Influence of Operating Load Spectra Shapes on Reliability Demonstration Test Planning

Achim Benz [a], Alexander Grundler[a], Martin Dazer[a] and Bernd Bertsche [a]
[a] Institute of Machine Components, University of Stuttgart, Germany,
achim.benz@ima.uni-stuttgart.de

Abstract:
Planning reliability demonstration tests is particularly complex if end-of-life tests are used instead of standard success run tests. In addition to the factors of cost and time, the probability of test success can be used to select the optimal test strategy. This has already been considered for a single-stage load in several papers. In real applications single stage loads are not applied. Using single stage loads also leads to ignoring the changing confidence level of the life model due to the placement of the test load levels. Recent research now also uses operating load spectra, deriving the probability of test success for the accumulated damage of the load spectrum. Considering the influence of this operating load spectrum on the probability of test success is therefore a logical step. To determine if there is an influence and how different parameters change the influence a simulation study with a wide parameter space is executed and analyzed. The study shows an influence where different parameters can affect the achievable probability of test success. This happens due to the interaction of the operating loads of the spectrum with the confidence level of the life model influenced by the test levels and test specimen.

Keywords: End of life test; reliability demonstration; probability of test success; operating load spectrum;

1. INTRODUCTION

In order to demonstrate a products reliability a physical test has to be performed. In practice different test boundaries influence the optimal test configuration. In addition to the specifications for the required reliability and confidence level, a project timeframe and a cost budget are usually specified.
Here we will focus on the End of Life (EoL) tests, which are more difficult to plan but generate more information about the failure mechanism. It is not easy to find the optimal test load levels and number of specimens to perform an EoL test within the cost and time constraints, due to their interdependence with the confidence level. With an additional optimization criterion, the probability of test success (P_{ts}), derived by simulations with prior knowledge of the life model facilitates the search for the optimal test configuration. In recent research the benefit of planning success run tests (SRT) and EoL tests with the optimization criterion P_{ts} was shown for single stage operating loads. However, in real application scenarios, operating load spectra are often present. An operating load spectrum leads to an accumulated damage from the individual loads. After implementing a method to calculate the P_{ts} for an operating load spectrum the question arises to how the shape of the operating load spectrum influences the P_{ts} used to find the optimal test strategy. The aim of this research is therefore the influence of the shape of the operating load spectrum on the optimal test planning with P_{ts}.

2. STATE OF THE ART AND RECENT RESEARCH

Nelson [1], Escobar [2] and Meeker [3], covered the basics of optimal test planning. They all provide guidance for selecting the height and number of load levels for the test configuration. But it needs to be kept in mind that all of these work by minimizing the variance. But with simulations using prior knowledge and an additional optimization criterion, the probability of test success (P_{ts}) developed by Dazer et al. [4] it is possible to determine the most economical test strategy. This is because the rate of success for the EoL test can be determined before the physical test itself is executed. The P_{ts} is therefore defined as the statistical power of a reliability demonstration test. The P_{ts} can thereby also be determined as a hypothesis [5], analogous to the well-understood and widely used confidence level

CI [6]. The hypotheses are defined via the lifetime quantiles t_{R_r} at required reliability R_r and the required lifetime quantile t_r for EoL tests [5, 7, 8]:

$$H_0: t_{R_\mathrm{r}} < t_\mathrm{r} \tag{1}$$

$$H_1: t_{R_\mathrm{r}} \geq t_\mathrm{r} \tag{2}$$

Using the required confidence level $C_r = 1 - \alpha$, the P_{ts} is the complement of the statistical error β (see e.g. [9]) of the test

$$P_\mathrm{ts} = 1 - \beta \tag{3}$$

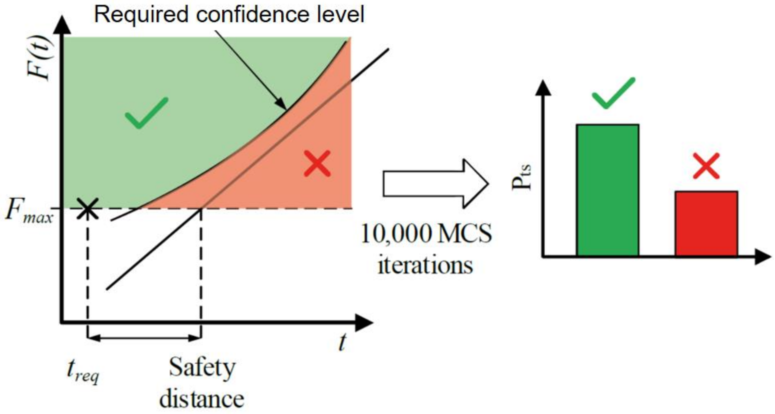
and can be calculated using the null and alternative distribution f_{H_0} and f_{H_1} of t_{R_r}, which represents the distribution of the lifetime quantile under validity of the respective hypothesis [7] for EoL tests:

$$C_\mathrm{r} = \int_{-\infty}^{t_\mathrm{crit}} f_{H_0}\, \mathrm{d}t_{R_\mathrm{r}} \tag{4}$$

$$P_\mathrm{ts} = \int_{t_\mathrm{crit}}^{+\infty} f_{H_1}\, \mathrm{d}t_{R_\mathrm{r}} \tag{5}$$

Deriving the P_{ts} by a Monte-Carlo-Simulation (MCS) the P_{ts} can be counted analytically by setting the number of successful simulated test runs in relation to the unsuccessful test runs, see Figure 1.

Figure 1: The concept of calculating the P_{ts} for an EoL test using an MCS, edited [4]



Information to the differences between SRT and EoL for computing P_{ts} can be found in references [4] and [10]. Work on this topic by Herzig et al. [11] extended the approach to include accelerated SRT and EoL testing for single stage loads. He also found that the upper test load for EoL test should always remain at the highest possible acceleration to find an optimal test strategy, by only varying the lower test load. In further work Herzig et al. [12] was able to determine minimum key design parameters using the P_{ts} approach planning efficient test strategies. Grundler et al. [6, 8] also worked with for single stage loads. In [8] he investigated uncertainty of the prior knowledge used to derive the P_{ts} and was able to determine a reduction of achievable P_{ts} due to an increased scatter of the lifetime quantiles. Grundler et al. [6] also considered multiple failure mechanisms and system levels to determine the optimal test strategy for real system configurations. Recent work on this topic by Benz et al. [13] extends optimal test planning with P_{ts} to include operating load spectra. Thereby, an accumulated damage is computed from the operating load spectra, for which the P_{ts} is derived by comparing it to a damage calculated from the prior knowledge.

Consideration of the influence of load spectra in conjunction with life models such as the Woehler model is necessary, as already shown by results in Haibach [14]. The amplitude collectives used for the calculation of the Woehler curve were varied and their influence on the course of the service life model was considered. Similarly, but always with the constant accumulated total damage D, different operating load spectra are also used in the following sections.

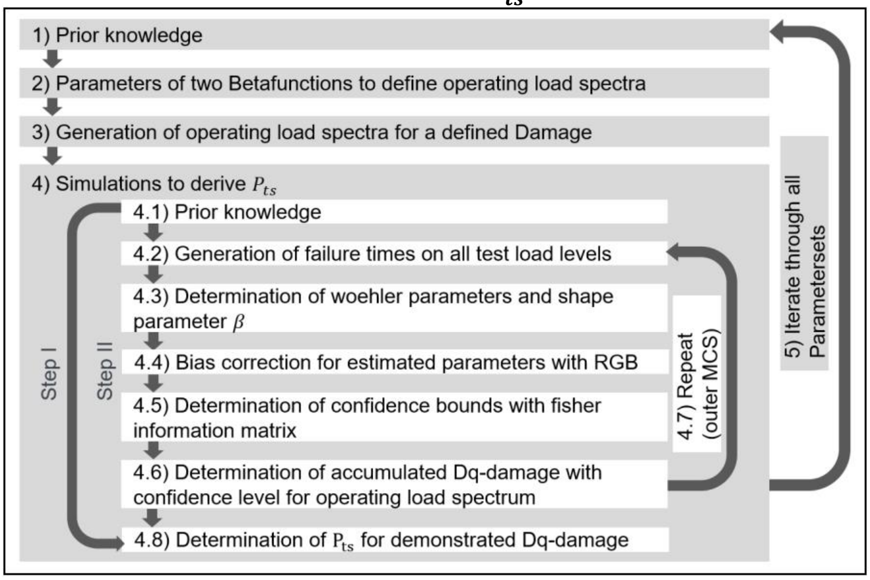
3. APPROACH

In the following section, the procedure for investigating the influence of the operating load spectrum on the optimization criterion P_{ts} used for planning EoL tests is presented. Three subsections are outlined. First, the general procedure to perform and evaluate P_{ts} for the different parameter settings using Monte-Carlo simulations. Second, how the operating load spectra are created to ensure repeatability of the experiments and to be able to represent all possible operating load spectra. Last, an overview of the parameter space for the analysis is presented.

3.1. GENERAL APPROACH

To investigate the influence of different operating load spectra on the P_{ts} used to design the optimal EoL test strategy, the following procedure is performed as shown in Figure 2. To derive the P_{ts} virtual tests are simulated on the basis of prior knowledge for a life model, in this case the Woehler model. Using an operating load spectrum, the damage is calculated first with the prior knowledge and then with the result of the virtual test to derive the P_{ts}. A more detailed description with formulas can be found in the paper Benz et al. [13]. In the following, the approach for investigating the influence of different operating load spectra is explained in detail as well as a brief introduction to the general calculation steps of the simulation for the P_{ts}, which was also extended by one step. The operating load spectrum is defined by means of two beta functions. In the simulation process, the operating load spectra (3) are first calculated for a given damage value from the available prior knowledge (1) and the beta functions (2). In more detail this is described in section 3.2.

Figure 2: General approach for the study of operating load spectra influence on P_{ts}



After that, virtual tests (4) are performed. The left string (Step I) shows the execution of a damage analysis and the determination of the accumulated damage of the operating load spectrum from the prior knowledge representing the knowledge of the real lifetime behavior of the considered system (failure mechanism). Step II is executed multiple times (4.7) to determine the 95% confidence limit of a virtual damage for each run. It represents multiple runs of EoL tests, resulting in a scattering confidence bound. For each inner MCS (Step II), Weibull distributed failure times are generated for the specified test load levels, (4.2), since the prior knowledge assumes a Weibull distributed failure behavior. From the failure times, the Woehler parameters $\hat{k}$, $\widehat{N_D}$ as well as the Weibull shape parameter $\hat{\beta}$ are determined by an MLE, (4.3). To avoid overestimation of the P_{ts}, a bias correction was introduced for the beta parameter estimated from the MLE (4.4).

The most common method is implemented called the reduced bias adjustment (RBA) method in Equation 6, see Abernethy [15].

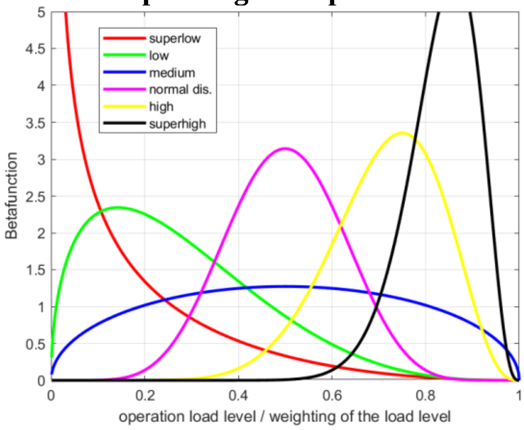
$$\hat{b}_{RBA} = \hat{b}_{MLE} \cdot \left(\sqrt{\frac{2}{n_f - 1}} \cdot \frac{\left(\frac{n_f - 2}{2}\right)!}{\left(\frac{n_f - 3}{2}\right)!} \right)^{3.52} \tag{6}$$

For the MLE estimated and bias corrected parameters, the Fisher Information Matrix (FIM) is set up, (4.5). With this information the accumulated Damage with confidence level can be derived for the operating load spectrum (4.6). Last the P_{ts} can be derived from all accumulated damage values from each inner Monte-Carlo-Iteration. In order to consider different parameter variations, these steps are performed for each parameter set (5) in each study. The parameters kept constant as well as the varied parameters of the individual simulation studies are described in section 3.3.

3.2. VARIATION OF LOADSPECTRA

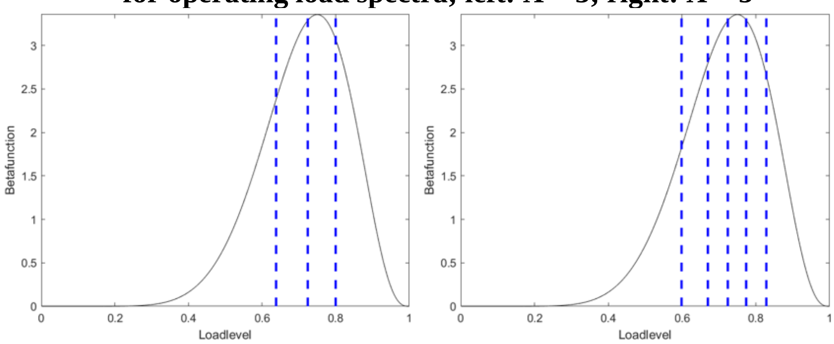
To accurately describe each operating load spectra, they are described with two beta functions. By specifying the parameters of the beta functions for each simulation study, the tests yield the same results in a reproducible manner. The beta function was chosen because it represents the range of values (0 - 1), which can indicate the load level in percent. Since all possible beta functions can be used, those used in the study are shown here in Figure 3.

Figure 3: Shapes of beta functions used to define operating load spectra



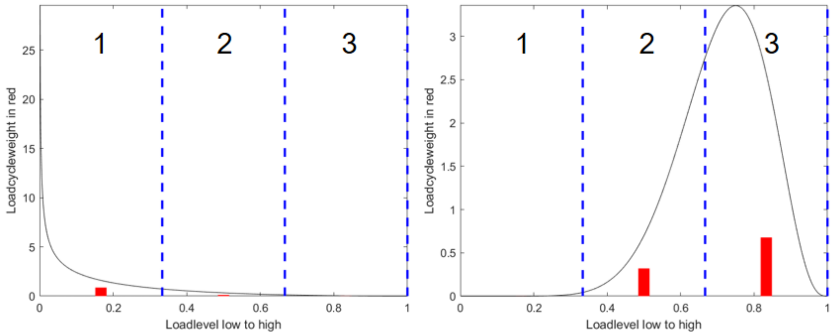
The first beta function is used to define the individual load levels. For this purpose, another parameter X is specified, which defines the number of load levels. Then the area under the beta function is divided into $X + 1$ equal areas. The intersection points of the dividing lines with the x axis define the load levels of the operating load spectrum, see Figure 4.

Figure 4: Definition of load level with beta function (A = 10, B = 4) for operating load spectra; left: X = 3; right: X = 5



In the next step, the load cycles of the load levels are determined with the life model given as prior knowledge and a damage D to be achieved. The second beta function indicates the weighting of the individual loads on the damage. For this purpose, the x-axis of the beta function is divided into X parts and the area thus enclosed by the beta function is calculated, see Figure 5. This area is then used as a proportion to the total area as a weighting for the respective load levels share of the accumulated damage.

Figure 5: Definition of load cycle allocation with beta function for three operating load levels; left: A = 0.5, B = 3; right: A = 10, B = 4



The load spectra created here are of course limited within the scope of the first analysis, however, all possible combinations can be represented with this approach. More about the simulation study will follow in the next section.

3.3. PARAMETERSPACE AND SIMULATIONSTUDY

The simulation study to investigate the influence of different shapes of operating load spectra is derived though Monte-Carlo-Simulations for each parameter combination. All parameters are presented in Table 1 with their value space if varied or the absolute values if constant.

Table 1: Parameter space

	Parameter		Value space
Changing parameters	Woehler slope k		3 - 15
	specimen on upper test load level n_H		5 - 50
	specimen on lower test load level n_L		5 - 50
	upper test load level σ_H		0.1 - 1
	lower test load level σ_L		0.1 - 0.9
	Amount of operation load level X		2 - 10
	Beta function parameters	superlow	A = 0.5 B = 3
		low	A = 1.5 B = 4
		medium	A = 1.5 B = 1.5
		norm. dist.	A = 8 B = 8
		high	A = 10 B = 4
		superhigh	A = 20 B = 4
Constant Parameters	Reliability requirement R		90%
	Confidence level CI		90%
	Woehler location parameter N_D		1
	Safety margin s		0.2
	Damage D		1

The study was performed using a Sobol sequence to cover the entire parameter space. However, this also leads to combinations that make little sense. For example, the upper test load level must always be above the lower test load level. This will be considered when evaluating the results in section 4. To investigate the shape of the operating load spectrum and not the accumulated damage caused from it, the operating load spectra are created so that the damage caused is always constant, which is why it was set to $D = 1$ in this study. Also, the Woehler location parameter N_D is held constant at 1 in order to investigate the influence of the slope k specifically without them interacting reciprocally. Other test planning parameters such as the reliability target, the confidence level and the safety margin are also kept constant.

4. RESULTS

In this section, the results of the simulations in the parameter space given above and the different operating load spectra are presented and discussed. The first results for the P_{ts} given here show the general effects of the operating load spectrum in interaction with the life model and its slope k. For this study, the number X of operating loads of the spectrum is kept constant at three and also the weighting of the operation loads is constant on medium and therefore nearly equal at all operation loads. As a result, every operating load of the spectrum is equally involved at the accumulated Damage D. It can be seen in Figure 8 on the left that the slope of the life model has a particularly strong effect on the P_{ts} when the "medium" load levels shown in blue are applied as an operating load spectrum. That occurs, when the operating load spectrum consists not only of high operating loads but also of low operating loads. This can be easily explained by the fact that the largest range of values of the life model is used here and therefore the slope has the biggest effect. Some of the other operating load spectra also show a slight influence of the slope k on the P_{ts}, but not to the same extent. It also shows that operating load spectra which only has loads that lie within the two test load levels σ_H and σ_L allow high P_{ts}, since here the confidence interval CI is the smallest and one can only interpolate between the two test levels. An operating load spectrum that is outside the extrapolated range only allows lower values for the P_{ts}. This behavior can be clearly seen in Figure 7 on the right.

Figure 7: Influence of the slope k on the P_{ts} for different operating load spectra; test levels: $\sigma_H = 1$; $\sigma_L = 0.8$;

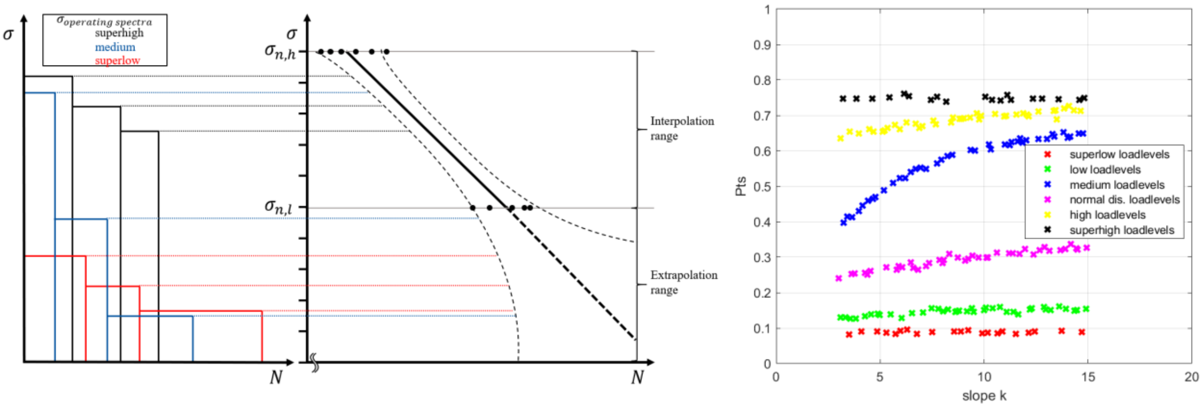
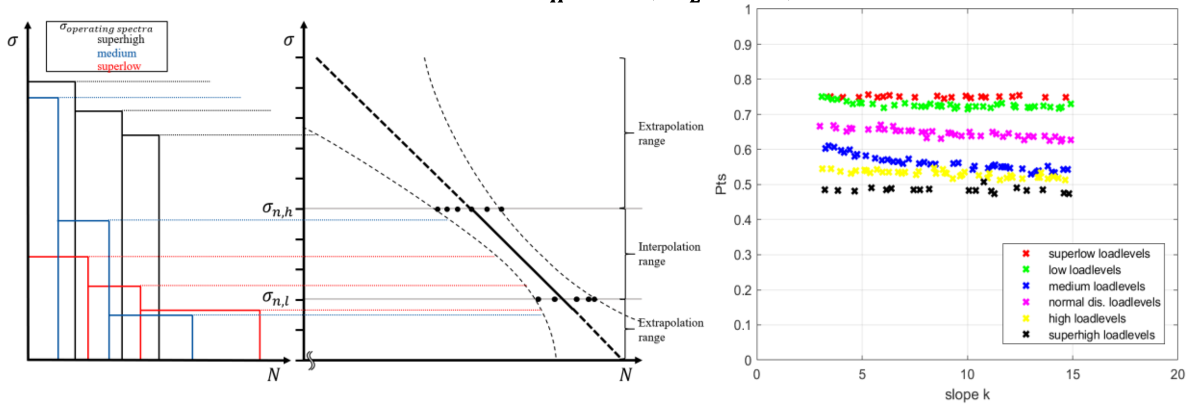


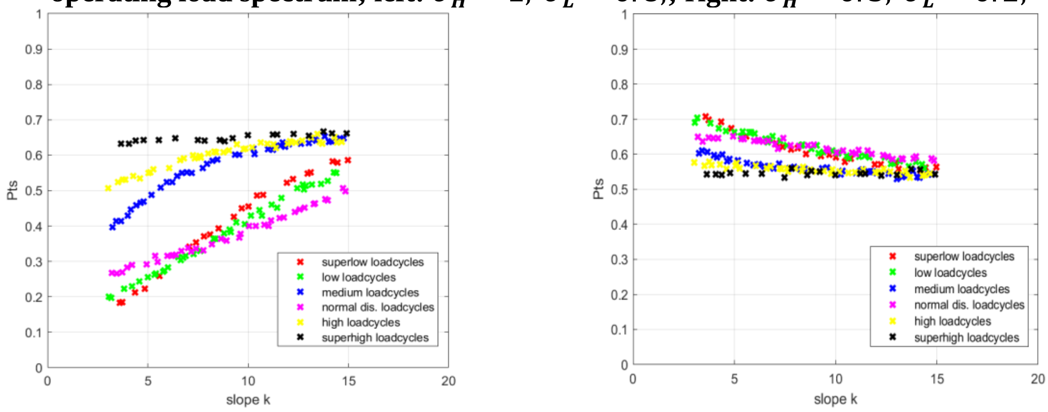
Figure 8 on the right shows that at low test levels, the correspondingly lower operating load spectra achieve a higher P_{ts} than the high operating load spectra. The influence of the slope k on the P_{ts} is lower. For the medium operating load spectrum, the slope k now even slightly reduces the achievable P_{ts}. Figure 8 on the left therefore shows that P_{ts} decreases the further away the operating load spectrum is from the test loads.

Figure 8: Influence of the slope k on the P_{ts} for different operating load spectra; test levels: $\sigma_H = 0.5$; $\sigma_L = 0.2$;



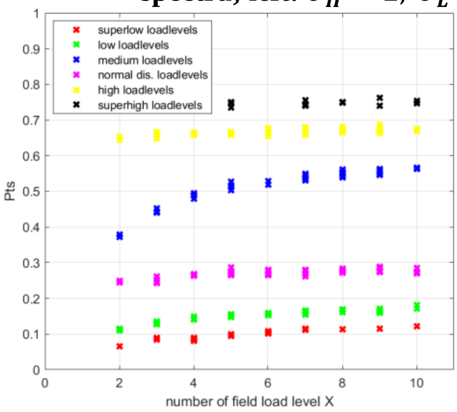
From Figure 9 on the left, the same dependence of the P_{ts} on the slope k is shown, but here the weighting of the individual operating load levels varies. It can be seen that a strong weighting of the high operating loads minimizes the influence of the life model on the P_{ts}. This is also because it makes the wider confidence regions of the life model extrapolated from the test levels less significant. In Figure 9 on the right, the test loads are set in the middle to low, which, as expected, makes the weighting less significant because the medium operating load spectrum has been evaluated and now does not need to be extrapolated as far.

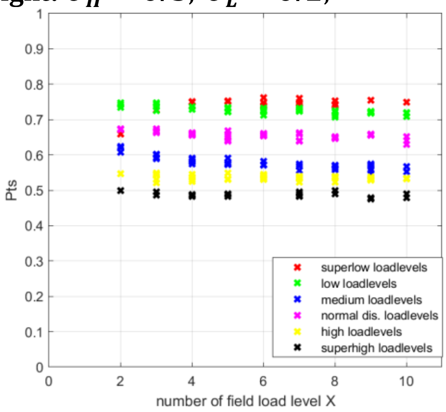
Figure 9: Influence of the slope k on the P_{ts} for different load cycle weights for medium operating load spectrum; left: $\sigma_H = 1$; $\sigma_L = 0.8$;; right: $\sigma_H = 0.5$; $\sigma_L = 0.2$;



Further results to be considered still concern the used operating load spectrum itself, by varying the number of operating load levels X. These directly affect the generation of the operating load spectra. In Figure 10, a relationship between the number X of operating loads and the P_{ts} can be seen in the left graph. This is due to the fact that, as the operating load spectrum is more evenly divided into individual loads, the influence of the loads lying in the extrapolated range of the service life model on the damage D is reduced. When the high test load is lower this influence weakens and reverses, see Figure 10 on the right.

Figure 10: Influence of the number of load levels X on the P_{ts} for different operating load spectra; left: $\sigma_H = 1$; $\sigma_L = 0.8$;; right: $\sigma_H = 0.5$; $\sigma_L = 0.2$;

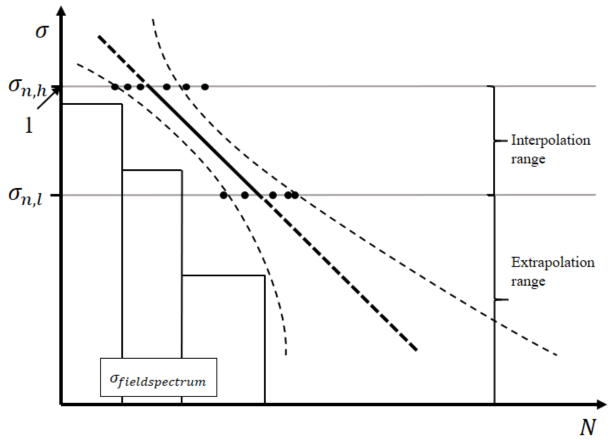




4.2. DISCUSSION

The strongly decreasing P_{ts} for operating load spectra with only low loads can be explained by the fact that in the area of the extrapolation of the service life model, see Figure 11, the confidence interval widens. To counteract this, currently only adjustments to the EoL test are known, which strongly increase the test time and costs like increasing the number of specimens on each test load level and also keeping the lower test level on the lowest operating load level of the spectrum.

Figure 11: Attempted explanation of the low P_{ts} for low operating load spectra



One possibility to perform the EoL test as economically as possible and to obtain high values for the P_{ts} for all possible operating load spectra is to increase the number of test levels. This means that an operating load spectrum can be better mapped by an optimized number of test load levels and thus achieve the reliability targets with a higher P_{ts}. In contrast, a single lower test load level is sufficient for a single stage operating load, which must also be close to the field load in order to keep the extrapolation error low.

5. CONCLUSION

In summary, the test load levels interact with the operating load spectrum directly through the confidence level of the life model. This shows that the more loads of the operating load spectrum lie within the narrow confidence level between the two test loads, the higher the P_{ts}. Likewise, heavily weighted individual operating loads of a spectrum can influence this interaction with the confidence level. Either the area with a narrow confidence level between the two test loads, which is conducive for the P_{ts}, can be weighted higher or, in order to lower the P_{ts}, the area outside, where a lot of extrapolation has to be done and thus a wide confidence level exists. As expected, the operating load

spectrum has a strong and the weighting of individual operating loads have a non-negligible influence. It was also shown that parameters like the amount of operating load of the spectrum can influence the P_{ts}. The performed simulation studies also show that by varying the number of test specimens and the level of the lower test load the optimal test strategy can be found.
As a next step, a larger parameter study would be suitable, which, however, would then no longer be evaluated analytically by hand, but would be used to train a neural network. This would allow the neural network to determine the optimal test strategy for an operating load spectrum without having to spend time and effort on a simulation study. Also, the introduction of more than two test loads has to be considered in order to achieve a test strategy with high P_{ts} within reasonable limits for the cost and time constraints.

References

[1] W. Nelson, "Accelerated Testing. Statistical models, Test plans and Data Analyses. Ed. Hoboken, NJ, USA, Wiley, 2004

[2] L. A. Escobar and W. Q. Meeker, "A review of accelerated test models." *Statist. Sci.*, vol. 21, no. 4, pp. 552–577, 2006

[3] W. Q. Meeker and L. A. Escobar, "Statistical Methods for Reliability Data". NY, USA, Wiley Series, 1998

[4] M. Dazer, et al., "Optimal planning of reliability life tests considering prior knowledge", Proc. of IEEE RAMS Conference, 2018

[5] A. Grundler , M. Dazer, T. Herzig; Statistical Power Analysis in Reliability Demonstration Testing: The Probability of Test Success. Appl. Sci. 2022, 12, 6190. https://doi.org/10.3390/app12126190

[6] A. Grundler, M. Dazer, and B. Bertsche, "Reliability-Test Planning Considering Multiple Failure Mechanisms and System Levels," in *Annual Reliability and Maintainability Symposium. 2020 Proceedings*, 2020.

[7] A. Grundler, M. Dazer, T. Herzig, and B. Bertsche, "Efficient System Reliability Demonstration Tests Using the Probability of Test Success," in *Proceedings of the 31st European Safety and Reliability Conference ESREL 2021*, 2021, pp. 1654–1661.

[8] Grundler, A. ; Dazer, M. ; Bertsche, B. : Effect of Uncertainty in Prior Knowledge on Test Planning for Brake Caliper using the Probability of Test Success : RAMS 2021 Conference (Annual Reliability and Maintainability Symposium), 24.05.-27.05.2021, Orlando, FL, USA

[9] J. Cohen, Statistical Power Analysis for the Behavioral Sciences, 2nd ed. New York: Lawrence Erlbaum Associates, 1988.

[10] M. Dazer, et al., "Planning of reliability life tests within the accuracy, time and cost triangle", Proc. Of IEEE ASTR Conference, pp. 50-57, 2016

[11] T. Herzig, et al., "Cost- and Time-Effective Planning of Accelerated Reliability Demonstration Tests - A New Approach of Comparing the Expenditure of Success Run and End-of-Life Tests", Proc. of IEEE RAMS Conference, 2019

[12] Herzig, T.; Dazer, M; Grundler, A.; Bertsche, B. Resource-efficient Gear Wheel Design considering early planning of Reliability Demonstration Tests, 2021

[13] Benz, A.; Grundler, A.; Herzig, T.; Dazer, M.; Bertsche, B: Reliability Demonstration Test Planning for Operating Load Spectra – an Approach for Identifying the Optimal Test Parameters considering Individual Cost and Time Constraints. RAMS 2022 Conference (Annual Reliability and Maintainability Symposium), 24.01.-27.01.2022, Tucson, AZ, USA.

[14] E. Haibach, „Betriebsfestigkeit", Heidelberg, Germany, Springer-Verlag Berlin Heidelberg, pp. 63-64, 2006

[15] R. B. Abernethy, „The New Weibull Handbook ", Fifth Edition, Florida, USA, 2006