USING CYCLICAL TREND MODELS TO PREDICT COMPONENT REMOVALS AND RELIABILITY FOR STAGGERED ENTRY INTO SERVICE PLATFORMS

Todd Paulos, Ph.D.[1], Annie Chung[2], and Curtis L. Smith, Ph.D.[3]

[1]*Manager, Reliability, Maintainability, and Safety Center of Excellence, Aerospace Group, Eaton: 9650 Jeronimo Road, Irvine, CA, 92618-2024, toddpaulos@eaton.com*

[2]*Independent Consultant, San Jose, CA, 95111, annie.chung928@gmail.com*

[3]*Idaho National Laboratory, PO Box 1625, MS 3850, Idaho Falls, ID 83415-3850, curtis.smith@inl.gov*

Modeling to engineers, economists, scientists, insurance companies, and others, is representing a complex world using mathematical relationships. We model for a variety of reasons, such as predicting sales, economic strength based on the price of oil, the rate that diseases spread, or how many component removals from an aircraft fleet will occur in a year. There are few limits to what we can model; all we need is a little understanding of the world, some basic math skills, and away we go. That is not to say that all models are simple, in fact some are amazingly complex, but no matter how complex the model is, it is a representation of an even more complex world. Additionally, with every model there is error and uncertainty. In this document, the authors compare a simplistic, exponential model to a more complex, cyclical model, when solving for the number of component removals from a fleet that had a staggered entry into service. This example is typical of the airline industry and military ground mobile vehicles, where accurate time or mileage on components are difficult to track.

I. INTRODUCTION

In terms of complexity, models can be anything from a simple relationship such as y = mx+b, a "back of an envelope" calculation, a multi-million dollar computer code, or anything in between. Although typically unrecognized, and no matter how simple or complex, there is always uncertainty and error present in the Model Of the World (MOW). Far too often reliability engineers hear the phrase "garbage in, garbage out". This phrase has arisen since many the reliability predictions haven't held "true"; however, what really occurred was that the error and uncertainty of the MOW were not understood, and the stakeholder expected more accuracy out of the analysis. Additionally, most stakeholders would say that the data is the key driver, but it is actually the MOW that drives the error and uncertainty, and they may not be entirely quantifiable.

II. THE WORLD

In aviation, Maintenance, Repair, and Overhaul (MRO) services is a multibillion dollar industry. The MRO industry is becoming increasingly competitive, with airlines, airline subsidiaries, manufacturers and repair shops competing for business. Airlines want to buy Cost Per Hour (CPH) agreements, or insurance policies, that guarantee costs on a flight hour basis. CPH agreements can run for 10-12 years, are difficult to predict with accuracy, and have significant implications on profit margins.

Airline operators prefer limiting the number of MROs/suppliers they use for several reasons. There are fewer companies to deal with, larger Bill of Materials (BOMs) provide the operators with more purchasing power, and it makes financial sense to understand costs on a per flight hour basis for future business planning. BOMs included in the CPH agreements potentially reach 100 or more components; however, the financial risk is typically dominated by only 10-15% of the components due to either the expected large number of returns for particular components, or because the cost to repair those components is very high. Operators may not be forthcoming with data either, since they may be either unwilling or unable, or hoping the insurance company/MRO sells them a policy in their financial favor. It may seem obvious when using 10+ years of data to predict 1 year in the future, but CPH agreements can be for 10-12 years. Using 10 years to predict 10 years into the future can lead to big differences in results depending on the complexity of the model.

For a simple example, consider a CPH BOM in which only three components, A, B, and C, dominate the financial risk. The other components are secondary, and can be assessed using the method in Section III.A. The 16 year removal history for components A, B and C can be seen in Table I. The fleet flight time and average age are shown in time units of years.

TABLE I. Removal History of Major Risk Components

	A	B	C	Flight Time (Years)	Avg. Fleet Age (Years)
Year 1	0	0	0	0.58	0.45
Year 2	0	2	0	2.64	0.59
Year 3	0	2	0	5.83	0.69
Year 4	0	2	0	7.7	1.07
Year 5	0	3	0	9.58	1.01
Year 6	0	7	0	15.07	1.22
Year 7	0	20	1	20.31	1.63
Year 8	0	3	13	25.93	2.2
Year 9	0	5	7	28.58	2.80
Year 10	0	5	1	30.20	3.26
Year 11	0	3	13	34.62	3.72
Year 12	8	17	3	38.81	4.14
Year 13	9	6	0	34.57	4.88
Year 14	6	13	4	29.92	5.75
Year 15	11	9	0	29.77	7.07
Year 16	3	7	1	30.28	8.45

Table II shows the flight hour estimates the operator expects to fly and the expected average age of the fleet. The third column may seem counter-intuitive in this example, with a fleet getting older and then younger, but that is common in the industry. As planes get older, they are replaced with newer or brand new aircraft, lowering the average age of the fleet.

TABLE II. Future 10 Year Profile

	Flight Time (Years)	Avg. Fleet Age (Years)
Future Year 1	14.88	9.64
Future Year 2	27.90	9.38
Future Year 3	26.53	9.28
Future Year 4	25.91	10.12
Future Year 5	24.38	10.49
Future Year 6	19.66	9.34
Future Year 7	14.63	8.24
Future Year 8	11.71	7.8
Future Year 9	8.63	7.19
Future Year 10	3.08	3.97

III. THE SOLUTIONS

Two different solutions will be presented and compared. The first will be a simplistic exponential MOW, which is typical in the airline operator industry as most reliability metrics are calculated using that form based on the ATA SPEC 2000 industry standard[1]. A second, more complicated approach using cyclical trend models is shown for comparison.

III.A. Exponential MOW

Exponential models for reliability metrics are used in the aviation industry as dictated by ATA SPEC 2000[1]. Although a simplistic approach, when a MRO receive parts, the only data that they know for sure is the day that it arrived and what was discovered during the repair; all other data, such as Time Since Repair (TSR) or Time Since Overhaul (TSO) is not consistently provided by the operators. ATA Spec 2000 defines reliability metrics as subsets of Mean Time Between Removals (MTBR). If an aircraft has 2 of a particular component (known as the Quantity Per Aircraft or QPA), the fleet flew

1,000 Flight Hours (FH) in the time period, and there were 2 removals, then the Mean Time Between Removals (MTBR) would be calculated by the reciprocal of the maximum likelihood estimate for the exponential rate, or

$$MTBR = \frac{Quantity\ Per\ Aircraft\ x\ Flight\ Hours}{Removals} \quad (1)$$

$$\frac{2\ x\ 1000}{2} = 1000\ Flight\ Hours \quad (2)$$

Examination of the data in Table I seems to show the removal distribution is not exponential (not a constant failure rate). It raises the question in these analyses, what time interval to use? Is 1, 3 or 5 years best, or maybe more? Table III shows the data and estimates for specific year MTBR values, and for the last 1, 3, 5 10 and 16 year look back estimates.

TABLE III. Yearly Mean Time Between Removal Estimates and Various Look-Back Periods

	A	B	C	Flight Time (Years)	Flight Time (Hours)	A MTBR	B MTBR	C MTBR
Year 1	0	0	0	0.58	5,081	infinite	infinite	infinite
Year 2	0	2	0	2.64	23,126	infinite	11,563	infinite
Year 3	0	2	0	5.83	51,071	infinite	25,535	infinite
Year 4	0	2	0	7.7	67,452	infinite	33,726	infinite
Year 5	0	3	0	9.58	83,921	infinite	27,974	infinite
Year 6	0	7	0	15.07	132,013	infinite	18,859	infinite
Year 7	0	20	1	20.31	177,916	infinite	8,896	177,916
Year 8	0	3	13	25.93	227,147	infinite	75,716	17,473
Year 9	0	5	7	28.58	250,361	infinite	50,072	35,766
Year 10	0	5	1	30.2	264,552	infinite	52,910	264,552
Year 11	0	3	13	34.62	303,271	infinite	101,090	23,329
Year 12	8	17	3	38.81	339,976	42,497	19,999	113,325
Year 13	9	6	0	34.57	302,833	33,648	50,472	infinite
Year 14	6	13	4	29.92	262,099	43,683	20,161	65,525
Year 15	11	9	0	29.77	260,785	23,708	28,976	infinite
Year 16	3	7	1	30.28	265,253	88,418	37,893	265,253
Last 3	20	29	5	89.97	788,137	39,407	27,177	157,627
Last 5	37	52	8	163.35	1,430,946	38,674	27,518	178,868
Last 10	37	88	43	302.99	2,654,192	71,735	30,161	61,725
ALL	37	104	43	344.39	3016856.4	81,537	29,008	70,159

Table III converts years to hours (365*24), and the exact calculation is shown instead of rounding for ease of reader duplication. Using the results from Table III, we can project a failure "rate" forward with the expected flight time, and predict what the number of removals will be on a yearly basis based on an exponential predictive model. The results are presented for components A, B and C based on the various look back periods. For component A, the yearly estimate and the total estimate over 10 years can vary by a factor of 2. For component B, the method really didn't affect the results at all, and for component C, the total can vary by a factor of 3 from smallest to largest. Now let's consider a more complex model.

TABLE III. Removal Predictions for Components Based on Exponential Model

		A				B				C			
	Flight Time (Years)	Last 3	Last 5	Last 10	All 16	Last 3	Last 5	Last 10	All 16	Last 3	Last 5	Last 10	All 16
Future Year 1	14.88	3.3	3.4	1.8	1.6	4.8	4.7	4.3	4.5	0.8	0.7	2.1	1.9
Future Year 2	27.9	6.2	6.3	3.4	3.0	9.0	8.9	8.1	8.4	1.6	1.4	4.0	3.5
Future Year 3	26.53	5.9	6.0	3.2	2.9	8.6	8.4	7.7	8.0	1.5	1.3	3.8	3.3
Future Year 4	25.91	5.8	5.9	3.2	2.8	8.4	8.2	7.5	7.8	1.4	1.3	3.7	3.2
Future Year 5	24.38	5.4	5.5	3.0	2.6	7.9	7.8	7.1	7.4	1.4	1.2	3.5	3.0
Future Year 6	19.66	4.4	4.5	2.4	2.1	6.3	6.3	5.7	5.9	1.1	1.0	2.8	2.5
Future Year 7	14.63	3.3	3.3	1.8	1.6	4.7	4.7	4.2	4.4	0.8	0.7	2.1	1.8
Future Year 8	11.71	2.6	2.7	1.4	1.3	3.8	3.7	3.4	3.5	0.7	0.6	1.7	1.5
Future Year 9	8.63	1.9	2.0	1.1	0.9	2.8	2.7	2.5	2.6	0.5	0.4	1.2	1.1
Future Year 10	3.08	0.7	0.7	0.4	0.3	1.0	1.0	0.9	0.9	0.2	0.2	0.4	0.4
TOTAL REMOVALS		**39.4**	**40.2**	**21.7**	**19.0**	**57.2**	**56.4**	**51.5**	**53.5**	**9.9**	**8.7**	**25.2**	**22.1**

III.B. Cyclical MOW

If the world is examined more closely, airline operators rarely bring a whole fleet of aircraft online at the same time. They tend to get their fleets through a build-up, such as acquiring a new aircraft at a rate of one a month for a period of time as the planes are delivered new from the factory. The term *staggered entry into service* is a more appropriate description of this world, and has been characterized in the literature before referencing aerospace, military ground vehicles, industrial components, and general business applications.[2-4] Although the term has been described before, to the authors' knowledge, a solution of this type has never been demonstrated anywhere else. Figure 1 shows a depiction of an aircraft fleet buildup, although this could easily be a military ground vehicle being introduced into service over a period of time, and a simplistic time line of a particular component on that platform being removed periodically. Figure 2 shows a typical removal trend from the fleet that would be seen in total number of components removed or repaired in this situation.

This model could be modeled using a simple regression fit to the data, however the use of OpenBUGS[5] was chosen in order to have more complex MOW models, use Bayesian p-values[6] for a goodness of fit tests instead of regression coefficients, and to calculate uncertainty values on the results. For this simulation, a few postulates were made. The first is that the number of removals in any year follows a Poisson process, the rate of the Poisson process can change from year to year, and the rate (based on Fig. 2) is a *function* of the *average age* of the aircraft fleet.

The basic simulation and trend model is shown below in blue font, the postulated prior distributions for parameters in the distribution are in red font (the distribution uses parameters a, b and c, not to be confused with components A, B and C), the removal data and the expected usage of aircraft fleet in the upcoming ten years is shown in purple font, the initial parameters used in for the Markov chain are shown in green font), the p-value calculation is shown in gold font. Several trend models, parameter distributions and initial parameters with p-value results are shown in pink font. The solution process is one of postulating a trend model and parameters, finding the p-value, and examining other trend models if a better fit is possible. One reason to use fewer parameters and hard code in others (such as the 1.5, 1.4, 1.3 and 1.2 values in the old trend models), is to limit the number of parameters the Gibbs' sampling routine has to solve for with such a limited data set, although for components B and C, 4 parameter models achieved reasonable p-values.

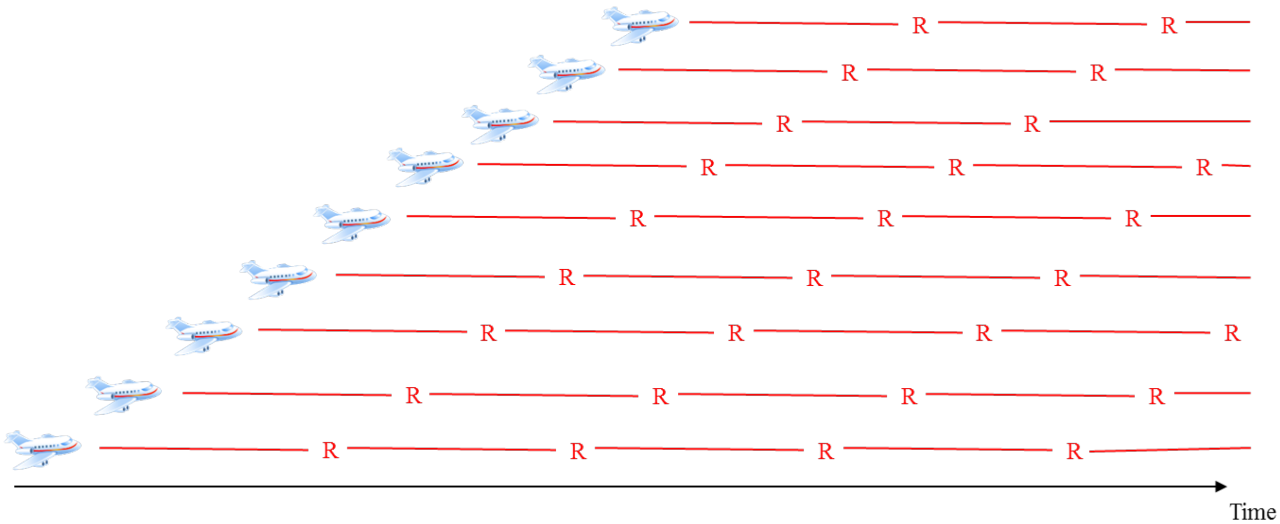

Fig. 1. Aircraft fleet buildup and periodic component removals

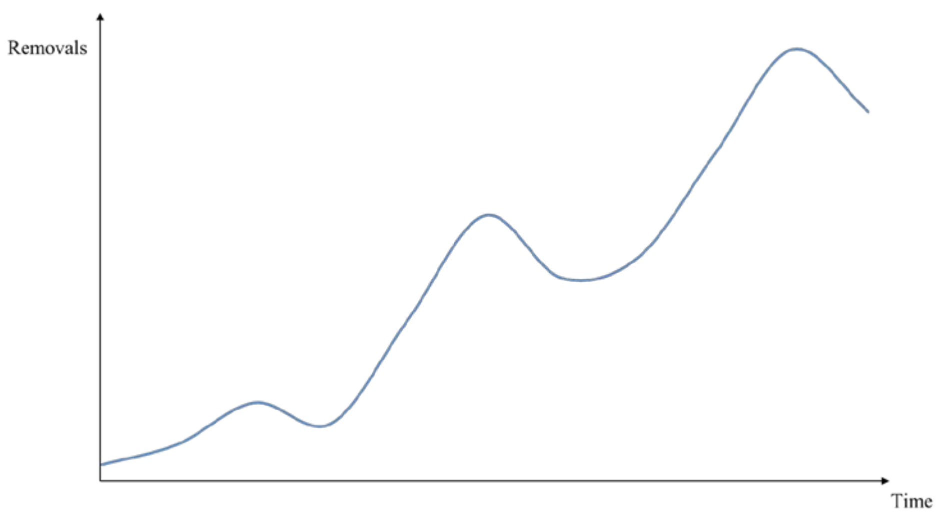

Fig. 2. Component removal trend from a platform that has staggered entry into service

For component A, the OpenBUGS script is as follows:

```
model   {
        for (i in 1:N)      {
        #Poisson likelihood function for each time interval
        x[i] ~ dpois(mu[i])
        #Poisson parameter for each time interval, lambda in /years
        mu[i] <- lambda[i]*years[i]
        #Trend model for lambda
        lambda[i] <- c*((cos(b*avg[i]-a)+1.2)*(1-exp(-i/60)))
        #Posterior predictive distribution
        x.pred[i] ~ dpois(mu[i])
        diff.obs[i]<-pow(x[i] - mu[i], 2)/mu[i]
        diff.pred[i]<-pow(x.pred[i] - mu[i], 2)/mu[i]
        }
```

```
        for (j in 1:M)     {
          returns[j] <- (c*((cos(b*age[j]-a)+1.2)*(1-exp(-(j+N)/60))))*fltyrs[j]
        }
#p-value calculation
        chisq.obs<-sum(diff.obs[])
        chisq.pred<-sum(diff.pred[])
        p.value<-step(chisq.pred - chisq.obs)
#Prior distributions
        a ~ dunif(2,4)
        b ~ dunif(.01,1)
        c ~ dunif(0.025,.75)
}
#Average is in years
Data
x[]     years[]  avg[]
0       .58      0.45
0       2.64     0.59
0       5.83     0.69
0       7.7      1.07
0       9.58     1.01
0       15.07    1.22
0       20.31    1.63
0       25.93    2.20
0       28.58    2.80
0       30.20    3.26
0       34.62    3.72
8       38.81    4.14
9       34.57    4.88
6       29.92    5.75
11      29.77    7.07
3       30.28    8.45
END
list(N=16)
list(fltyrs = c(14.88,27.90,26.53,25.91,24.38,19.66,14.63,11.71,8.63,3.08), age =
c(9.64,9.38,9.28,10.12,10.49,9.34,8.24,7.8,7.19,3.97),M=10)
Inits
list(a=2,b=.75,c = .35)
list(a=4,b=.5, c=.70)
#       OLD TREND MODELS
#       lambda[i] <- c*((cos(b*avg[i])+a)*(1-exp(-i/60))) p=.1445
#       lambda[i] <- c*((cos(b*avg[i]-a)+1.5)*(1-exp(-i/60))) p=.1623
#       lambda[i] <- c*((cos(b*avg[i]-a)+1.4)*(1-exp(-i/60))) p=.18
#       lambda[i] <- c*((cos(b*avg[i]-a)+1.3)*(1-exp(-i/60))) p=.1883
#       lambda[i] <- c*((cos(b*avg[i]-a)+1.2)*(1-exp(-i/60))) p=.1928
#       lambda[i] <- c*((cos(b*avg[i]-a)+1.2)*(1-exp(-i/60))) p=.2498
```

Table IV shows the mean values, standard deviations, and 5th, 50th and 95th percentiles of the various parameters in the trend model, the p-value, and the predicted future returns of Component A for this trend model. This predictive model estimates 35.5 returns for the next ten years (by summing the returns for each year over the 10 year period). Comparing the results to Table III, which estimated between 19 and 40 for this component based on the look-back period, the simple models could be off by as much as a factor of almost 2 (e.g., when using all the data, 19 for the simple model versus 35.5 for the more complex model). In the interest of space, the trend models and results for component B and C, are shown below in Tables V and VI. Components B and C had a best fits with the trend model abs(a+b*sin(c*avg[i])+d*cos(c*avg[i])). This is interesting in that components B and C are of the same type.

TABLE IV. OpenBUGS Results for Component A Removal/Return Model

	Mean	5th	50th	95th
p.value	0.2538			
a	3.705	3.172	3.782	3.983
b	0.5448	0.3832	0.5562	0.6693
c	0.5116	0.3726	0.506	0.6719
returns[1]	2.299	0.6237	2.088	4.772
returns[2]	4.946	1.577	4.613	9.572
returns[3]	5.087	1.725	4.79	9.597
returns[4]	3.848	0.8463	3.243	9.114
returns[5]	3.262	0.7191	2.514	8.549
returns[6]	4.182	1.367	3.918	8.004
returns[7]	4.297	2.376	4.237	6.472
returns[8]	3.832	2.408	3.802	5.389
returns[9]	3.104	2.181	3.084	4.115
returns[10]	0.6705	0.4514	0.6584	0.9306

In Table V, the expected number of returns is 55.6, which is close to the simplistic estimate. Although some may feel this might suggest to use the simple model, what it shows is that in some cases the complex model matches a simpler model, but use of the complex model seems to provide as-good-as or (much) better results as compared to the simpler model. Further, since the analysis time is similar for either case, the model that works for all cases should be preferred.

TABLE V. OpenBUGS Results for Component B Removal/Return Model

	Mean	5th	50th	95th
p.value	.1001			
a	0.06623	-0.3726	-0.06122	1.062
b	0.1265	-0.6042	0.2752	0.5668
c	-16.06	-47.17	-2.285	2.343
d	0.03242	-0.6443	0.03716	0.6441
returns[1]	5.3	1.416	5.501	8.382
returns[2]	8.332	1.042	8.146	15.58
returns[3]	8.76	2.025	8.361	16.77
returns[4]	7.849	0.9724	7.907	13.89
returns[5]	8.44	1.527	8.027	16.42
returns[6]	6.702	1.512	6.657	11.36
returns[7]	3.95	1.032	3.922	7.19
returns[8]	2.747	0.237	2.526	5.873
returns[9]	2.636	0.2485	2.523	4.577
returns[10]	0.9524	0.2706	0.9613	1.419

In Table VI, the expected number of returns is 32.6, which exceeds the worst case exponential estimate by 30%, and could have been more than 3 times the predicted value if the look back period was 3 years.

TABLE VI. OpenBUGS Results for Component C Removal/Return Mode

	Mean	5th	50th	95th
p.value	0.1236			
a	-0.0669	-0.1771	-0.06125	0.05315
b	-0.0491	-0.1941	-0.07524	0.158
c	-2.649	-8.554	-1.681	1.702
d	0.07721	-0.2194	0.1745	0.2664
returns[1]	4.249	2.623	4.269	5.84
returns[2]	5.948	0.6363	6.529	9.663
returns[3]	4.882	0.2366	5.646	8.586
returns[4]	4.837	0.4984	5.302	8.141
returns[5]	3.543	0.4327	3.182	8.199
returns[6]	3.918	0.2499	4.421	6.647
returns[7]	2.409	0.7038	2.14	4.954
returns[8]	1.584	0.06961	1.888	2.901
returns[9]	0.735	0.1204	0.6468	1.63
returns[10]	0.4581	0.03694	0.514	0.7615

IV. CONCLUSIONS

There are several conclusions that can be learned from this example of predicting removals of components that have a staggered entry into service. Although this is a simple example, it is not a given that components behave as an exponential, and choosing one, or any other statistical model for convenience, can have far reaching effects on the results. In using a simplistic model to predict 10 years into the future, even using 16 years of past data may lead to results that are quite in error, and could have costly implications. This paper demonstrated the use of an OpenBUGS-based cyclical trend model to reasonably predict returns/removals of items based on the average age of the fleet, whether it be the aircraft example shown, military vehicles, farm machinery, or any platform that has a staggered entry into service, and usage time/mileage is difficult to obtain. One surprising find during this work was that similar types of components often had similar or identical trend model forms with only the parameters of that model changing. This allows us to reduce the number of unknowns in future predictions, and to use smaller data sets to obtain reasonable results.

ACKNOWLEDGMENTS

The authors would like to thank the many people at Eaton for making this effort possible, and in particular the Aerospace Reliability, Maintainability and Safety team, who make every day more interesting than the last.

REFERENCES

1. http://www.spec2000.com/Files/S2K_Ch8_2004.1.pdf
2. NATIONAL RESEARCH COUNCIL, *Improved Operational Testing and Evaluation and Methods of Combining Test Information for the Stryker Family of Vehicles and Related Army Systems*, pp. 51-52, Phase II Report, Panel on Operational Test Design and Evaluation of the Interim Armored Vehicle, Committee on National Statistics, The National Academies Press, Washington, DC (2004)
3. STEPHEN P. JOHNSON, *System Health management with Aerospace Applications*, Chapter 14, Stephen P. Johnson et al., Wiley, Hoboken, NJ (2013)
4. GERALD J. HAHN and NECIP DOGANAKSOY, *The Role of Statistics in Business and Industry,* Chapter 9, John Wiley & Sons, New York, NY (2011)
5. http://www.openbugs.net/w/FrontPage
6. DANA KELLY and CURTIS SMITH, *Bayesian Inference for Probabilistic Risk Assessment: A Practitioner's Guidebook*, pp. 46, Springer, London, United Kingdom (2011)