ALEATORY UNCERTAINTY APPEARED IN STRONG NONLINEAR BEHAVIOR SUCH AS SLOPE COLLAPSE

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Nonlinear phenomenon like slope failure involves unpredictable uncertainties, which means that we cannot reproduce same result in an experiment even though every condition is carefully set in the same way. This paper discusses this type of uncertainties observed in numerical simulations and experiments. Numerical simulations of slope failure test due to gravity or sinusoidal excitation are carried out by MPS (Moving Particle Simulation) method, which is derived mathematically from a wave motion equation. Stiffness is selected to study influence of input uncertainty in the numerical simulation because it affects from linear (elastic) response to strong nonlinear response (collapse). Deformation due to small gravity, namely weak nonlinear or linear behavior, the output uncertainty (standard deviation of displacement of slope top) is proportional to input uncertainty. In strong nonlinear behavior, the output uncertainty is constant irrespective of uncertainty level of input data when the input uncertainty is less than a certain level. Dynamic centrifuge tests are also carried out in order to study the behavior of slope failure caused by sinusoidal excitation. Its numerical simulations by MPS method are performed. Uncertainties due to initial locations of particle (so-called initial packing of DEM) are illustrated.

I. INTRODUCTION

Linear behavior can be reproduced in experiments to some extents. It can be assumed that input uncertainty is proportional to output uncertainty. Nonlinear phenomenon like slope failure, however, involves unpredictable uncertainties, which means that we cannot reproduce same result even though every condition is carefully set in the same way. We sometimes encounter a case in which very small change of computation or experiment condition causes the significant difference in response when strong nonlinearity is involved. This type of uncertainty is difficult to predict, which is considered to be aleatory uncertainty. Traditional reliability theory, which assumes input uncertainty is proportional to output uncertainty, cannot explain this phenomenon.

Conventional FEM (Finite Element Method) has a difficulty to simulate strong nonlinear behavior such as failure or collapse. DEM (Distinct Element Method) is one of the most successful methods to simulate the slope collapse. Its drawback is that it is an empirical method, in which values of spring and dashpot to connect each particle must be determined empirically. In addition, theoretically DEM does not satisfy a wave motion equation. MPS (Moving Particle Simulation) method is derived mathematically from a wave motion equation. MPS method was proposed for fluid material at first (Ref.1, 2), then that for elastic body was developed (Ref.3). Most significant feature of MPS method is that distribution of displace instead of strain is used in the formulation. When a slope collapses, many small discontinuities, cracks, are developed at first, and then they are gradually concentrated to a couple of slip lines. The expression of discontinuity is very important in failure of geo-material. Most of methods such as FEM, MPM (Material Point Method) or SPH (Smoothed Particle Hydrodynamics), are formulated based on distribution of strain. Since a discontinuity cannot be expressed in term of strain, a special treatment is required to model the discontinuity. On the other hand, the displacement-based method like DEM easily expresses the discontinuity. The formulation of MPS method is very close to that of DEM. Consequently MPS method has capability similar to DEM with respect to failure phenomenon. MPS-DEM method are proposed to simulation failure behavior of geo-material (Ref.4, 5)

This paper discusses uncertainties of output, which is response of slope such as deformation or slip lines (area) caused by gravity, in numerical simulations by MPS-DEM method. Small to large uncertainties are given to input parameter (stiffness) of the slope model or initial location of particles (so-called packing in DEM). Uncertainties observed in dynamic slope failure centrifuge test and its simulations by MPS-DEM method are also shown and discussed.

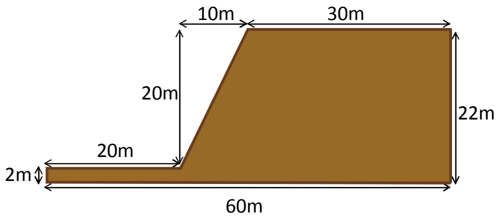


Fig. 1. Model for the collapse simulation

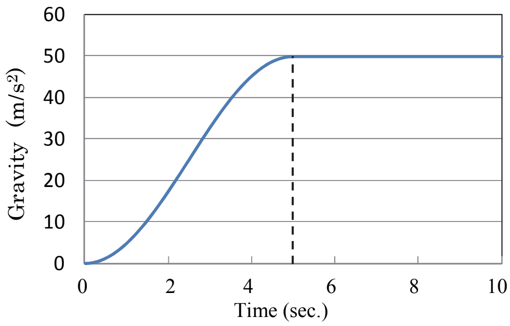
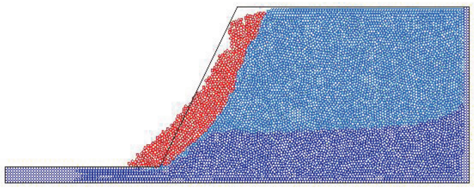
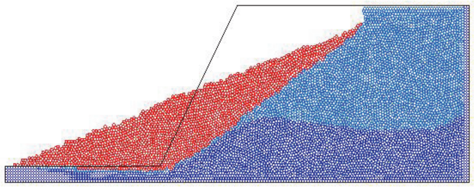


Fig. 2. Application of gravity



(1) 4 second



(2) 10 second

○ < 0.1m, ○ 0.1 to 0.5m, ○ >0.5m

Fig. 3. Collapse of slope due to gravity

II. INPUT UNCERTAINTIES V.S. OUTPUT UNCERTAINTIES IN NUMERICAL SIMULATIONS

II.A. Conditions for numerical simulation

In order to discuss output uncertainties caused by input uncertainties, simulation of slope failure due to gravity is carried out by MPS-DEM method with a slope model shown in Figure 1. The height of slope is 25 [m], slope gradient is 1:0.5. Figure 2 shows time history of gravity applied up to 50 [m/s^2] during 5 seconds. The downward force 50 [m/s^2] is around five times of ordinary gravity, which can be interpreted that the height of slope is gradually increased from 0 [m] to 100 [m] under ordinary gravity. The Young's modulus E is assumed to be dependent on depth d.

$$E = E_0 + C_E d \qquad (1)$$

E_0 is Young's Modulus at the slope surface, and C_E is coefficient to the depth d from the slope surface. The material properties are summarized as follows,

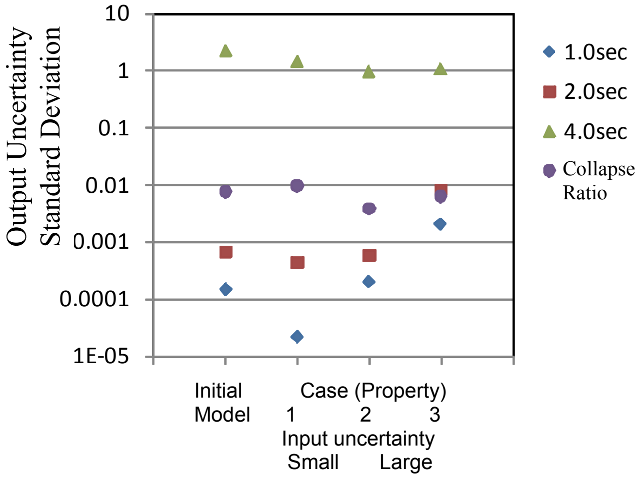
Young's Modulus (E_0)	50000 [kPa]	Coefficient (C_E)	500 [kPa/m]
Poisson's Ratio	0.35	Damping Ratio	0.05
Cohesion	80.0[kPa]	Internal Friction	30 [degree]

The radius of particle of MPS-DEM is 0.167 [m], the number of particle is 7380, time increment is 5.0 [10^{-5} sec.]. The formulation of the MPS-DEM can be summarized in (Ref.4, 5).

II.B. Discussion on nature of uncertainties

Figure 3 shows collapse process of the slope due to the growing gravity. The color of each particle indicates displacement from initial location. The red color shows the particles which move more than 0.5 [m]. "collapse ratio" is defined as the ratio of particle which moves more than 0.5 [m] from initial position, namely the ratio of red particles to all particles at 10 second. In the illustrated case in Figure.3, the "collapse ratio" is 0.27.

Stiffness is selected to study influence of input uncertainty because it affects from linear (elastic) response to strong nonlinear response (collapse). The uncertainties of E_0 and C_E in Eq.(1) are considered. In Case 1, coefficient of variance (COV) of E_0 and C_E are 0.1 and 0.4%. In Case 2, 3, the COVs are 1.0, 4.0 and 10.0 and 40.0% respectively. The COVs of Case 2 and 3 are 10, 100 times of that of Case 1. Figure 4 shows the standard deviation of response (output) due to the



COV of Young's Modulus (%)

	E_0 Young's Modulus	C_E Coefficient
Case 1	0.1	0.4
Case 2	1.0	4.0
Case 3	10.0	40.0

$$E = E_0 + dC_E$$

d : depth

Fig. 4. Uncertainty of Response, displacement at slope top and collapse ratio

uncertainties of stiffness. The vertical displacement from initial location at slope top (shoulder) and collapse ratio are focused in order to discuss the uncertainties of the response. The displacement at 1.0 second is mainly due to elastic deformation in which strong nonlinearity is not involved yet. Standard deviation of Case 2 and 3 is 10, 100 times of Case 1 as shown in Figure 4. The standard deviation of displacement is proportional to COV of stiffness. The Standard deviation of Case 1 and 2 are, however, almost same at 2 second. The standard deviations at 4 second are almost same among all cases. The standard deviations of collapse ratio of all cases are also almost same (at 10 second).

In DEM (Discrete/Distinct Element Method) calculation, a regularly placed particle model like lattice (grid) shows strange behavior because the regularity of the placement affects the failure behavior strongly. Therefore random packing of particle, which is modelling as to the initial locations of particle, is very important. The same goes for MPS-DEM method. The initial locations are determined by using random numbers such that particle number density is as uniform as possible. The description of detailed procedure is omitted due to the limitation of the space. Several models with different random number are made and standard deviation of response are calculated, which are indicated as "initial model" in Figure 4. The standard deviations by "initial model" are almost same as other cases when the nonlinearity is strong, such as "4 seconds", "collapse ratio".

III. UNCERTAINTIES APPEARED IN CENTRIGUGE TEST

III.A. Outline of the test

Centrifuge test of slope failure and its uncertainties are discussed (Shuku et al. 2015). Numerical simulations of the slope failure test by MPS method are performed and compared with the test results. This is, however, a preliminary report because the centrifuge test is still ongoing, and input parameters for the simulation by MPS-DEM method are temporarily determined by try-and-error.

A centrifugal apparatus at Okayama University was used to conduct a series of centrifuge tests. The centrifugal apparatus is the smallest one that nominal radius is 0.30 m and can generate a maximum acceleration up to 560 G. Figure 5 shows the shape of model slope. The summary of procedure is outlined as follows.

(1) Slurry of kaolin clay was cast into the box. Two container boxes filled with the kaolin slurry were prepared in one set of experiment.
(2) Those two boxes, which have the same weight, are mounted on the buckets and are accelerated to 300 G over 5 minutes. The slurry in the boxes was consolidated by its self-weight for two hours.
(3) The two container boxes were taken from the bucket of centrifuge apparatus, and the model slope (Figure 5) was carefully made by cutting the consolidated kaolin clay.
(4) One of the slope model and a counter weight box were mounted on the bucket, and the centrifugal acceleration was raised up to 150 G, then the model was held at a constant for two minutes to allow equalization of the load. At the end of the testing, the centrifuge was slow down gradually until the spinning is full stopped.

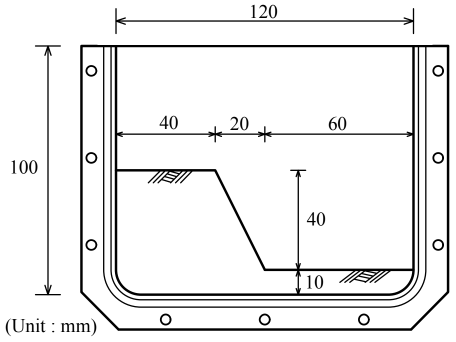


Fig. 5. Device of slope failure centrifuge test

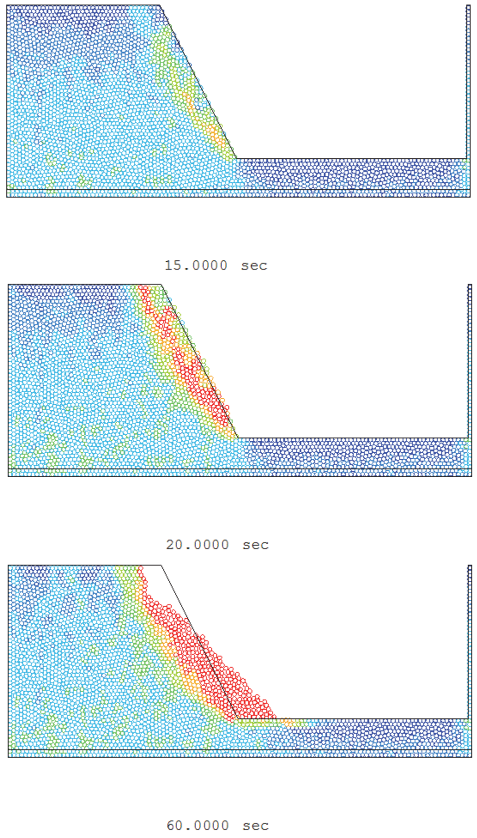


Fig. 7. Process of slope failure, distribution of maximum shear strain (MPS-DEM method)

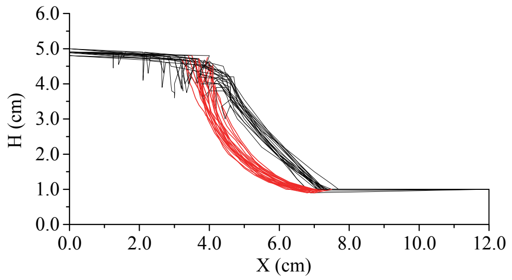


Fig. 6. Failure lines obtained by centrifuge test

Failure process and deformation of the model slopes were observed with a digital video camera during the experiment. In addition, failure lines observed along the front side of containers were carefully traced onto a piece of sheet and then digitized. This model tests under the same experimental conditions were conducted twenty times. All failure lines observed at the end of the tests are shown in Figure 6. The observed failure behavior can be summarized as follows.

(1) A tension crack at the middle of the crest appeared at first.
(2) The toe of the slope was displaced to the front. Although the slip line was not able to be observed with digital video camera, it can be assumed that slip failure of the slope occurred at this time.
(3) The slope continued to deform and other tension cracks which closes to slope shoulder occurred. Finally the slope collapsed.

Please see (Ref.6) for the detail of the centrifuge test.

III.B. Numerical simulation of the centrifuge test

Numerical simulations of the slope failure centrifuge test are performed by MPS-DEM method. The model for the simulation has same configuration but the size is different. The height of the slope for the simulation is 40m. The gravity 150G in the centrifuge test is equivalent to 0.147cm/s^2 in the simulation because of similarity rule. The gravity is gradually increased up to 0.147cm/s^2 during first 20 seconds and is kept constant 40 more seconds. Strength parameters are determined by try-and-error such that the simulation results agree with the test as a preliminary study.

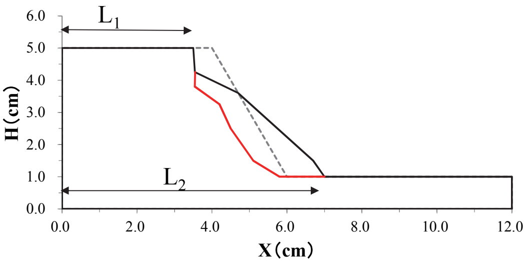


Fig. 8. An example of failure line and locations of the line, L_1 and L_2

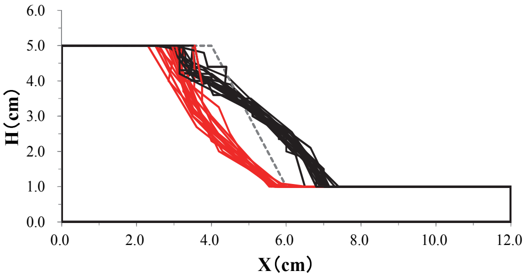


Fig. 9. Failure lines obtained by numerical simulation (MPS-DEM method)

TABLE I. Comparison of L_1 and L_2 (cm)

	L_1		L_2	
	mean	St.Dev	Mean	St.Dev.
Test	3.78	0.27	7.23	0.20
Simulation	2.98	0.38	7.04	0.21

St.Dev. : Standard Deviation

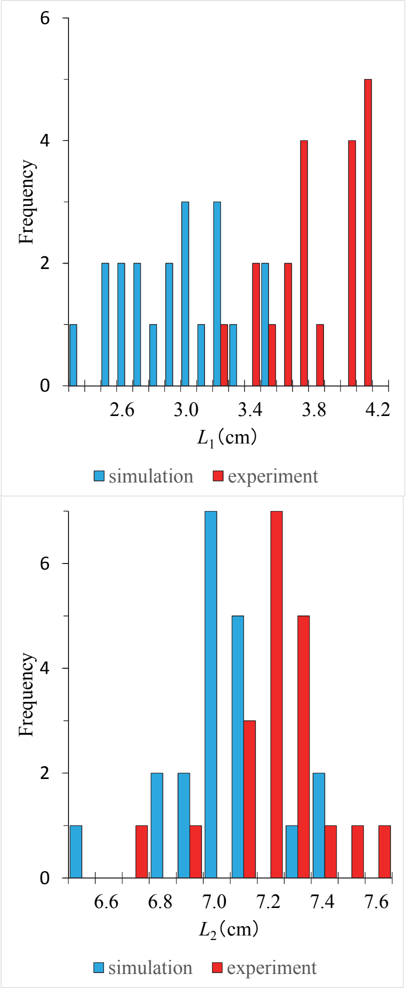


Fig. 10. Comparison of failure line locations between centrifuge test and numerical simulation (MPS-DEM method)

The simulation results at 15, 20, 60 seconds are shown in Figure 7. The color of each particle shows maximum shear strain. At first large strain particles are observed at foot of the slope and developed toward top of the slope. The large strain area suggests a potential slip line. The line reaches to the top of the slope at 15 seconds. Finally the slope collapsed along the line. From the final shape of the slope and distribution of maximum shear strain, the failure line is estimated as shown in Figure 8.

Twenty models as to initial particle location are made with different seed of random number. Slope failure simulations with the twenty models are performed by MPS-DEM method under same condition except the initial particle location. All the slip lines obtained by the simulations are shown in Figure 9. The characteristic locations of slope failure L_1 and L_2, which are defined in Figure 8, are focused in order to discuss the uncertainty in the slope failure. The comparison of the locations L_1 and L_2 between the centrifuge test and the simulation is shown in Figure 10. The slip lines observed in the simulation are generally larger than those of the centrifuge test. Especially mean of L_1 by the test is larger than that of the simulation. It means that the slope failure areas estimated by the simulations are relatively larger than the test. The mean and standard deviation of locations L_1 and L_2 are summarized in TABLE I. The standard deviation by simulation is also larger than that of the test. It seems that it is attributed to the relatively large estimation of failure area. Since the simulation does not agree very

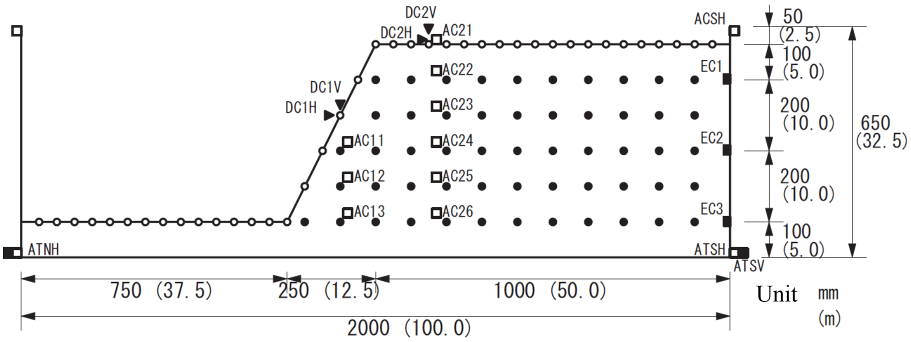


Fig. 11. A slope model for centrifugal dynamic test,
numbers noted in brackets stand for size converted in 1G

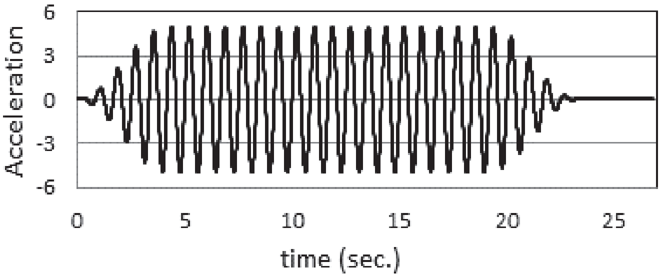


Fig. 12. Time history of input motion, 0.5G

well with the test, especially the size of slip failure, simulation conditions and algorithm should be improved. It is, however, noted that magnitude of uncertainty in slip lines are roughly similar.

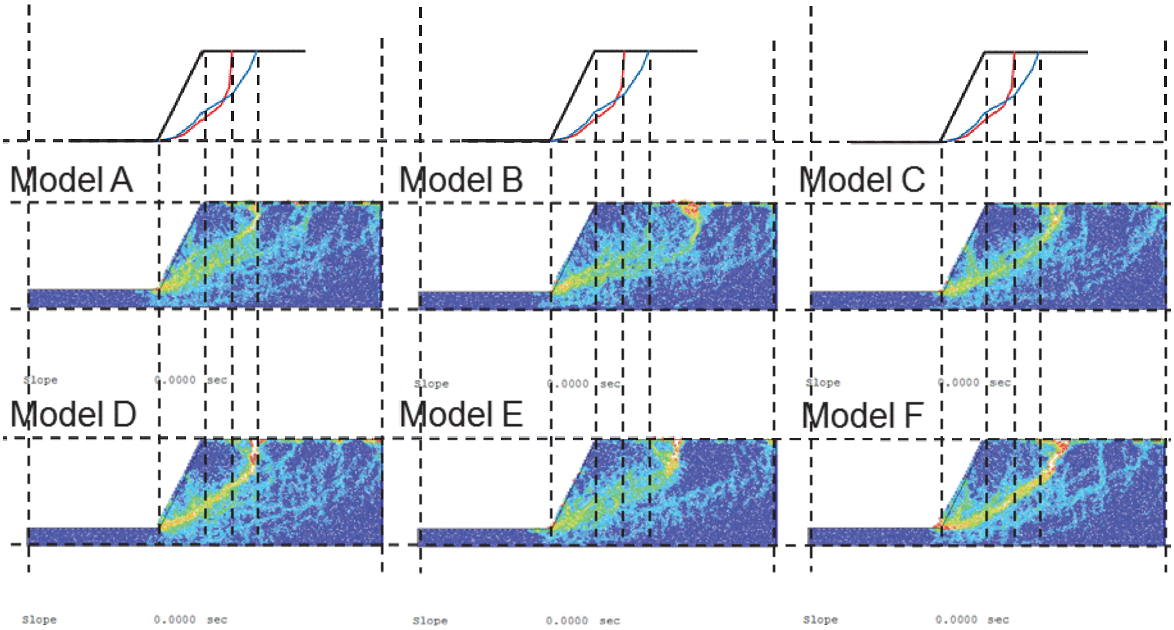
IV. UNCERTAINTIES APPEARED IN DYNAMIC CENTRIGUGE TEST

IV.A. Outline of the dynamic centrifuge test

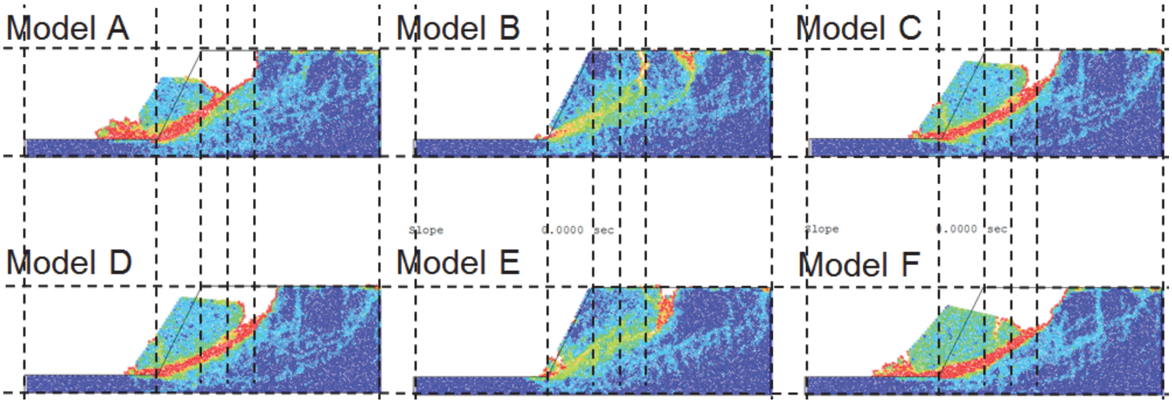
The dynamic centrifuge test was carried out under 50G gravity in order to study the behavior of slope failure caused by earthquake (Ref.7). The outline of slope model for the test is shown in Figure 11. Cement-mixed soil is used to model a mudstone slope. The height of slope is 50 cm under 50G, which can be converted to be 25 m under 1G. Teflon sheet and silicon coating are implemented at the boundary between the slope model and the rigid soil vessel to reduce shear stress to the depth direction (out of plane direction). A sinusoidal motion shown in Figure 12 is used as an excitation input to the shaking table after static 50G force is applied to vertical direction. The frequency of the sinusoidal wave is 1.2Hz. The maximum amplitude of the excitation motion (maximum acceleration) is increased at 0.1G interval. Figure 5 shows a time history with 0.5G amplitude as an example. The slope model collapsed in the experiment, when maximum acceleration of input motion is around 0.5G.

IV.B. Numerical simulation of the dynamic centrifuge test

Six models as to initial particle location, A to F, are made with different seed of random number. The simulations are carried out against 300, 400, 500, 600gal sinusoidal wave sequentially. Figure 13 shows distributions of residual maximum shear strain and shapes of slope of the six models after excitation of 400 gal (cm^2/sec) and 500gal sinusoidal wave. Slip lines by the experiment, which was carried out twice, are shown at the top of the figure. Figures of 300 and 600gal excitation are omitted in this paper due to the space limitation. After the excitation of 400gal sinusoidal wave, similar general trend is observed among all models, but the detail of the distribution depends on the model. Model A and D have a small potential



(1) After 400 (cm^2/sec) excitation



(2) After 500 (cm^2/sec) excitation

Fig. 13. Shape and residual maximum shear strain of slope after excitation by MPS-DEM

slip line which is the most concentrated strain line. Model B and D have large slip lines, Model C and F have intermediate lines. During the 500 gal excitation as shown in Figure 13(2), the slope models are collapsed along the potential slip lines except Model B and E. Model B has two potential slip lines. The shallower line is clearer than the deeper. Model B and E are collapsed during the excitation of 600gal sinusoidal wave. Model B collapsed along the shallower line. The simulation results show good agreement with the experiment results though the slip lines by simulation are slightly larger than those of the experiment.

V. CONCLUSIONS

Uncertainties observed in experiments and numerical simulations are illustrated and discussed. A part of uncertainties in strong nonlinear phenomenon can be interpreted as aleatory type. The features of the uncertainty are inferred as shown in Figure 14 from the simulation results discussed in Chap.II. In weak nonlinear or linear behavior, the output uncertainty is proportional to input uncertainty. In strong nonlinear behavior, the output uncertainty is constant irrespective of uncertainty level of input data when the input uncertainty is less than a certain level. The range of the constant uncertainty depends on nonlinearity in the phenomenon. When strong nonlinear phenomenon is involved, we should be prepared for this type of unpredictable uncertainties. It is recommended to perform several cases of simulation or experiment to interpret strong nonlinear phenomenon like Chap.III and IV. It is difficult to reduce this uncertainty even though very detailed information of the model and input motion are provided.

Centrifuge test and its simulation shown in Chap.III are still ongoing. We are examining the material for the experiments and input parameters for the simulations. Detailed quantitative evaluation of uncertainty in the slope failure will be future topics.

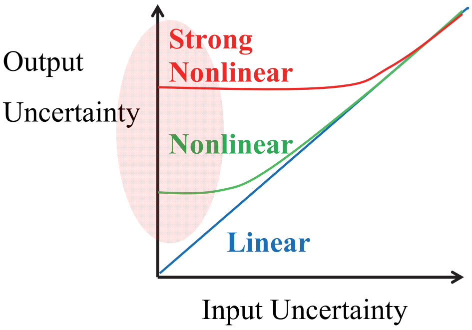


Fig. 14. Aleatory uncertainty involved in nonlinear behavior

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