Optimal Sampling Placement Based on Value of Information Considering Seismic Hazard

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Sampling placement has been determined empirically in practice. This paper discusses optimal additional sampling planning along a river for liquefaction countermeasure based on Value of Information (VoI) considering seismic hazard. The proposed VoI-based objective function for optimal placement quantifies the risk reduction of decision making error considering the uncertainty of estimation in a Gaussian random field. Seismicity, namely the probability of exceedance of scenario earthquake is considered in the VoI formulation. Optimal number of additional sampling is also evaluated from total cost, i.e., sum of observation cost and VoI. The balance of penalties and observation cost determines the optimal placement and the number though the penalties are difficult to determine rationally. The difference of seismicity reflects the optimal number. The seismicity is the higher, the number of optimal sampling is the more. The results show that optimal sampling placement may be obtained with a feasible computational cost. Optimal number of additional sampling is also evaluated depending on the seismicity of the site.

I. INTORODUCTION

Sampling placement has been determined empirically. Some researches try to establish a quantitative method to obtain optimal sampling placement. Most of the studies, however, considered only the uncertainty of the estimation. The optimal observation placement problem contains two aspects, minimization of the relevant uncertainties (maximization of the accuracy) and minimization of total costs. Several measures of uncertainty, such as covariance matrix or information entropy have been used in optimal observation placement problems (Ref. 1). The various norms of the parameter or prediction covariance matrix may be used to express overall uncertainty. For example, Ref. 2 use information entropy to study observation scheme for ground deformation prediction, while Ref. 3 use geometric mean of reduction ratio of standard deviation of model or response parameters of a slope.

Traditional measures of uncertainty, such as covariance matrix or information entropy, however, do not depict the significance of uncertainty if the consequence due to the uncertainty is not considered. Ref. 4 describe intensively the theory of Value of Information (VoI) in decision making under uncertainty. VoI can be interpreted to be expectancy of risk reduction or benefit obtained by the information. Ref. 5 points out that no theory that involves just the probabilities of outcome without considering their consequences could be adequate to a decision maker, and shows an application of VoI-based decision making to a bidding problem. Ref. 6 introduce an example of application of VoI to project decision making with decision tree. Ref. 7 propose a Bayes decision procedure model with VoI concept optimizing the process of post-earthquake emergency response in highly uncertain conditions to prevent secondary damage by emergency shut-off of lifeline services. Ref. 8, 9 and 10 describe intensively concept and application of VoI in maintenance problem of infrastructures. Ref. 11 discuss the application of VoI-based method to structural health monitoring. Ref. 12 discuss decision making framework for earthquake early warning with VoI concept. Ref. 13 point out that VoI based method seeks to minimize expected cost attributable to uncertainty, and helps decision makers to decide whether the data should be collected or not. New samplings information reduces the variance of parameters, however, quantification of reduction in variance is not enough to answer the question whether sampling (observation, measurements) should be collected or not. To answer these questions, we need to estimate the worth of the information content in data, i.e., the value of information (VoI).

Ref. 14 proposes an efficient method based on VoI to answer where additional sampling points should be placed and how many sampling should be collected in a Gaussian random field. In this paper we consider seismicity of the site in the formulation of VoI based objective function and discuss how the seismicity affect the optimal number of additional sampling.

II. VALUE OF INFORMATION FOR OPTIMAL SAMPLING PLACEMENT

II.A. Linear inverse problem and Kriging

Prior information as to random variable vector $\boldsymbol{x}$ is given as,

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \boldsymbol{w} \qquad (1)$$

where $\bar{\mathbf{x}}$ and $\boldsymbol{w}$ are mean and random component of prior information. Here, it is postulated that the observation $\boldsymbol{z}$ is expressed as a linear function of $\boldsymbol{x}$ and is contaminated with a Gaussian noise $\boldsymbol{v}$ as follows.

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} \qquad (2)$$

$\boldsymbol{v}$ and $\boldsymbol{w}$ are Gaussian random variable vector with zero mean and their covariance matrices $\boldsymbol{R}$, $\boldsymbol{M}$. Best posterior estimate (MAP) and its covariance matrix are,

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \boldsymbol{P}\boldsymbol{H}^T\boldsymbol{R}^{-1}(\boldsymbol{z} - \boldsymbol{H}\bar{\boldsymbol{x}}) \qquad (3) \qquad \boldsymbol{P} = (\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H} + \boldsymbol{M}^{-1})^{-1} \qquad (4)$$

Kriging is a probabilistic interpolation method in a Gaussian random field (e.g. Ref. 15; Ref. 16), and is derived as a special case of above mentioned linear inverse problem (Ref. 17). Assume that the observation vector $\boldsymbol{z}$ and parameters $\boldsymbol{x}$ are the same physical parameters at discrete spatial points in the Gaussian random field.

$$\boldsymbol{x}^T = \left\{\boldsymbol{x}_1^T, \boldsymbol{x}_2^T\right\} \qquad (5) \qquad \boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} = \begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\end{bmatrix}\begin{Bmatrix}\boldsymbol{x}_1 \\ \boldsymbol{x}_2\end{Bmatrix} + \boldsymbol{v} \qquad (6)$$

where $\boldsymbol{x}_1$ denotes variables at observation site; $\boldsymbol{x}_2$ denotes parameters at the region to be estimated. The observation equation Eq.(2) becomes, $\boldsymbol{I}$ denotes unit matrix; $\boldsymbol{0}$ denotes zero matrix. By substituting Eq.(5) , (6) into Eq.(3), (4), we have,

$$\begin{Bmatrix}\boldsymbol{x}_1 \\ \boldsymbol{x}_2\end{Bmatrix} = \begin{Bmatrix}\bar{\boldsymbol{x}}_1 \\ \bar{\boldsymbol{x}}_2\end{Bmatrix} + \begin{bmatrix}\boldsymbol{M}_{11} \\ \boldsymbol{M}_{12}^T\end{bmatrix}\left[\boldsymbol{M}_{11} + \boldsymbol{R}\right]^{-1}\left\{\boldsymbol{z} - \bar{\boldsymbol{x}}_1\right\} \qquad (7) \qquad \boldsymbol{P} = \begin{bmatrix}\boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{12}^T & \boldsymbol{P}_{22}\end{bmatrix} \qquad (8)$$

$$\boldsymbol{P}_{ij} = E[\boldsymbol{M}_{ij} - \boldsymbol{M}_{i1}(\boldsymbol{M}_{11} + \boldsymbol{R})^{-1}\boldsymbol{M}_{1j}] \qquad (9)$$

It is noted that Prior covariance matrix $\boldsymbol{M}$ is separated into $\boldsymbol{M}_{11}$, $\boldsymbol{M}_{12}$, $\boldsymbol{M}_{21}$, $\boldsymbol{M}_{22}$ corresponding to $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. Prior covariance matrix $\boldsymbol{M}$ is formulated often based on auto-correlation function. Several types of auto-correlation function are proposed. In this paper, the following equation is used.

$$R(d_1, d_2, d_3) = \sigma^2 \exp\left[-\left\{\left(\frac{d_1}{a_1}\right)^2 + \left(\frac{d_2}{a_2}\right)^2 + \left(\frac{d_3}{a_3}\right)^2\right\}\right] \qquad (10)$$

where, d_1, d_2, d_3 stand for a distance, a_1, a_2, a_3 stand for an auto-correlation distance in each direction in three dimensional space; σ^2 is variance of the field.

II.B. Quantification of VoI in a Gaussian random field

It is assumed that observation is performed to obtain useful information to make decision by comparing estimator x with threshold limit value x_0, e.g., to judge contaminated soil or ordinary soil by comparing poisonous material concentration x and its threshold limit value x_0, or to judge necessity of liquefaction countermeasure on embankment along a river by comparing liquefaction potential x with its threshold limit value x_0.

Statistical test has two kinds of error. A type I error (or error of the first kind) is the incorrect rejection of a true null hypothesis. A type II error (or error of the second kind) is the failure to reject a false null hypothesis. Referring to these error types, we define two types of false decision making.

i) Decision error type 1
Judge $x < x_0$ when true $x > x_0$ (e.g., to judge that liquefaction countermeasure is not necessary when it is necessary actually)

ii) Decision error type 2
Judge $x > x_0$ when true $x < x_0$ (e.g., to judge that liquefaction countermeasure is necessary when it is not necessary actually)

The probabilities of decision error type 1, 2 are denoted as P_1, P_2 (= 1-P_1). The risk of the decision error can be calculated with penalties per unit area C_1, C_2 for the decision errors and the probabilities. Naturally we should make decision to take lower risk.

$$J = \sum_i L_i = \sum_i \min\left(C_1 P_{1,i}, \quad C_2(1 - P_{1,i})\right) \qquad (11)$$

Suffix i indicates a region for estimation of risk. Total risk is calculated by summing up the risk over the area for the estimation.

Let's have an example that we have estimator x=3 when threshold limit value x_0=3. It is assumed that the estimator involves uncertainty and its mean is 3. It is also assumed that penalties of the error type 1, 2 are 10, 2 respectively. These assumed values are only for the illustration, and do not have any actual meaning. If the estimator is judged to be less than the threshold value, the probability of error is 0.5, and its risk is 5. If the estimator is judged to be larger than the threshold value, the probability of error is also 0.5, and its risk is 1. The former and latter are called as risk 1 and 2 respectively. Since the smaller risk should be taken naturally, we should take risk 2. Fig. 1 shows the risk we should take for estimator of which mean is 0 to 4. It is assumed that the estimator is Gaussian and its standard deviation is 0.4.

When the mean of estimator is 3, the risk 1 and 2 are plotted at 1 and 2 respectively. When the mean becomes small, risk 1 also becomes small, on the other hand risk 2 becomes large. The point x_c that risk 1 is equivalent to risk 2 indicates a threshold value for the judgement under uncertainty. We should judge that the estimator is larger than threshold limit value x_0 when the mean of estimator is larger than the threshold value for the judgement x_c. The difference between the x_c and x_0 expresses safety margin. The threshold for judgement x_c is determined by uncertainty of the estimator and the ratio of penalty 1 and 2, C_1, C_2.

In general, it is difficult to compute VoI so that MC approach is proposed(Ref. 10; Ref. 13; Ref. 18). VoI can be, however, computed easily in updating of Gaussian random field, i.e., Kriging, described in 2.2. It is assumed that observation data at new locations are obtained at each observation step.

$$\boldsymbol{Z}^{k^T} = \left\{ \boldsymbol{z}^{1^T}, \boldsymbol{z}^{2^T}, \cdots, \boldsymbol{z}^{k^T} \right\} \qquad (12)$$

where $\boldsymbol{z}^k$, $\boldsymbol{Z}^k$ represent observation data at step k and up to step k. Mean vectors at three types of places are obtained by referring Eq.(7),

$$\begin{Bmatrix} \bar{\boldsymbol{x}}_1^k \\ \bar{\boldsymbol{x}}_2^k \\ \bar{\boldsymbol{x}}_3^k \end{Bmatrix} = \begin{Bmatrix} \bar{\boldsymbol{x}}_1^0 \\ \bar{\boldsymbol{x}}_2^0 \\ \bar{\boldsymbol{x}}_3^0 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{M}_{11}^0 \\ \boldsymbol{M}_{12}^{0\,T} \\ \boldsymbol{M}_{13}^{0\,T} \end{bmatrix} \left[\boldsymbol{M}_{11}^0 + \boldsymbol{R}_1^k \right]^{-1} \left\{ \boldsymbol{Z}^k - \bar{\boldsymbol{x}}_1^0 \right\} \qquad (13)$$

where $\bar{\boldsymbol{x}}_1^k$ represents a mean vector at places where the observation $\boldsymbol{Z}^k$ is given; $\bar{\boldsymbol{x}}_2^k$ is a mean vector at places where new observation $\boldsymbol{z}^k$ will be given; $\bar{\boldsymbol{x}}_3^k$ presents a mean vector at area where decision error risk is evaluated. Their covariance matrices are given as:

$$\boldsymbol{M}_{ij}^k = \boldsymbol{M}_{ij}^0 - \boldsymbol{M}_{1i}^{0^T} (\boldsymbol{M}_{11}^0 + \boldsymbol{R}_1^k)^{-1} \boldsymbol{M}_{1j}^0 \qquad (14)$$

It is noted that locations of $\boldsymbol{x}_2$ are those of observation points at k+1 step, and the observation data is not obtained yet. As

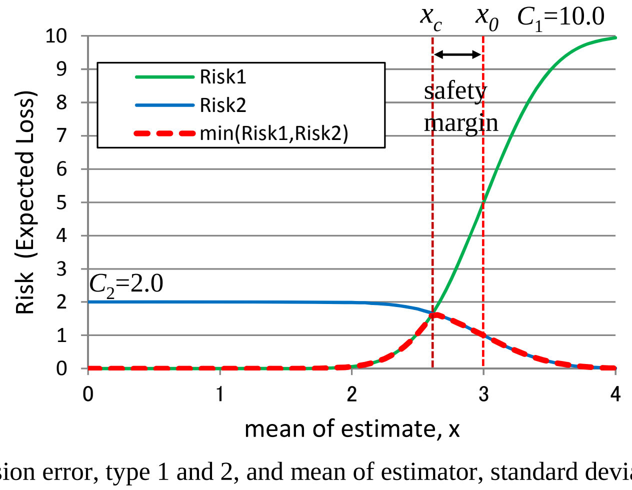


Fig. 1. Risk of decision error, type 1 and 2, and mean of estimator, standard deviation of estimator=0.4, penalties C_1=10, C_2=2

mentioned above penalty is imposed on false decision making. The risk can be evaluated from the product of probability of false decision making and the penalty.

$$L(\bar{x}_{3,i}^k, \sigma_{3,i}^k) = \min\left(C_1 P_{1,i}, \quad C_2(1-P_{1,i})\right), \quad \text{where,} \quad P_{1,i} = P_s \Phi(\beta_i), \qquad \beta_i = \frac{\bar{x}_{3,i}^k - x_o}{\sigma_{3,i}^k} \qquad (15)$$

Φ is the standard Normal (Gaussian) cumulative distribution function; $\sigma_{3,i}^k$ is standard deviation of $x_{3,i}^k$ which can be obtained from diagonal component of covariance matrix $\boldsymbol{M}_{33}{}^k$ shown in Eq.(14); P_s is probability of exceedance of specific level of seismic motion. The total risk at the decision making area is given by:

$$J^k = \sum_i L(\bar{x}_{3,i}^k, \sigma_{3,i}^k) \qquad (16)$$

The decision error risk is reduced by the new information $\boldsymbol{z}^{k+1}$. After we obtained observation vector $\boldsymbol{z}^{k+1}$, the mean and covariance matrix of $\boldsymbol{x}_3$ is updated as:

$$\bar{\boldsymbol{x}}_3^{k+1} = \bar{\boldsymbol{x}}_3^k + \boldsymbol{M}_{23}^{k\,T}\left[\boldsymbol{M}_{22}^k + \boldsymbol{R}_2^{k+1}\right]^{-1}\left\{\boldsymbol{z}^{k+1} - \bar{\boldsymbol{x}}_2^k\right\} \quad (17) \qquad \boldsymbol{M}_{33}^{k+1} = \boldsymbol{M}_{33}^k - \boldsymbol{M}_{23}^{k\,T}(\boldsymbol{M}_{22}^k + \boldsymbol{R}_2^{k+1})^{-1}\boldsymbol{M}_{23}^k \qquad (18)$$

Naturally value of the new information $\boldsymbol{z}^{k+1}$ is not given yet. Therefore $\boldsymbol{x}_2{}^k$ instead of $\boldsymbol{z}^{k+1}$ is used in Eq.(17). The expectancy of risk reduction is defined as VoI. The expectancy of risk considering observation data in next step $\boldsymbol{z}^{k+1}$ is

$$\text{VoI} = E[J^{k+1} - J^k] = E[J^{k+1}] - J^k \qquad (19) \qquad E[J^{k+1}] = \sum_i \int L(\bar{x}_{3,i}^{k+1}, \sigma_{3,i}^{k+1})\, p(\boldsymbol{x}_2^k)\, d\boldsymbol{x}_2^k \qquad (20)$$

Integration with respect to $\boldsymbol{x}_2{}^k$ is required, but it cannot be performed analytically. When dimension of $\boldsymbol{x}_2{}^k$ is high, numerical integration is not practical to implement. Thanks to reproductive property of Gaussian, the numerical integration can be always reduced to one-dimensional numerical integration. Consequently VoI can be calculated easily even if the dimension of $\boldsymbol{z}^{k+1}$ (the number of additional observation points) is large, e.g., more than 10.

When the dimension of vector $\boldsymbol{z}^{k+1}$ is low, it is not difficult to optimize the location of new observation. You can determine the optimal location by evaluating VoI at every possible combination of locations. It is, however, difficult to evaluate them due to "curse of dimensionality" when the dimension of the vector $\boldsymbol{z}^{k+1}$ is high. In this paper PSO (Particle Swarm Optimization) is introduced to optimize a set of location of new observation with respect to VoI. PSO is one of global optimization methods, which was proposed by Ref. 19. It is said that PSO is a simple method with a few parameters to for optimization but efficient for optimization with regard to real number variables.

III. OPTIMAL ADDITIONAL BORING FOR LIQUEFACTION COUNTERMEASURE

III.A. Liquefaction along river and liquefaction potential PL

Ref. 20 report damages caused by liquefaction along a river in the 2011 off the Pacific coast of Tohoku Earthquake. The optimal additional boring planning is studied in term of VoI in the region STA 30-35, where the number of existing boring is 18, relatively small. STA indicates the distance (km) from its estuary. At each existing boring point PL value is evaluated. PL is a liquefaction potential index to estimate the severity of liquefaction degree at a given site (Ref. 21). The value of PL is the larger, the site has higher possibility of liquefaction. The logarithm of PL along the river is modeled as a Gaussian random field with mean, standard deviation and auto correlation distance, 1.0, 0.3 and 200m. Standard deviation of observation error is 0.087. These parameters are determined based on the PL values calculated at all existing boring site by Ref. 20.

It is assumed that liquefaction countermeasure is implemented to each 100m length unit when its PL value is larger than 15, which corresponds to threshold limit value x_0 =15. It is also assumed penalty C_1, C_2 are 10, 2 respectively. PL value at each unit is estimated by using Kriging illustrated from PL values at 18 existing boring sites which is shown in Fig. 4(1) (denoted as "old" in the figure). The probability of exceedance of specific level of seismic motion P_s, which is caused by scenario earthquake assumed in order to evaluate PL, is considered as 30% in this calculation. The probability of exceedance P_s is defined in a specified period, e.g., 50 years. The mean of estimate and threshold x_c for judgement considering uncertainty are also shown in the figure. The units where the countermeasure is judged to be implemented are also shown in the figure (denoted as "P.of C.").

III.B. VoI of additional boring data

It is required to make decisions of the liquefaction countermeasure under uncertainty and risk. Decision makers sometimes have the possibility to gather further information prior to making the decision. Such additional information reduces the uncertainty and thus facilitates improved decision making. It is assumed that additional boring investigation in the region is allowed to improve the decision making. The locations of additional boring are determined such that the risk reduction by additional information is maximized in terms of VoI.

An optimal set of sites for additional boring are evaluated as a solution of optimization problem by PSO (Particle Swarm Optimization). The objective function is VoI, the variables for optimization are coordinate of the additional boring sites. Fig. 5 shows the obtained optimal set of boring sites when the number of additional boring is 6. The sites where are far away from existing boring sites are basically selected for the additional boring.

When necessity of countermeasure is judged based on existing boring data only, the area for countermeasure implementation is shown deterministically, namely probabilities of countermeasure (P.of C.) are 0 or 1. When we consider additional boring data, countermeasure probabilities between 0 and 1 are indicated as shown in Fig. 5(1), because information obtained in future is taken into account. Figure 5(1) also indicates a distribution of standard deviation of the mean of estimator PL. Note that the means are Gaussian random variables because $\boldsymbol{x}_2{}^k$ is used instead of $\boldsymbol{z}^{k+1}$ in Eq.(17). Fig. 5(2) shows standard deviation of estimator, threshold for judge x_c with and without additional boring ("prior", "post" in the figure). Fig. 5(3) shows distributions of risk. Standard deviation of estimator around new boring site shown in Fig. 5(2) is reduced, consequently risks are also reduced. The threshold for judgement x_c is increased because the uncertainty of estimator is reduced. The sum of differences between the risk with and without additional boring information corresponds to VoI, which is -7.08 in this case.

Optimal placement of additional boring and its VoI are evaluated to the cases that the number of additional boring is 0 to 15. When we assume more number of additional boring, the deterministic area (probability is 0 or 1) for the liquefaction countermeasure area is decreased, because the judgement of necessity of countermeasure is based on the numerical value of observation (PL) obtained in future. It is also indicated that the expectancy of total area of countermeasure is the less, when the more additional boring is taken.

III.C. Optimal number of additional boring

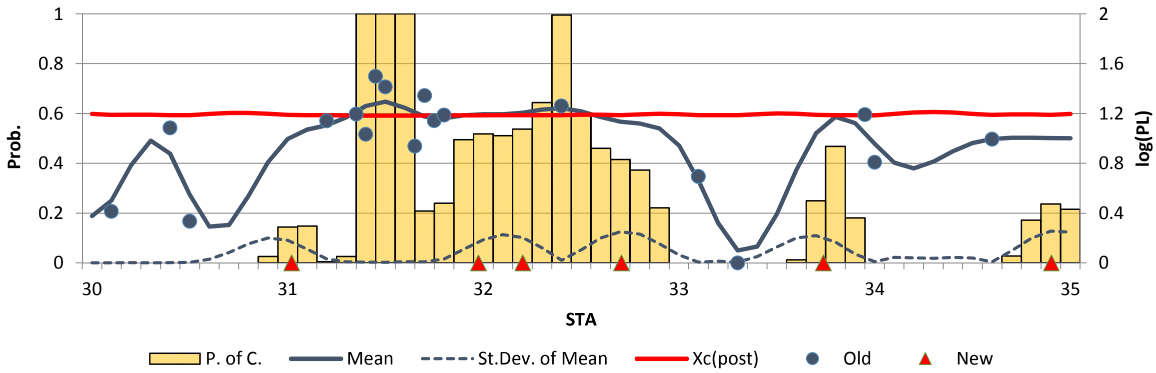
Fig. 6 shows the distribution of obtained VoI for the cases of which the number of additional boring is 0 to 15. When the number of additional boring is 0, naturally the numerical value of VoI is 0 because VoI indicates the expectancy of reduction of risk by the new information. The VoI decreases (its absolute value increases) with respect to the number of additional boring. The relation is not linear but convex downward, which means the value of new information gradually decreases. Total cost is evaluated by adding observation cost to VoI. Observation cost is the expense which is necessary for single boring and related investigation or test. When the observation cost is 1, then a curve of total cost shown in Fig. 6(1) is obtained. It is a convex downward curve, and has a minimum point at 2. If we assume observation cost is 2, then the optimal number is 6 as shown in Fig. 6(2). When observation cost is the higher, the optimal number is the less.

These optimal number is determined by the balance of penalty and observation cost. Many researches have studied the best safety level (how safe is safe enough) from total cost or life cycle cost considering risk of disaster, e.g., Ref. 22. Despite the difficulty of evaluating the consequence of disaster quantitatively, this type of research provides useful insights for the balance of safety and economy. The proposed method in this paper is also expected to provide useful insights for the balance of quantity of observation and economy in the same manner.

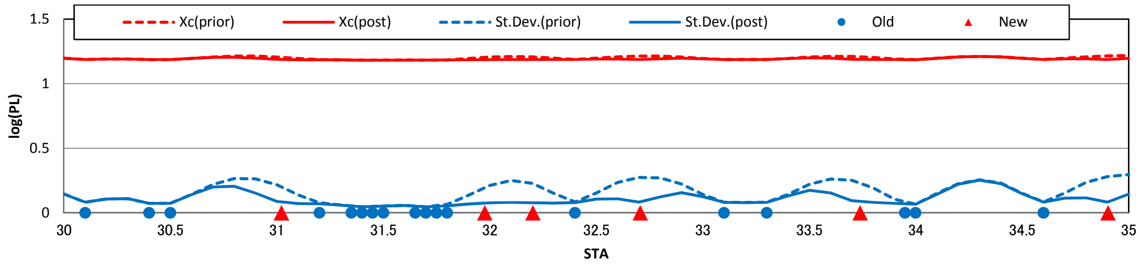
For comparison, a site with higher seismicity is considered. The probability of exceedance of specific level of seismic motion P_s, is 30% in the previous example. As a high seismicity site, P_s, is assumed to be 100% in specific period. Fig. 7 shows the obtained optimal set of boring sites when the number of additional boring is 6. Fig. 8 shows distribution of obtained VoI for the cases of which the number of additional boring is 0 to 15.

IV. CONCLUSION

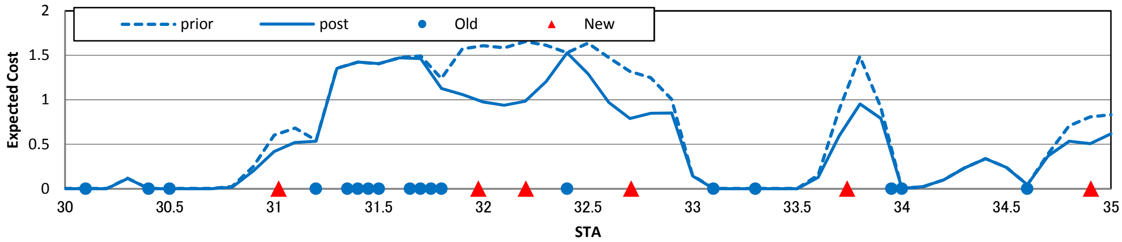
This paper discusses an efficient method to obtain optimal sampling (observation, boring) placement considering seismicity of site, which is based on Value of Information (VoI) in a Gaussian random field. VoI is closely related to traditional uncertainty measures such as variance and covariance, information entropy. VoI, however, contains cost attributable to the uncertainty to assess the usefulness of observation information considering the consequence due to the uncertainty. The proposed method is applied to additional boring placement for liquefaction countermeasure. Optimal number and placement of additional sampling are evaluated depending on the assumed seismicity of the site. One of the difficulties in practical applications of VoI lies in the determination of parameters like penalties. This will be future topics to be discussed.



(1) Mean of estimated PL and threshold for decision after new boring is considered (post) x_c, probabilities of implementation of liquefaction countermeasure (P.of C.)

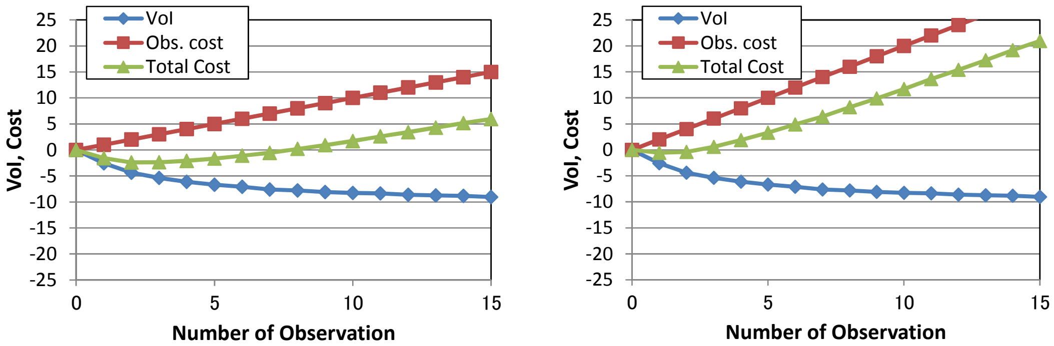


(2) Threshold for decision x_c of prior and post, standard deviation of estimator (St.Dev.)



(3) Distribution of risk of decision error of prior and post

Fig. 5. Locations of additional six boring and distributions of related parameters, low seismicity, Ps=0.3



(1) Observation Cost=1 (O.N.=2, T.C.=-2.39) (2) Observation Cost=2 (O.N.=1, T.C.=-0.57)

Fig. 6. The number of additional boring, VoI and Cost, low seismicity, Ps=0.3 (O.N.: Optimal number of observation, T.C.: Total Cost)

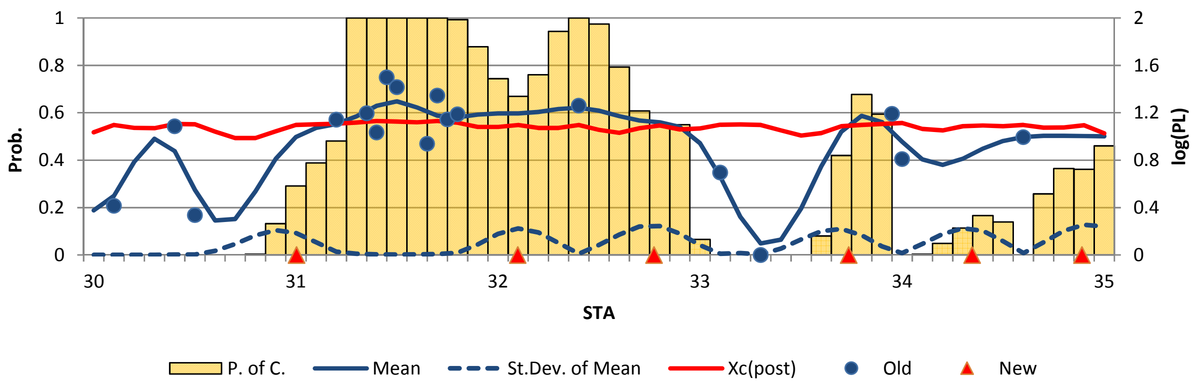
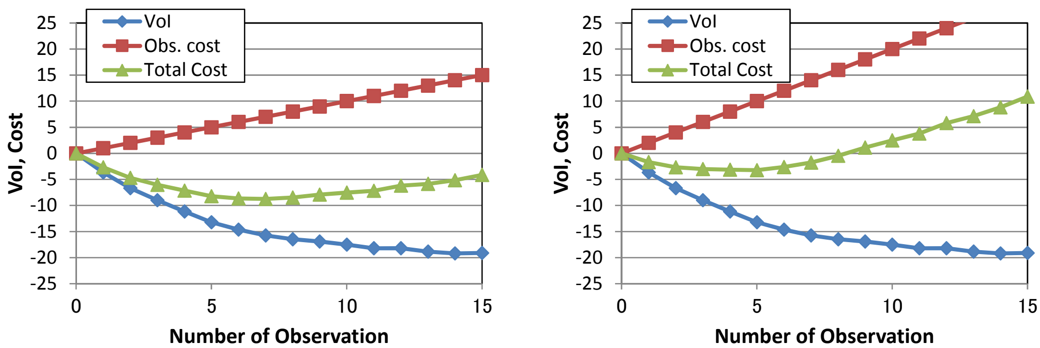


Fig. 7. The optimal sampling placement when the number of additional boring is 6, high seismicity, Ps=1.0



(1) Observation Cost=1 (O.N.=8, T.C.=-8.75) (2) Observation Cost=2 (O.N.=5, T.C.=-3.22)

Fig. 8. The number of additional boring, high seismicity Ps=1.0
(O.N.: Optimal number of observation, T.C.: Total Cost)

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