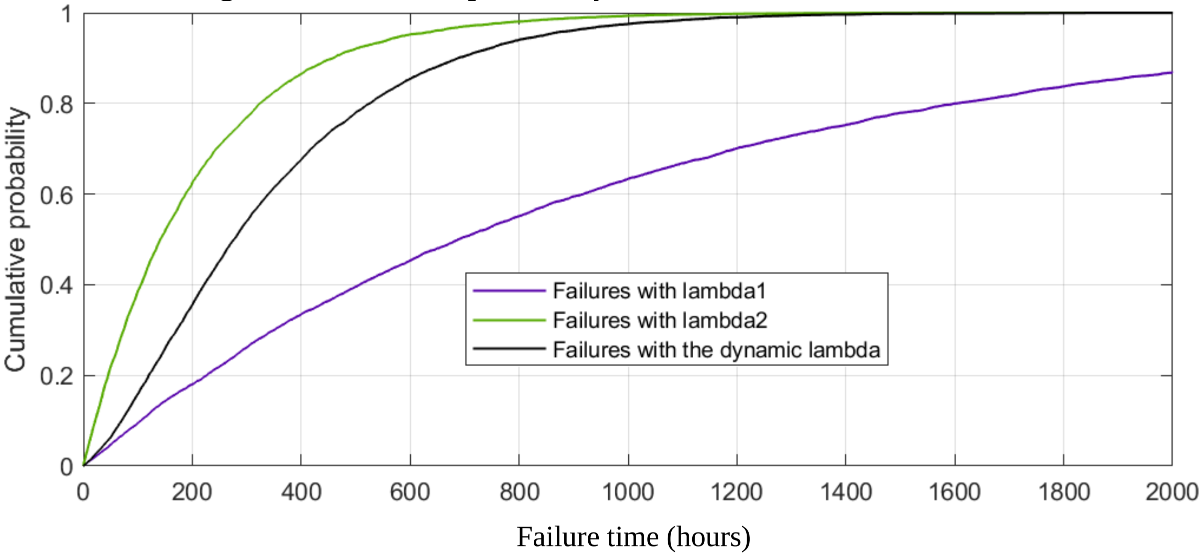


Figure 7. Cumulative probability of failure time from 10,000 runs



5. CONCLUSION

As exemplified in the methodology and examples, discretion between scenarios is critical for appropriately estimating the time to failure. If the inappropriate methodology is used, it can lead to skewed results due to an under or overestimation of the time to failure of the component.

Acknowledgments

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References

[1] Idaho National Laboratory, "EMRALD," Accessed April 12, 2022. https://github.com/idaholab/EMRALD.

[2] J. K. Knudsen and C. L. Smith, "Space Propulsion System Phased-Mission Probability Analysis Using Conventional Pra Methods," Idaho National Laboratory, 2006, Idaho Falls.

[3] N. Kopytko, "Spineless attacks on nuclear power plants could increase," Bulletin of the Atomic Scientists, 2015, Chicago.

[4] M. Alamaniotis, L. H. Tsoukalas, and V. Agarwal, "Predictive Based Monitoring Of Nuclear Plant Component Degradation Using A Support Vector Regression Approach," Idaho National Laboratory, 2015, Idaho Falls.

[5] F. M. Dekking, H. P. Lopuhaä, C. Kraaikamp and L. E. Meester, "A Modern Introduction to Probability and Statistics: Understanding Why and How," Springer, 2005, London.

[6] M. Engelhardt and L. J. Bain, "Introduction to Probability and Mathematical Statistics," Duxbury Press, 1992, Belmont.

[7] V. Agarwal, Y. A. Ballout and J. C. Gehin, "Fission Battery Initiative," Idaho National Laboratory, 2021, Idaho Falls.