ENHANCED MONTE CARLO SIMULATION-BASED SPATIAL RELIABILITY ANALYSIS OF RC PLATE SUBJECT TO CHLORIDE-INDUCED CORROSION

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Numerical handling of random field has been considered challenging due to infinite number of spatial random variables which are mostly correlated to each other. To overcome this issue, random field modeling approaches such as Karhunen–Loève (KL) expansion have been used to parameterize the random field and approximated into finite number of independent random variables. Engaging the beneficial features of KL random fields, we propose an efficient method for spatial reliability estimates. This research consists of two distinctive parts: (1) Implementation of KL random field to reduce the dimension and achieve statistical independence between main random variables and (2) to obtain spatial reliability distribution using recently developed enhanced Monte Carlo method. As an example, our scheme is adopted for the statistical prediction of chloride-induced cover crack propagation in a RC structure to show the potential applicability to various engineering systems.

I. INTRODUCTION

As infrastructures, which once served pioneering effort for brisk industrial development, are being aged, proper management of the deteriorated structures has emerged as a critical priority for urban communities. Since we have several times witnessed the catastrophic outcomes of deteriorated structure exposed to natural or man-made hazards, most of the developed countries are investing a huge amount of budget to mitigate potential influences of structural deterioration on the society. Among a variety of structural deterioration, corrosion is considered one of the most common and serious ones, especially in reinforced concrete structures. It mainly initiates by surface chloride ingress and is followed by rusting of rebar and concrete crack propagation, eventually yielding loss of structural integrity. Corrosion is a complicated process which is heavily influenced by various uncertainties in material, geometric and environmental conditions.

Prompt detection of deterioration and proactive maintenance actions could increase social safety and thus save huge amount of cost. However, accurate prediction of future corrosion state is challenging mainly due to intrinsic randomness in deterioration factors and complicated mechanism of corrosion. Hence, the prognosis can be improved by quantifying the uncertainty by means of reliability analysis. There were several attempts to incorporate probability theory to predict spatial corrosion damage in RC plate. For example, Peng and Stewart proposed a time-dependent reliability analysis technique to predict spatial corrosion[1] and Keßler *et al.* calculated spatial corrosion probability of RC plate using novel observation involving method called potential mapping[2]. They used Monte Carlo simulation to get spatial distribution of reliability. Monte Carlo simulation is known as most robust and accurate estimation method provided computing power is sufficient. However for the rare event, which in many case yields significant outcomes in engineering systems, they are often highly inefficient or sometimes it is impractical to conduct desired number of simulations. To overcome the issue, efficient structural reliability analysis tools are developed, such as FORM/SORM[3], adaptive importance sampling[4,5] and subset simulation. But in case of spatial reliability analysis, these conventional structural reliability analysis methods are inappropriate. It is due to spatially varying limit state expressions, which makes optimal reference points unattainable, where it is indispensable for each methods. To get the spatial reliability distribution, containing rare-occurrence points, Straub modified adaptive importance sampling by grouping adjacent locations and selecting reference importance density optimal to one of the points in each group[6]. This method provides efficient way to obtain spatial reliability but is only feasible for gradually varying limit states. In this paper, to deal with same problem, we adopted enhanced Monte Carlo simulation method based on parametric reliability extrapolation, recently developed by Naess *et al.*[7]. They proposed an innovative way to utilize reasonable number of Monte Carlo simulation results for estimating exceedingly small probability. The method was originally developed for system reliability analysis and to the best of the author's knowledge, this is first time to be directly applied to calculate spatial reliability distribution of a random field.

Another concern dealing with random field is its continuity, which requires proper discretization for numerical handling. The geometric discretization is most intuitive method, but yields high dimensionality. To avoid this, several series expansion method are popularly used in the structural reliability fields, such as Karhunen-Loève (KL) expansion, orthogonal series expansion (OLE), and expansion optimal linear estimation (EOLE) [8]. Among them, KL expansion, which relies on spectral decomposition of auto covariance function, is known as the most effective approximation representation. By reducing the stochastic dimension, it becomes easier to conduct reliability analysis. Further, even though it is beyond the scope of this paper, it facilitates to incorporate the Bayesian inference by reducing the dimension of random field into finite number of random variables, which probability density can be directly updated from observation and Bayes equation.

In this research, we propose an efficient framework to estimate the spatial reliability of RC plate under corrosion cracking. In the next section, a model to describe corrosion-induced cover crack propagation mechanism is illustrated. In Section III, we briefly review the formulation of KL-expansion. The random fields for initial environmental and geometrical factors are then used for spatial reliability analysis which requires special approach of enhanced Monte Carlo, as explained in Section IV. Through the numerical demonstration at Section V, we will discuss the relevance of proposed reliability analysis method on chloride-induced RC plate, and concluding remarks will be provided in Section VI.

II. CORROSION-INDUCED COVER CRACK PROPAGATION MODEL

Corrosion-induced cracking is a chemical-physical process that involves three phenomenon including diffusion of corrosion ion, accumulation of corrosion product and crack propagation.

II.A. Modelling of chloride ion diffusion

Fick's second law[9] is commonly used to model diffusion of chloride ion through concrete cover. From Fick's second law, at location (x,y) in a 2D domain, the corrosion initiation time T_{int} for the rebar under concrete cover thickness C_{cover} can be derived as

$$T_{\text{int}} = \frac{C_{\text{cover}}^2}{4D_{cl^{-1}}[erf^{-1}(1-\frac{Cl_{cr}}{C_s})]^2} \qquad (1)$$

where Cl_{cr} is the critical chloride content for corrosion initiation in kg/m^3; C_{cover} is the cover thickness at location (x,y) in m; $D_{cl^{-1}}$ is the chloride diffusion coefficient in m^2/s; $erf(\cdot)$ is the Gauss error function; and C_s is the surface chloride concentration in kg/m^3.

II.B. Accumulation of corrosion product

The corrosion induced concrete cover cracking is directly related with the volume expansion of corrosion production and the resulting tensile force exerted to surrounding concrete. Once the corrosion process is initiated, the volume of rust production increases. Due to impedance in diffusion of ions and electrons through the rust layer, the rate of rust production reduces along with the corrosion progress. The assumed rate of rust production J_r is[10]

$$J_r = dM_r / dt = \beta / M_r \qquad (2)$$

where M_r is mass of rust production in kg/m; t is time in s; and coefficient $\beta=3.328\times10^{-10}\pi D_b\cdot i_{corr}$ (Ref. 11) in which i_{corr} is the corrosion rate in A/m^2, and D_b is the rebar diameter in m. So the mass of rust products can be determined by integration of Eq. (2) through time. The mass of consumed steel can also be calculated by $\Delta M_s=r_m\Delta M_r$, where r_m is the ratio of molecular weight of iron to that of rust products ranging from 0.523 to 0.62. The density of rust $\rho_r=\rho_s/(r_v r_m)$, where r_v is the relative volume of the rust oxides to the intact iron, and ρ_s is the density of steel. So the volume of rust production can also be calculated. The corrosion rate i_{corr} can be predicted using Liu and Weyers' empirical equation[10] which is developed based on 5 years' accelerated corrosion tests.

II.C. Modelling for splitting crack propagation

As accumulation of rust production expands, the resulting tensile stress exerted to surrounding concrete may exceed the tensile strength of concrete, the concrete cracking is initiated and then propagates through the cover thickness to cover surface. This splitting crack propagation is modeled using a well-recognized analytical model that can evaluate the cracking propagation as well as the reduction of material properties[11].The volume changes due to relative difference between iron and the primary oxides are incorporated to compute the inner pressure caused by rebar corrosion.

II.C.1 Boundary value problem setting

The cover concrete is idealized as a thick-walled cylinder. Uniform corrosion is assumed around the rebar so that the pressures build up evenly. Neglecting the volume of rust product in the crack, the oxide layer thickness t_r can be calculated from

$$t_r = \sqrt{R_{rb}{}^2 + \frac{\Delta V_r}{\pi}} - R_{rb} \qquad (3)$$

in which R_{rb} is the reduced radius of steel rebar; and ΔV_r is the volume change of rust production formed per unit length of rebar. Therefore, the inner boundary of the cover will move by the prescribed radial displacement $u_{r|Rb}=R_r-R_b$ (where R_r is the rust front, i.e. $R_r = R_{rb}+t_r$; and R_b is the original rebar radius). Accordingly, stress is developed along the inner boundary.

The cover concrete is anisotropic due to cracks developing in its tangential direction. Assuming a plane stress condition, all the Poisson ratios associated with deformations of the cover concrete in axial direction (z direction) of rebar will be neglected. From a general boundary value problem given for deformation of anisotropic thick-walled cylinder subjected to inner displacement boundary conditions $u_{r|R_b}=R_r-R_b$ and the outer stress boundary condition $\sigma_{r|Cc}=0$, the governing differential equation of cover concrete subjected to rebar corrosion induced cracking is simplified as

$$\frac{d^2u_r}{dr^2} + \frac{1}{r}\frac{du_r}{dr} - \frac{u_r}{r^2}\frac{E_\theta}{E_r} = 0 \qquad (4)$$

where u_r is the radial displacement of cover concrete at a certain point through cover thickness; E_θ and E_r are elastic modulus of cover concrete for tangential and radial directions, respectively. Due to the complexity in material constitutive models, Eq. (4) will be solved by finite difference technique following the procedures in Ref.11 for radial displacement u_r at each discretization point through the cover thickness. Based on the solution of $u_r(r)$, strains can be computed as $\varepsilon_r=du_r/dr$ and $\varepsilon_\theta = u_r/r$. Meanwhile, the cracking width w can be calculated by $w=2\pi r\varepsilon_\theta$. From the stress-strain relationship for plane stress $\sigma_r=(E_r\varepsilon_r+\nu_{r\theta}E_\theta\varepsilon_\theta)/(1-\nu_{r\theta}\nu_{\theta r})$ and $\sigma_\theta=(E_\theta\varepsilon_\theta+\nu_{\theta r}E_r\varepsilon_r)/(1-\nu_{r\theta}\nu_{\theta r})$, the stresses can be calculated. Comparing the tangential stress σ_θ with tensile strength of concrete f_{ct}, we can check whether concrete cracking takes place at a certain location through the cover thickness. If crack happens, E_θ and E_r will be updated using calculated strains ε_θ and ε_r based on the constitutive relationships of concrete selected. At the next time step in the corrosion process, the crack front moves toward the cover surface with the rust production accumulation. Then the new cracking condition of cover concrete can be evaluated using the same process introduced above with the updated inner boundary condition.

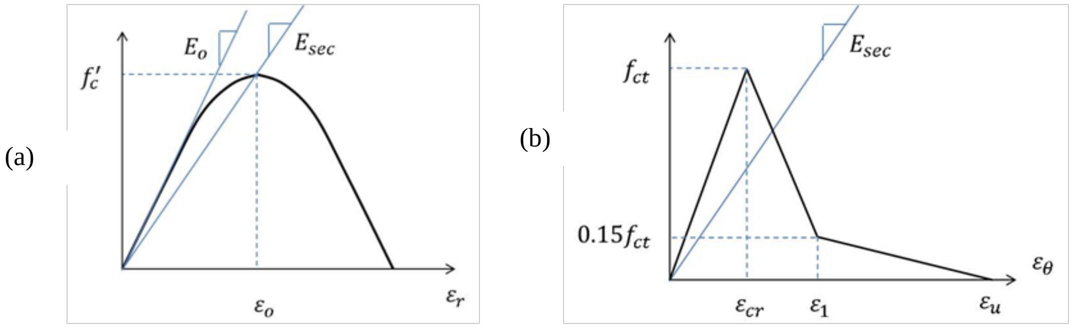


Fig. 1. Stress-strain relationships for compressive stress and tension stress (redrawn following Ref.11)

II.C.2 Anisotropic material constitutive relationship for corrosion cracking

For the cover concrete in the corrosion process, the stresses in the radial direction are given in compression while stresses in the tangential direction are in tension. The selected stress-strain relationship for compression is a Hognestad-Type parabola model[11] as depicted in Fig. 1(a). In the model, the initial modulus is defined as $E_0=2f_c'/\varepsilon_o$, where ε_o=0.002 and f_c' is the compressive strength. The stress-strain relationship shown in Fig. 1(b) is used to describe the tensile softening of cover concrete subjected to cracking. In this constitutive relationship, the bilinear curves describing the post-cracking behavior of concrete are defined by two points (ε_1, $0.15f_{ct}$) and (ε_u, 0). The values of ε_1 and ε_u are determined by the fracture energy which are given as 0.0003 and 0.002, respectively, in this paper.

III. THEORIES OF KARHUNEN-LOÈVE EXPANSION

To account for the uncertainty inherent in environmental, material, and geometric conditions of structure, it is essential to describe corrosion factors and resulting outcomes as random fields. However, for numerical adaptation, proper discretization to represent random field into finite number of random variables are required. Among the methods, approximation based on spectral representation approaches such as Karhunen-Loève (KL) expansion are known to be the most effective among others in terms of minimizing global mean square error. In form of KL-expansion, a random field can be separated into deterministic part and stochastic part, and the stochastic part are expended as a Fourier-type series:

$$w(\mathbf{x},\boldsymbol{\theta}) = \overline{w}(\mathbf{x}) + \sum_{n=0}^{\infty} \sqrt{\lambda_n}\, u_n(\boldsymbol{\theta})\varphi_n(\mathbf{x}), \qquad (5)$$

where $\overline{w}(\mathbf{x})$ is the mean function of the random field, $\{u_n(\boldsymbol{\theta})\}$ is the set of KL random variables with distribution parameter $\boldsymbol{\theta}$, and λ_n is the eigenvalue of eigenvector $\varphi_n(\mathbf{x})$ of the covariance function

$$C(\mathbf{x}_1,\mathbf{x}_2) = \sum_{n=0}^{\infty} \lambda_n \varphi_n(\mathbf{x}_1)\varphi_n(\mathbf{x}_2). \qquad (6)$$

Since the covariance function is symmetric and positive semi-definite, the eigenvalue and eigenvector can be obtained by solving

$$\int_D C(\mathbf{x}_1,\mathbf{x}_2)\varphi_n(\mathbf{x}_2)dx_1 = \lambda_n \varphi_n(\mathbf{x}_2). \qquad (7)$$

This equation is called the Fredholm integral equation of the second kind. Solving the equation is difficult in general, however, there are some numerical methods available. In case of Gaussian random field $w(\mathbf{x},\boldsymbol{\theta})$, KL random variables follow standard normal distribution[8,12].

By rearranging Eq. (5) in descending series of eigenvalues, and truncating the series after M-th term, it is possible to obtain an approximate random field:

$$w(\mathbf{x},\boldsymbol{\theta}) \approx \overline{w}(\mathbf{x}) + \sum_{n=0}^{M} \sqrt{\lambda_n}\, u_n(\boldsymbol{\theta})\varphi_n(\mathbf{x}). \qquad (8)$$

Every input of corrosion propagating model illustrated in Section II can be modeled as approximated random fields using KL-expansion with M_w random variables $\{u_n(\boldsymbol{\theta})\}$. As a result, random fields of corrosion factors – $f_{\mathrm{ct}}(\mathbf{x})$, $C_{\mathrm{cover}}(\mathbf{x})$, $D_{cl^-}(\mathbf{x})$, and $C_s(\mathbf{x})$ introduced in in Section II.A. – can be approximately reduced to a set of random variables $\{\mathbf{u}^{\mathrm{D}},\mathbf{u}^{\mathrm{Cs}},\mathbf{u}^{\mathrm{Ccover}},\mathbf{u}^{\mathrm{fct}}\}$.

IV. RELIABILITY EXTRAPOLATING USING ENHANCED MONTE CARO SIMULATION

By assuming input random fields follow Gaussian distributions and representing them as KL-expansion, we were able to achieve powerful dimension reduction, by which model continuous random fields by uncorrelated standard normal random variables. However, we often come back to the initial problem of high dimensionality when we attempt to calculate spatial reliability distribution, and therefore need a special technique.

For the deterioration-related reliability assessment of a spatial domain, the limit state function can be introduced at any location **x**, as the safety margin which has form of

$$g(\mathbf{x}) = C(\mathbf{x}) - D(\mathbf{x}), \qquad (9)$$

where $C(\mathbf{x})$ is capacity and $D(\mathbf{x})$ is demand of the system. Using the KL-expansion modeling approach, one set of sample will generate a complete description of spatial deterioration outcomes and the limit state function values. It is straightforward to apply crude Monte Carlo simulation to compute the reliability in the entire domain. However, it is noted that the approach may require an exceedingly large number of samples if the domain includes points where the probability to estimate is too low. On the other hand, advanced structural reliability estimation techniques such as FORM[3], SORM[3], adaptive importance sampling[4,5], and subset simulation are also infeasible for the problem. It is because those methods usually gain efficiency by finding stochastically important region in the random variable space identified for a given limit state function. In this case, however, by having spatially varying limit state function, spatial locations in the domain may have different important regions in the random variable space. To overcome this issue, an efficient sampling scheme based on Monte Carlo simulation such as Latin Hypercube sampling can be adopted. In this research, recently developed enhanced Monte Caro simulation-based reliability extrapolation method proposed by Naess *et al.* [7] is adopted. In the method, by reformulating the reliability problem with auxiliary scaling parameter λ, the authors demonstrated and verified that behavior of the small-scale probability regularly follows,

$$p_f(\lambda) \underset{\lambda\to 1}{\approx} q\exp\{-a(\lambda-b)^c\}, \quad \text{for} \quad \lambda_o \le \lambda \le 1, \qquad (10)$$

for a suitable choice of λ_o, where $p_f(\lambda)$ is auxiliary probability term that can be obtained from Monte Carlo based reliability estimation in use of reformulated safety margin:

$$M(\lambda) = M - \mu_M(1-\lambda), \qquad (11)$$

where M is the safety margin of original reliability problem and μ_M is expectation of M. In Eq. (10), a, b, c and q are constant parameters that could be estimated by interpolating auxiliary probability $p_f(\lambda)$. By focusing on the facts that **(a)** as λ goes to 1, the $p_f(\lambda)$ approaches to the actual probability, and **(b)** the estimating probability for safety margin with lower value of λ is less time-consuming than directly obtaining the actual probability, the strategy is to obtain actual probability by extrapolating Eq. (10) into λ=1 with optimally fitted parameters a, b, c and q. To get the sample points of Eq. (10), λ is discretized in domain of $\lambda_o<\lambda<1$, and each corresponding $p_f(\lambda)$ is simulated. However, since a set of safety margin sample can be reused in Eq. (11), repetitive sampling procedure is unnecessary for the interpolation. Therefore, the reliability of rare failure event can be acquired more efficiently compared to crude Monte Carlo method. Detailed explanation are descripted in Ref. 7.

V. NUMERICAL EXAMPLES

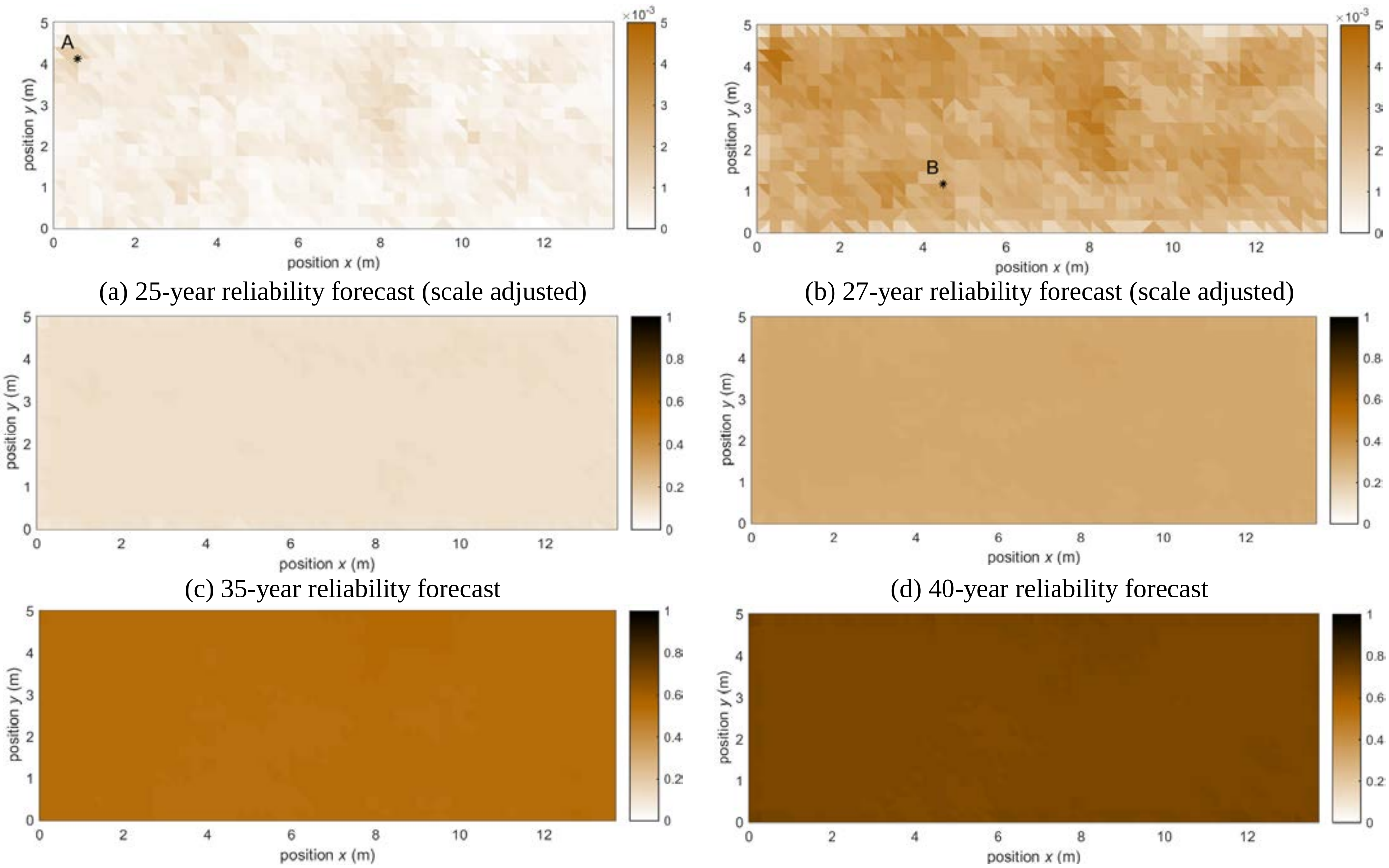
To demonstrate the proposed methodology, a corrosion induced rectangle RC-plate is studied. The width and length of plate are about 13m and 5m, respectively, and the depth is 0.275m. Corrosion mechanism is assumed to follow the models illustrated in Section II with stochastic properties of input variables in Table 1. Initial random fields of corrosion factors are modeled to have homogeneous mean, due to difficulty in pre-estimating the spatial discrepancy before operation, and the autocorrelation function is assumed as

$$C(\mathbf{x_1},\mathbf{x_2}) = \sigma^2 \exp\left(-\frac{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{L_D}\right). \qquad (12)$$

where $\mathbf{x_1}=(x_1,y_1)$ and $\mathbf{x_2}=(x_2,y_2)$ represents two separate points in domain, and σ is the standard deviation, and L_D is the correlation length. Taking the natural logarithm of the random fields representing corrosion factors $\{D_{cl^-},C_s,C_{\text{cover}},f_{\text{ct}}\}$, the random fields can be parameterized to standard normal random variables using KL-expansion. For each random field, 40 random variables are used to get a reasonable level of accuracy. Enhanced MCS was conducted by sampling of a set of random variables $\{\mathbf{u}^{D},\mathbf{u}^{C_s},\mathbf{u}^{C_{cover}},\mathbf{u}^{f_{ct}}\}$ in KL series in order to forecast spatial reliability along the time axis.

TABLE 1. Stochastic Parameters of Corrosion Factors

Variables	Unit	Distribution Type	Mean	Standard Deviation	Correlation Length (m)
Chloride ion diffusion coefficient $D_{cl^{-1}}$	cm/s^2	LN	2.0×10^{-8}	1.5×10^{-8}	2
Surface chloride concentration C_s	kg/m^3	LN	3.5	0.5	2
Concrete cover depth C_{cover}	cm	LN	1.03	2.15×10^{-5}	2
Tensile strength of concrete f_{ct}	MPa	LN	3.3	0.5	2



(a) 25-year reliability forecast (scale adjusted)

(b) 27-year reliability forecast (scale adjusted)

(c) 35-year reliability forecast

(d) 40-year reliability forecast

(e) 45-year reliability forecast

(f) 50-year reliability forecast

Fig. 2. Predicted reliability of excessive crack growth

Fig. 2(a)-(f) present the predicted spatial reliability of excessive crack growth on the RC plate with assumed stochastic parameters and models of KL random variables. The crack growth criteria is set as 0.4mm which corresponds to durability limit[13]. 20,000 samples are used for Monte Carlo simulation, and for discretized location having small-scale probability, the enhanced Monte Carlo simulation introduced in Section IV is applied. Inspecting the figures, one can notice that the predicted probability of crack growth are very small until 35-year, and then increases linearly as also shown in Fig. 3. Following initial assumption of homogeneous distribution of corrosion factors, the predicted corrosion damage also appears to be nearly homogeneous.

The adequacy of reliability extrapolation based on enhanced Monte Carlo simulation is verified graphically. Point A and Point B in Fig. 2(a) and Fig. 2(b), respectively, are selected as arbitrary representative points having small probability. Fig. 4(a) and 4(b) show the fitted parametric curve of Eq. (10) and attributed empirical probability estimates at point A and point B, respectively. It can be observed that the assumption of the probability trend following Eq. (10) are well satisfied in this

KL-expansion based 2D reliability analysis. Due to excessive computational time, the comparison with crude Monte Carlo simulation is not executed.

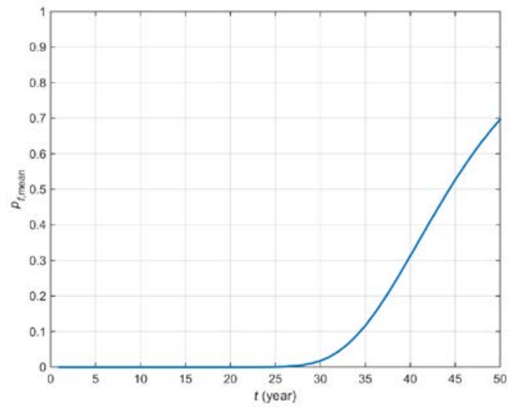
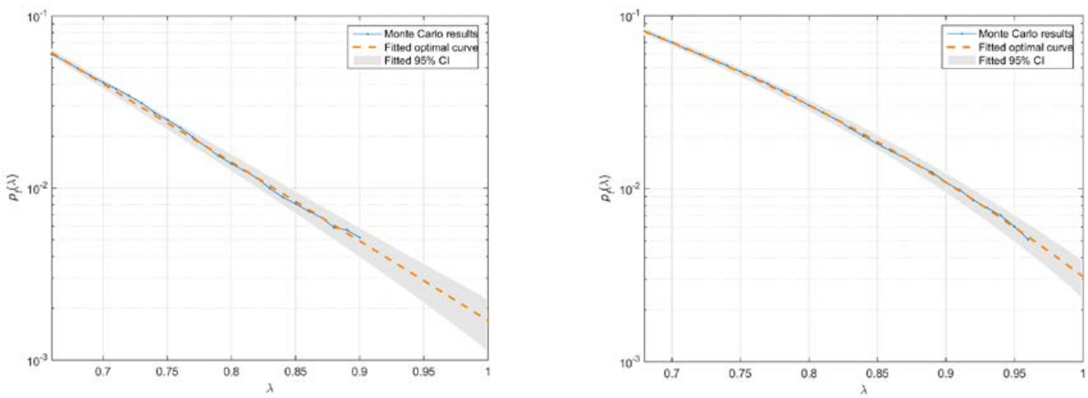


Fig. 3. Spatial mean of predicted crack growth probability along time axis



(a) Representative point A (b) Representative point B

Fig. 4. Reliability extrapolation using enhanced Monte Carlo simulation

VI. CONCLUSION

In this research, authors proposed an efficient method for spatial reliability analysis for corrosion-induced RC-plate. This paper described the corrosion-induced cracking process considering chemical-physical based phenomenon including diffusion of corrosion ion, accumulation of corrosion product and crack propagation. By introducing Gaussian random fields of corrosion factors and its Karhunen-Loève expansions, the initial environmental and geometrical factors associated with corrosion damage are easily and efficiently discretized. Spatial reliability approximation method using the enhanced Monte Carlo simulation are successfully adopted to obtain spatial reliability distribution in more effective sense. The proposed reliability analysis method will be applied to updating of knowledge based on operational condition. In the future, finding the potentially vulnerable region of infrastructure can be usefully applied to risk analysis and further to estimate urgency and cost of maintenance actions as well as identifying its priority.

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REFERENCES

1. L. PENG and M. G. STEWART, "Spatial time-dependent reliability analysis of corrosion damage to RC structures with climate change," *Magazine of Concrete Research*, **66**, 22 (2014)
2. S. KEßLER, J. FISCHER, D. STRAUB, and C. GEHLEN, "Updating of service-life prediction of reinforced concrete structures with potential mapping," *Cement and Concrete Composites*, **47** (2014)
3. A. DER KIUREGHIAN, First-and second-order reliability methods. In: E. NIKOLAIDIS, D. M. GHIOCEL and S. SINGHAL, Edts. *Engineering design reliability handbook*, CRC Press, Florida, USA (2005) [chapter 14].
4. N. KURTZ and J. SONG, "Cross-entropy-based adaptive importance sampling using Gaussian mixture," *Structural Safety*, **42** (2013)
5. Z. WANG and J. SONG, "Cross-entropy-based adaptive importance sampling using von Mises–Fisher mixture for high dimensional reliability analysis," *Structural Safety*, **59** (2016)
6. D. STRAUB, "Reliability updating with inspection and monitoring data in deteriorating reinforced concrete slabs." *Proc. ICASP11* (2011).
7. A. NAESS, B. J. LEIRA, and O. BATSEVYCH, "System reliability analysis by enhanced Monte Carlo simulation," *Structural safety*, **5**, 31 (2009)
8. B. SUDRET and A. DER KIUREGHIAN, *Stochastic finite element methods and reliability: a state-of-the-art report*, pp. 17-26, CA: Department of Civil and Environmental Engineering, University of California, Berkeley, USA (2000).
9. M.G. STEWART and D.V. ROSOWSKY, "Time-dependent reliability of deteriorating reinforced concrete bridge decks," *Structural Safety*, **20**, 1 (1998)
10. Y. LIU and R. E. WEYERS, "Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures," *ACI Materials Journal*, **95**, 6 (1998)
11. S.J. PANTAZOPOULOU and K.D. PAPOULIA, "Modeling cover-cracking due to reinforcement corrosion in RC structures," *Journal of Engineering Mechanics-ASCE*, **127**, 4 (2001)
12. R. G. GHANEM, and P. D. SPANOS, *Stochastic finite elements: a spectral approach*, pp. 1-62, Courier Corporation, New York, USA (2003)
13. M. G. STEWART and J. A. MULLARD, "Spatial time-dependent reliability analysis of corrosion damage and the timing of first repair for RC structures," *Engineering Structures*, **7**, 29 (2007)