A DISCUSSION OF FAILURE MODE MODELING OF COMPLEX COMPONENTS AND OVERALL COMPONENT RELIABILITY

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There are many solution forms that mathematicians, engineers and scientists use for ease and simplicity. Everything is a sphere, behaves like an exponential random variable, and can be solved with a closed form solution. Are things really that simple in a complex world? Unfortunately, the answer is often no, but solution forms are typically chosen for convenience instead of exactness. This particularly holds true for the reliability engineering discipline. Few engineers understand the implications of choosing a model or solution form for convenience, and results are presented as being exact or a reasonable approximation.

Every industry that utilizes risk or safety practitioners, or reliability or maintainability engineers, deals with complex components. Whether they are called Line Replacement Units (LRUs), Orbital Replacement Units (ORUs), components, widgets, etc., they are typically complex devices, encompassing more than one technology, and have many failure modes. Even something as simple as a wire can break, short, arc, or have a hot spot; more complex components, such as valves, actuators, pumps, have subassemblies that are complex in their own right, and can have many failure modes that are seen during failure inspections.

This document discusses how to analyze failure modes of components, and to prevent the over-estimation of reliability gains from component improvements.

I. INTRODUCTION

Every heavy industry in the free world is competitive. Original Equipment Manufacturing (OEM) companies, whether in the aviation, transport, aerospace or industrial sectors, will not stay in business long if they do not make a reliable and affordable product. There are reliability engineers everywhere analyzing data, trying to understand how the components or systems operated, how they failed and what they can do better. The continuous improvement cycle is truly a way of life in most industries. As industries advance, so does the technology used in the components. Components can be highly complex mini-systems in their own right as engineers strive for miniaturization.

This document focuses on analyzing failure modes of components at the level of independence. In other words, if the analyst is worried about a valve failing to open or close, the analysis can be done at the valve fail level, with a percentage of failures going into the respective open and closed buckets. The causes for a valve failing to open or close would be the same; the randomness would be in which position the valve failed. If the analyst is trying to understand the race of components within the valve, such as the solenoid, position sensor, valve, body, etc., then a more realistic approach to this type of modeling is presented in this document. Although this is a more complex modeling effort at this level, this would be necessary when an engineer is analyzing cost-benefit returns of component improvements, as the more simplistic methods tend to result in grossly inaccurate and optimistic results.

I.A. Simplicity Leads to Error and Uncertainty

Many engineers do not understand error or uncertainty. They are not topics readily taught in most engineering curriculums. Engineers typically do not understand the implications of saying "assume this behaves like an exponential random variable," or any other random variable that is easily solvable using a computer, calculator, or even the back of an envelope. The use of a simplifying assumption or a simplistic model, without understanding the applicability and repercussions, will bound a complex problem into a smaller solution space with less error and uncertainty. In real world applications, over simplifying problems leads to bad designs, or design "improvements" that are basically no improvement at all, with companies wondering why their investment is not reaping the benefits they expected. The world is, simply, not simple.

Take the common automobile, for example. No one buys a new automobile thinking that the new automobile will see as many problems over a period of time as an old automobile. Automobiles should have very few things go wrong before 50,000 or even 100,000 miles, but at some point in time they become problematic with failure of components occurring more often. Clearly, the automobile does not have a constant failure rate, and neither do many of the components that comprise an automobile, or components that make up any type of system.

If you ask a reliability engineer what the formula for reliability is of anything, invariably they will respond with the simple equation:

$$Reliability = R = e^{-\lambda t}. \tag{1}$$

However, many engineers do not understand that the use of Eq. (1) implies that the exponential random variable is applicable to the situation, the failure rate is constant, and it is memoryless, all of which is easily dispelled by the automobile example. Although immediately an analyst would think how to estimate λ for Eq. (1), a better question might be how to determine what model of the world is more applicable. Most Probabilistic Risk Assessment (PRA) practitioners are taught that the uncertainty in the model of the world dominates over the parametric uncertainty (Ref. 1 is one example), and that holds true across the engineering disciplines. The model chosen to represent the world has a significant impact on the error and uncertainty.

On a side note, the authors have never seen a constant failure rate of anything, despite many years of trying to find that example. There are, of course, many other engineers who have the same feeling that nothing behaves like an exponential (see Ref. 2), but the majority of the engineering world, reliability engineering included, still treats everything as if it is an exponential for simplicity and ease.

Eq. (1) can be expanded beyond the single item of interest. Now instead of one component, consider a set of components,

$$X_1, X_2, \cdots, X_n \tag{2}$$

that are independent and exponentially distributed random variables with failure rates of

$$\lambda_1, \lambda_2, \cdots, \lambda_n. \tag{3}$$

Then the minimum reliability of the set, or

$$Minimum\{X_1, X_2, \cdots, X_n\} \tag{4}$$

is also an exponential distribution, with parameter

$$\lambda_{Minimum} = \lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_n. \tag{5}$$

Thus, for a set of components, all independent and exponentially distributed, the *minimum reliability* is

$$R_{Min}(X_1, X_2, \cdots, X_n) = e^{-[(\sum_i^n \lambda_n)t]}. \tag{6}$$

This is a very simple equation, especially when the analyst is faced with analyzing hundreds of items. But what if some or none of the components are exponential in nature? How should this be treated?

I.B. The Race

Consider a different perspective for the failure modes; or in this case, we will look at a complex component with various subcomponents, all of which behave independently of the others. In the automobile example, we can define failure if the automobile is unable to start or run. There are a lot of component failures that can lead to that end state, such as a battery loss, starter loss, transmission loss, etc. There could be hundreds, if not thousands, of little items that could cause a typical automobile to fail. However, for demonstration purposes, let's consider only the supercharger with four subcomponents as the system. A picture depicting this simplistic model can be seen in Fig. 1.

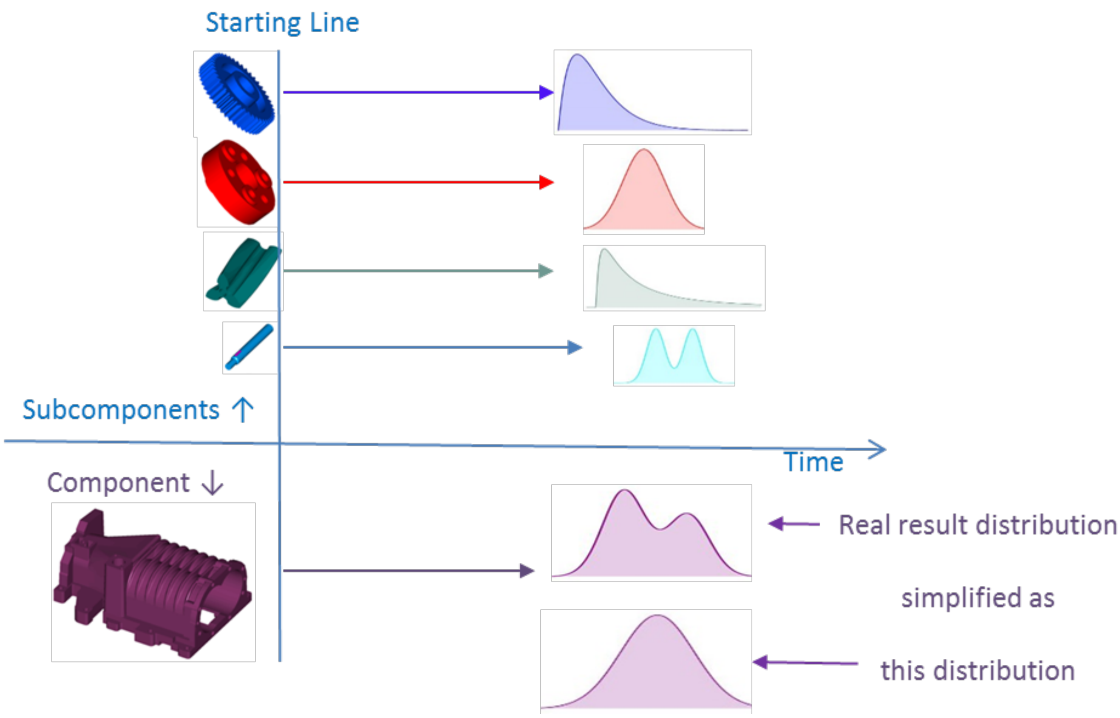

Fig. 1. Race of components to the failure finish line.

In the upper part of Fig. 1 we see a small set of subcomponents, all fresh from the factory, ready to begin the race from brand new to failure. The system, or component level item, is also shown in the bottom part of the figure. Every subcomponent has a different failure distribution associated with it, as shown. They could be normal, lognormal, gamma, Weibull, multimodal, or many others. The component level will have a failure distribution associated with it, but given that it is failure modeled as $1\text{-}R_{min}$, it is not easy to model using closed form distributions (although most likely an exponential or Weibull would be selected for simplicity). A common mistake is to say the component level failure rate is a sum of the subcomponent failure rates, which as shown in Fig. 1, is not the case. The failures that one would see at the top level could be estimated with an exponential failure rate, but the failure distribution really has no "rate" at all. The top level failure distribution is the minimum of the set of subcomponent failure distributions, and it is probably *not* a closed form, parametric solution.

I.C. The Data

In the real world, interpreting the data obtained from situations described in Fig. 1 is not simple. The real world is a complex place, with many operators, engineers and technicians working together to keep things moving smoothly, reliably and safely. In the automotive industry, there is no telling in most cases what the product went through before failure, and the warranty investigations only look at the end effect. Data is not always clear and distinct, and will need interpretation before being used to understand and predict component reliability. As with any engineering analysis, understanding the data can be a monumental task, but is necessary due to the profound implications on the results (see Ref. 1, Chapters 3 and 5).

II. SIMPLISTIC FAILURE MODE EXAMPLE

The term failure mode will be used for the remainder of the document, but as stated previously, it can represent a subcomponent as well when modeling a complex component.

II.A. Simple Example

Consider a simple component with three failure modes. The analyst is trying to determine the failure rate of each failure mode based on test data. Consider that nine tests are performed until the component fails, and the failure mode is defined as one of the three failure modes occurring. Each test is with a completely new assembly, with no used parts from previous tests. Three failures of each of the three components are observed at various times, in hours (see Table I).

TABLE I. Failure Times to Different Failure Modes of Nine Trials

A	B	C
100	75	125
75	100	150
125	200	150

Table I should actually be written with the understanding that when the failure of one subcomponent occurs, the test is halted for the other two. The data is said to be *censored*, and a better way to get that across can be seen in Table II.

TABLE II. Failure Times to Different Failure Modes with Censored (“>” Time) Data (Time in Hours)

Trial	A	B	C
1	**100**	>100	>100
2	>75	**75**	>75
3	>125	>125	**125**
4	**75**	>75	>75
5	>100	**100**	>100
6	>150	>150	**150**
7	**125**	>125	>125
8	>200	**200**	>200
9	>150	>150	**150**

In Table II, the rows depict each trial, the time to first failure mode or component that failed first (shown in bold), and which two components survived (denoted by the “>”). Now let’s examine different ways of analyzing the data.

II.B. Simple Solution

The simplest solution is to treat each failure mode as being exponential, in which case the analyst would “bin” the various failures by group, and estimate the rates and Mean Time Between Failures (MTBF) using total time, as shown in Table III.

From an engineer’s perspective, it is strange to see that the three failure modes all have different average times of failure (100hr, 125hr, and 142hr, respectively), yet have the same MTBF — one that is much higher than seen in any of the tests. This is an example of what has been called the “Bayesian Anomaly” but is really a case of not properly treating the knowledge dependence (in this case the censored data) properly. A further analysis using Minitab[3] to estimate the goodness of fit for the data to an exponential is shown in Fig. 2.

TABLE III. Simple Solution

Failure Mode	Number	Total Time	Failure Rate (/hrs)	MTBF
A	3	1100	0.0027	367
B	3	1100	0.0027	367
C	3	1100	0.0027	367

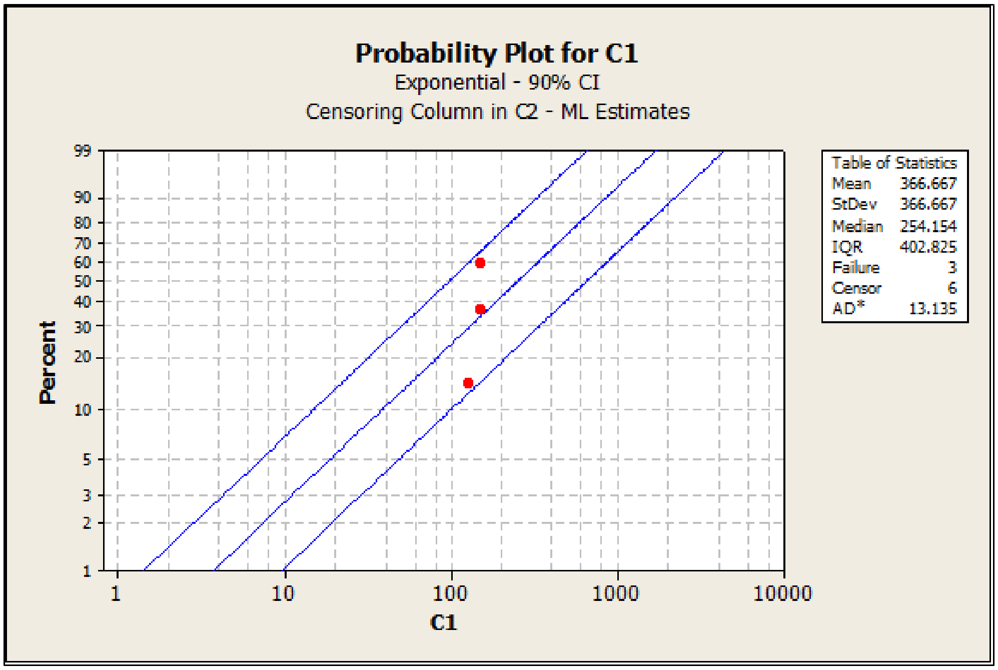

Fig. 2. Minitab data fit for failure mode A

The results for failure mode A do not show a good fit for the data (the other two, although not shown, are even worse fits for an exponential), as seen by both the modified Anderson-Darling statistic. For an exponential distribution, the Anderson-Darling value of 9.599 already has a p-value of below 0.003[4], and this Anderson-Darling statistic is even higher, making the p-value even lower. Additionally, a visualization of the data also does not lead one to believe this is a good data fit. This is as expected, as there are many references in the literature that describe the bias of using censored data with small numbers of actual failures (Refs. 5, 6 and 7). However, in real world experience, the number of censored data points may be an order or magnitude or larger than the known failures, and many of the distributions with the data seen do not fit a parametric distribution at all. Software programs calculate Maximum Likelihood Estimations through a variety of approximations, which further add error and uncertainty to the results.[8]

II.C. Alternative Solution

An alternative solution is to model the race of the failure modes through simulation to determine the failure mode distributions. For this example, the program OpenBUGS[9] is used, although there are many alternatives available, the authors chose this tool due to familiarity; the concept is simple enough to program in any number of statistical packages. Note that BUGS stands for Bayesian inference Using Gibbs Sampling; OpenBUGS is the follow on to WinBUGS, and is a program for Bayesian inference using Markov Chain Monte Carlo (MCMC) techniques.

Prior to developing the analysis, the analyst needs to dissect the data to the appropriate failure mode level, as was done in the simple solution. It would also be helpful to understand what potential forms the data distributions could take, but in this example, with three data points, almost any continuous distribution would fit. In more complex problems, it would be best to test for a statistical fit of the data using a program such as Minitab (easy case), or to create a histogram of what is seen in the real world (more difficult, but more exact case).

The following script was written for OpenBUGS to run the simulation:

```
model    {
# Loop through the observed and censored times
for (i in 1:N)       {
T.A[i] ~ dnorm(mu.A, tau.A)C(lowerA[i], )
T.B[i] ~ dnorm(mu.B, tau.B)C(lowerB[i], )
T.C[i] ~ dnorm(mu.C, tau.C)C(lowerC[i], )
}
# Replicate the posterior model for failure times
TA ~ dnorm(mu.A, tau.A)
TB ~ dnorm(mu.B, tau.B)
TC ~ dnorm(mu.C, tau.C)
# Find the minimum times
AB <- min(TA, TB)
T.TE <- min(AB, TC)
# diffuse priors
mu.A ~ dflat()
mu.B ~ dflat()
mu.C ~ dflat()
tau.A ~ dunif(0,100)
tau.B ~ dunif(0,100)
tau.C ~ dunif(0,100)
}
data
list(T.A=c(100,NA,NA,75,NA,NA,125,NA,NA), T.B=c(NA,75,NA,NA,100,NA,NA,200,NA),
T.C=c(NA,NA,125,NA,NA,150,NA,NA,150),
lowerA=c(100,75,125,75,100,150,125,200,150), lowerB=c(100,75,125,75,100,150,125,200,150),
lowerC=c(100,75,125,75,100,150,125,200,150), N=9)

inits
        list(mu.A=367, mu.B=367, mu.C=367, tau.A=2, tau.B=2, tau.C=2)
        list(mu.A=300, mu.B=300, mu.C=300, tau.A=2, tau.B=2, tau.C=2)
```

Note the use of the “C,” or censored option in the OpenBUGS script. Additional information on censored data within OpenBUGS may be found in Chapter 10 of Ref. 10. The Time to Top Event (T.TE, or component failure time), is the minimum of the failure times for failure modes A, B and C, denoted T.A, T.B and T.C, respectively. OpenBUGS has a slightly different routine for finding the minimum; it cannot do it from a set, but has to do piecewise comparisons as shown in the code.

In this coding example, normal distributions were chosen due to the tightness of the data, and initial parameters were chosen for diffuseness. A sample size of 200,000 trials was run (discarding the first 1,000 for burn in), although this simple model achieved convergence much sooner. The results of the simulation are shown in Table IV. The important results to note are TA, TB and TC; the other information is for the OpenBUGS users to compare results. TA, TB and TC, show times of failure at 176, 174 and 166 hours, respectively. This is quite a difference compared to the simple solution, which showed that all three failure modes have the expected mean time to failure of 367 hours.

III. RESULTS AND IMPLICATIONS

The results between the two approaches are quite different, and yet when the problem is compounded by more failure modes, the answers will get further and further apart. When engineers use this data to estimate the effect on design changes, the results can be quite different as well, and can lead to questionable engineering management decisions. Consider the case where failure modes A and B can be completely eliminated through redesign, and someone asks the question “what will the reliability gains be?" In the simple model, only failure mode C would remain, and from Table III, we see that is a failure rate of 0.0027 per hour or a MTBF of 367 hours. In the complex model, from Table IV we see a time to failure of 166 hours. We see a difference of over 100% between the two reliability estimates. This error is commonly seen in many industries among many engineers, and this is why the reliability gains predicted for redesigns of components typically result in smaller reliability improvements.

TABLE IV. OpenBUGS Simulation Results

Node	mean	5th	50th	95th
T.TE	118	30	125	182
TA	176	52	167	327
TB	174	67	167	304
TC	166	104	163	235
mu.A	176	127	167	254
mu.B	174	131	167	239
mu.C	166	140	163	200
tau.A	3.3E-04	5.1E-05	2.7E-04	8.2E-04
tau.B	4.1E-04	7.5E-05	3.4E-04	9.6E-04
tau.C	1.3E-03	2.5E-04	1.1E-03	3.0E-03

Somewhere we have forgotten as an industry that there is a difference between analyzing for the minimum of distributions versus solving for the sum of distributions. Some analysts believe they can sum up the distributions of the piece parts for a top level distribution. Examples are presented in literature that show the simplistic approach following an exponential, then the failure modes are analyzed and treated as Weibulls or other distributions, then added up again. This makes no sense and has no mathematical basis. Used in many engineering disciplines, the convolution to the statistician/reliability engineer is the sum of two probability density functions. The convolution of two functions, g(t) and h(t) is written as:

$$f(t) = (g * h)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau \quad (7)$$

$$= \int_{-\infty}^{\infty} g(t-\tau)h(\tau)d\tau \quad (8)$$

The most well-known convolution involves two normally distributed random variables where the sum of two normally distributed random variables is also a normally distributed random variable, with the resulting mean the sum of the means and the resulting variance the sum of the variances. One way to prove this is through the computation of the convolution. There are many derivations on this computation in published material (for one example, see Ref. 11). Even the convolution of two exponential random variables is not what they seem. Again, there are many examples in the literature (such as Wikipedia, ReliaWiki, or Ref. 12). The sum of two exponential random variables with failure rates λ_1 and λ_2 is not of the form

$$f(t) = \lambda e^{-\lambda t} \quad (9)$$

But is instead of the form

$$f(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad (10)$$

Lastly, the convolution of two Weibull distributions has no closed form solution, and needs to be done numerically. Without realizing how to properly do sums of probability distributions, engineers will produce meaningless results. Binning data points into bins, analyzing distributions, considering improvement factors, and then summing this all back into one distribution, through simple addition, is mathematically incorrect, and the more complex approach, as presented in this document, is necessary.

IV. CONCLUSIONS

In this paper we have shown an approach to calculating the contributions of failure modes to a higher level assembly using analysis techniques instead of simplistic techniques that fit an exponential distribution for convenience. The document has shown that with just a few failure modes, the differences are already large, and when considering a complex component with 10, 20 or even more failure modes, the differences between the methods will be extremely large.

The simplistic modeling approach presented in this document is one of the most common reliability engineering mistakes made. There is a difference between the sum of two distributions and their minimum. As the world is complex and does not behave like the exponential distribution, there is no mathematical basis for binning data points, developing separate distributions, sometimes different distributions, and summing up everything again. These simplistic methods lead to the wrong conclusions about product improvements, and can cause large financial mistakes to be made.

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