PROBABILISTIC SEISMIC RISK ASSESSMENT OF LIFELINE NETWORKS USING CROSS-ENTROPY-BASED ADAPTIVE IMPORTANCE SAMPLING

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Because of its complexity, reliability analysis of lifeline-network usually employs a sampling-based approach. Monte-Carlo simulation (MCS) provides a straightforward method to deal with interdependence between structural components and their cascading failures in the lifeline network system, but its computational cost might be expensive if the probability of the event of interest is too low. To overcome this issue, an adaptive importance sampling (AIS) method was recently developed to identify a near-optimal sampling density by minimizing Kullback–Leibler cross entropy (CE) measuring the difference between the best importance sampling (IS) density and the sampling density model in use. This cross-entropy-based adaptive importance sampling (CE-AIS) drastically improves efficiency of MCS by using the near-optimal sampling density. To facilitate its applications to probabilistic seismic risk assessment (PSRA) for lifeline-network, we propose a sophisticated sampling technique which is suitable to evaluate the probabilities of multiple network performance states caused by earthquake. The proposed method does not rely on any heuristic intuition to perform importance sampling, and concurrently obtains the probabilities of multiple post-disaster consequences of the lifeline network in a way that they converge in a speedy manner. The results of the numerical example demonstrate that our approach, termed as CE-based "concurrent" AIS (CE-CAIS) make the probabilities of multiple system events converge to the exact values evenly well in terms of the level of coefficients of variation of the estimates. The proposed method is expected to be useful for a variety of hazard risk assessment for complex systems and provide new insights into the simulation-based PSRA.

I. INTRODUCTION

Probabilistic evaluation of the seismic risk regarding critical infrastructure such as lifeline networks is crucial to prepare a proper post-hazard resilience plan for potential earthquake disasters. In particular, such evaluations help facilitate risk-informed decision-making regarding recovery plan and resource preparation at the community level. However, carrying out probabilistic seismic risk assessment (PSRA) regarding post-disaster performance of networks often encounters several technical challenges. As observed from many previous disasters, the damage of structural components may undermine the performance of the lifeline network. Therefore, the individual structural failures and their joint-occurrence need to be incorporated into system-level analysis. Since seismological properties at the sites and their spatially correlated ground-motion intensities induce a significant degree of uncertainties and complexity, the fragility calculations of structural components need to employ a probabilistic model of seismic hazard and consider the statistical dependence of the component failures. These complexities make PSRA of lifeline networks challenging or time-consuming.

Monte-Carlo Simulation (MCS) is a straightforward simulation-based approach often used for reliability analysis of complex systems. Although MCS is widely used for PSRA of lifeline networks, this methodology is not desirable if the probability of the event of interest is too low but still catastrophic to the urban community; in this case, a large number of simulations will be required to obtain a reliable estimation by MCS. As one of the research efforts to overcome this issue, the post-hazard analysis of traffic flow capacity was recently performed using a non-simulation-based method called, the matrix based system reliability (MSR) method[1]. As another non-simulation-based approach, a selective recursive decomposition algorithm was also developed to obtain narrow bounds on the reliabilities of disconnection between sources and sinks of water/gas distribution network[2]. A multi-scale analysis scheme using network clustering algorithm[3] was also developed to apply the approach in (Ref. 2) to larger-size networks. On the other hand, a technique was developed to use importance sampling (IS) and data reduction by shifting sampling densities and employing *k*-means clustering for traffic flow analysis[4]. Recently, an optimization-based approach was proposed to select scenario-maps which reduce the number of simulations and computational cost by introducing a proxy measure to depict the features of damaged network[5]. Although these research efforts significantly improved the efficacy of PSRA for lifeline networks, non-simulation-based approaches have intrinsic limitations in terms of the complexity of the lifeline network topology, size and inherent uncertainties while simulation-based approaches may be time-consuming or have uncertain level of confidence in the estimated results.

To address this critical research need, this paper proposes an alternative simulation-based approach for lifeline networks which may have multiple states after the earthquake event, e.g. traffic flow capacity under seismic hazard. For this purpose, the authors extend and modify the cross-entropy-based adaptive IS (CE-AIS)[6-8]. This adaptive IS based method provides a near-optimal IS density not relying on any heuristic judgement while drastically reducing the number of samples required to obtain reliable estimates on the probabilities. In particular, the proposed method can obtain reliabilities of multiple network state scenarios at once, therefore, termed as CE-based "concurrent" AIS (CE-CAIS). A numerical example will demonstrate robustness and applicability of the proposed approach.

II. NETWORK SYSTEM ANALYSES

Under seismic hazard, the fragility of a road component, e.g. bridge or tunnel, is defined as the conditional probability of exceedance over the damage state DS_i given the value of ground motion intensity. For an example network illustrated in Fig. 1(a), suppose bridges are the only road components that can affect the post-disaster performance of the network. For example, a typical fragility model used for bridge is expressed as[9]

$$P(damage \geq DS_i \mid Sa) = \Phi\left(\frac{\ln Sa - \ln \overline{Sa_{DS,i}}}{\beta_{DS,i}}\right) \tag{1}$$

where Sa is a ground-motion intensity called, spectral acceleration at the location of the structure, $\overline{Sa_{ds,i}}$ is the median value of spectral acceleration at which the bridge reaches the threshold of the damage state DS_i, $\beta_{DS,i}$ is the standard deviation of the natural logarithm of the spectral acceleration of damage state DS_i, and $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution.

Then, the conditional probability of being in the i-th damage state DS_i is computed as

$$P(damage = DS_i \mid Sa) = \Phi\left(\frac{\ln Sa - \ln \overline{Sa_{DS,i}}}{\beta_{DS,i}}\right) - \Phi\left(\frac{\ln Sa - \ln \overline{Sa_{DS,i+1}}}{\beta_{DS,i+1}}\right) \tag{2}$$

where DS_{i+1} denotes the $(i+1)^{\text{th}}$ damage state. For instance, in the fragility function example shown in Fig. 1(b), the conditional probability that the structure is in the "moderate" damage state (DS_3) is the difference between the probabilities exceeding the "moderate" damage state (DS_3) and the "extensive" damage state (DS_4).

From the total probability theorem, the marginal probability of being in the i^{th} damage state of the network component is expressed as

$$P(damage = DS_i) = \int P(damage = DS_i \mid Sa)\, f_{Sa}(Sa)\, dSa \tag{3}$$

where $f_{Sa}(\cdot)$ denotes the probabilistic density function of Sa at the site. Extending the component-level fragilities to the network performance, the probability that the system shows a performance level SS is

$$\begin{aligned} &P(system = SS) = \int P(system = SS \mid \mathbf{Sa})\, f_{\mathbf{Sa}}(\mathbf{Sa})\, d\mathbf{Sa} \\ &\text{where } P(system = SS \mid \mathbf{Sa}) = \sum_{\forall \mathbf{DS} \in \{\mathbf{DS} \mid G(\mathbf{DS}) = SS\}} P(\mathbf{damage} = \mathbf{DS} \mid \mathbf{Sa}) \end{aligned} \tag{4}$$

where **damage** and **Sa** are vectors of the possible damage states of the geographically distributed network-components and ground-motion intensities at the sites, respectively. In Eq. (4), $G(\cdot)$ represents a function that yields the system state that corresponds to the given component damage state. Fig. 1(a) illustrates a hypothetical traffic network revised from those in Ref. 1,10,11 while Fig. 1(b) shows the fragility curves of Bridge 1 as an example; the combinations of these component-level conditional probabilities are used to evaluate a seismic risk of lifeline network.

In this example, **Sa** in Eq. (4) can be considered as common source random variables (CSRV)[1,12-14] representing the sources of 'environmental dependence' or 'common source effects'. Thus, one can achieve conditional independence between component-level damage events through given outcomes of random variables **Sa**. The matrix-based system reliability (MSR) analysis approach[1,12-14] provides an efficient way to construct the vector of conditional probabilities for systemic event in Eq. (4) with simple vector operations.

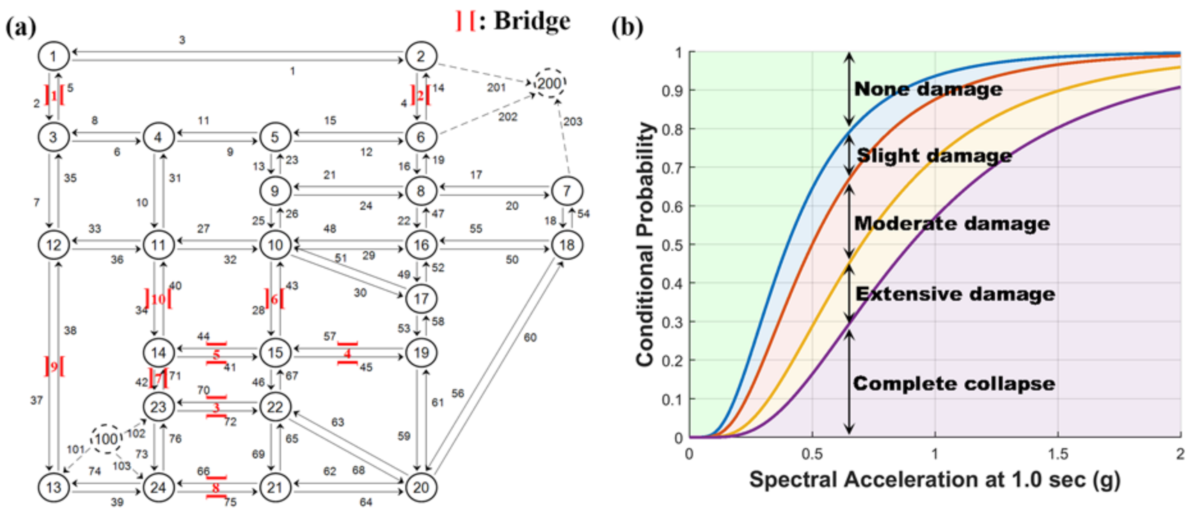

Fig. 1. (a) A hypothetical traffic network used in the numerical example; and (b) the fragility curves of Bridge 1.

III. STOCHASTIC MODELING OF GROUND-MOTION INTENSITIES IN UNCORRELATED STANDARD NORMAL SPACE

For a probabilistic seismic hazard analysis, the ground-motion intensities at the sites are often predicted using so-called ground motion prediction equations (GMPE). A general form of GMPE is given as

$$\ln Y_{kl} = f(M_l, R_l, \boldsymbol{\lambda}_k) + \sigma_{kl}\varepsilon_{kl} + \tau_l \eta_l \tag{5}$$

where Y_{kl} is the chosen ground-motion intensity at site k for the earthquake event l, and $f(M_l, R_{kl}, \boldsymbol{\lambda}_l)$ is the estimated mean value of Y_{kl} by GMPE given as a function of the magnitude M_l of event l, a seismological distance R_{kl} to site k in earthquake l, and other explanatory variables $\boldsymbol{\lambda}_k$ assigned at site k. In the same equation, ε_{kl} is the intra-event residual to represent site-to-site uncertainty within the same event l, η_l is the inter-event residual to represent event-to-event uncertainty shared by all sites, and σ_{kl} and τ_l are deterministic values to represent the standard deviations of the intra-event and inter-event residuals, respectively. While η_l is constant for all sites, ε_{kl} varies from site to site, and shows statistical dependence for pairs of nearby sites. A correlation coefficient matrix is composed with a selected auto-correlation model that represents the so-called spatial correlation, often described as a function of distance between sites[15,16]. To facilitate the application of CE-AIS to PSRA, the random variables in (5) are transformed to the uncorrelated standard normal space, called "u-space"[17]. As a result, the probability of being in a certain system state (SS) is described as

$$P(SS) = \int P(SS \mid \mathbf{Sa})\, f_{\mathbf{Sa}}(\mathbf{Sa})\, d\mathbf{Sa} = \int P(SS \mid \mathbf{u})\, f_{\mathbf{u}}(\mathbf{u})\, d\mathbf{u} \tag{6}$$

where $f_{\mathbf{u}}(\mathbf{u})$ denotes the joint probability density function of uncorrelated standard normal random variables.

Fig. 2(a) shows 9 hypothetical active faults threatening the example network in Fig. 1(a). The geometric and seismic properties are adopted from (Ref. 18). In Fig. 2(b), 500 earthquake events are generated using the u-space, and their moment magnitudes and locations of hypocenters are shown; this illustration is to visually check whether our sampling in u-space is valid.

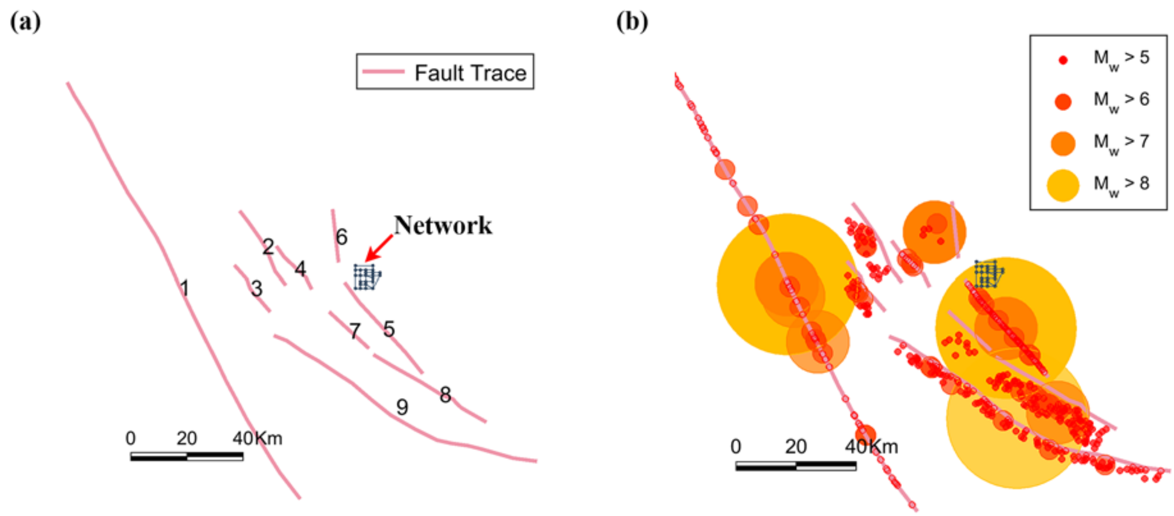

Fig. 2. (a) 9 active faults threatening the hypothetical traffic network; and (b) 500 artificial earthquake events generated using the hazard model in the uncorrelated standard normal space.

IV. CONCURRENT ADAPTIVE IMPORTANCE SAMPLING

For the risk assessment of the network system which has multiple post-hazard states, a vector of the probabilities needs to be evaluated instead of a scalar shown in (6). Using Monte Carlo simulation (MCS) approach, the vector of the probabilities can be estimated as

$$\mathbf{P} = \begin{Bmatrix} P(SS_1) \\ \vdots \\ P(SS_O) \end{Bmatrix} = \int \begin{Bmatrix} P(SS_1 \mid \mathbf{u}) \\ \vdots \\ P(SS_O \mid \mathbf{u}) \end{Bmatrix} f_{\mathbf{u}}(\mathbf{u}) d\mathbf{u} \approx \frac{1}{N} \sum_{n=1}^{N} \begin{Bmatrix} P(SS_1 \mid \mathbf{u}_n) \\ \vdots \\ P(SS_o \mid \mathbf{u}_n) \end{Bmatrix} \tag{7}$$

where O is the number of the system states, and N is the number of the generated samples.

Importance sampling (IS) aims to improve the efficiency of MCS by employing an alternative sampling density $h(\mathbf{u};\mathbf{v})$ with the distribution parameters $\mathbf{v}$, i.e.

$$\int \begin{Bmatrix} P(SS_1 \mid \mathbf{u}) \\ \vdots \\ P(SS_K \mid \mathbf{u}) \end{Bmatrix} f_{\mathbf{u}}(\mathbf{u}) d\mathbf{u} = \int \left[\begin{Bmatrix} P(SS_1 \mid \mathbf{u}) \\ \vdots \\ P(SS_K \mid \mathbf{u}) \end{Bmatrix} \frac{f_{\mathbf{u}}(\mathbf{u})}{h(\mathbf{u};\mathbf{v})} \right] h(\mathbf{u};\mathbf{v}) d\mathbf{u} \tag{8}$$

To evaluate the integral in (8) with a simulation-based approach, it is desired that the multiple probabilities of post-hazard events are obtained concurrently so as to guarantee convergence evenly well for all possible states. A disadvantage of using MCS in this case is that a large number of samples are required to secure the reliable estimate of rare but still destructive scenarios while probable but less harmful one converges quickly. Implementation of conventional IS is also problematic in such a situation because the optimal sampling densities may be significantly different depending on scenarios. This study proposes a concurrent adaptive importance sampling (CAIS) method to overcome these limitations especially inherent in PSRA of lifeline networks. This technique helps identify a near-optimal IS density for the entire set of post-disaster utilities of the system.

In the proposed approach, the optimal IS density function for (8) is defined as to minimize the sum of the squares of the coefficient of variations (c.o.v) of the estimates in $\mathbf{P}$. Because each element in $\mathbf{P}$ can have a different level of the likelihood, the c.o.v is hereby used instead of the variance, which is a proper measure for a single value integral[19]. Based on this definition, the optimal density for concurrent IS, $h^*(\mathbf{u})$ is now derived as

$$h^*(\mathbf{u}) = \frac{\mathcal{P}(\mathbf{u}) f_{\mathbf{u}}(\mathbf{u})}{\int \mathcal{P}(\mathbf{u}) f_{\mathbf{u}}(\mathbf{u}) d\mathbf{x}} \quad \text{where} \quad \mathcal{P}(\mathbf{u}) = \sqrt{\sum_{o=1}^{O} \left[w_o P(SS_o \mid \mathbf{u}) \right]^2} \qquad (9)$$

where w_o is the o-th element of the unit vector proportional to the vector including the inverses of the estimated $P(SS_o)$ for o=1, 2, …, O. Although the exact evaluation of w_o is not possible, reasonable estimates on those weights are obtainable from a small number of pre-samples; a generated ground-motion scenario provides conditional probabilities of systemic events in this case.

V. CROSS-ENTRPOY BASED CONCURRENT ADATIVE SAMPING (CE-CAIS)

The theoretically optimal IS density function $h^*(\mathbf{u})$ is derived as (9), but this form is still impractical because it is required to evaluate the exact value of **P**. However, it is possible to obtain a near-optimal IS density by using an adaptive IS method, i.e. by minimizing the Kullback-Leibler cross-entropy (CE) – a measure of the difference between the optimal density $h^*(\mathbf{u})$ and the IS density model $h(\mathbf{u}; \mathbf{v})$. Through a few rounds of pre-sampling, the density model parameters estimated from the previous round, denoted by **t**, are updated to **v** such that the estimated CE between $h^*(\mathbf{u})$ and the sampling density $h(\mathbf{u}; \mathbf{v})$ is minimized.

By applying CE-AIS to the optimal density derivation in (9), a near-optimal IS density is found such that the following condition is satisfied:

$$\frac{1}{N} \sum_{i=1}^{N} \mathcal{P}(\mathbf{u}_i) W(\mathbf{u}_i; \mathbf{t}) \nabla_{\mathbf{v}} h(\mathbf{u}_i; \mathbf{v}) = 0 \qquad (10)$$

where $W(\mathbf{u};\mathbf{t})$ is the so-called likelihood ratio, and $\nabla_{\mathbf{v}} h(\mathbf{u}; \mathbf{v})$ denotes partial derivatives of IS density model $h(\cdot)$ with respect to distribution parameter **v**.

This study uses the closed-form updating rules recently derived for a CE-AIS approach[7] that employs a nonparametrc multimodal distribution model called the Gaussian mixture (GM). The results of numerical tests confirmed that CE-AIS-GM is not sensitive to the level of probability or nonlinearity of limit-state surfaces while drastrically improving the efficiency of the simulation. Such a nonparameteric model based CE-AIS approach was recently extended to high dimensional reliability problems by employing the von Mises–Fisher mixture density model instead of a Gaussian mixture[8].

VI. NUMERICAL EXAMPLE: A HYPOTHETICAL NETWORK

As a numerical example, let us consider a hypothetical traffic network surrounded by 9 active faults (Fig. 1(a) and 2(a)). For the sake of simplicity, it is assumed that the damage of bridges is the only factor that may reduce the traffic capacity of associated links after an earthquake event; this eventually decreases the entire traffic flow capacity between the source and terminal nodes. The bridge fragility models in (Ref. 9) are used, and the bridge types were randomly selected. Each link capacity is assumed to be decreased as 100%, 75% and 50% of the full capacity for "none or slight", "moderate or extensive", and "collapse" damage states, respectively, except for bridge 6, 10 for which the reduced capacity is 100%, 75% , 50%, 25%, and 0% for each of the five separate states. The spectral accelerations at the natural period 1.0 sec are used to compute the conditional probability of damage as in (Ref. 9); Magnitude-scaling relationship in (Ref. 20) with parameters used in appendix *E* of (Ref. 17), and GMPE in (Ref. 21) are adopted in this example. The post-disaster reduction of the flow capacity between nodes {13, 23, 24} to nodes {2, 6, 7} after an earthquake event is of concern. Two sets of subjunctive nodes and links (dashed lines) are introduced to handle this multi-sources/sinks problem; node 100 is assigned as a master source to nodes {13, 23, 24}, and node 200 as a mater sink from {2, 6, 7} in this example.

The whole procedure of PSRA of this network is summarized as below.

- Step 1: Perform deterministic maximum flow analyses to identify possible network utility scenarios and their corresponding component damage combinations
- Step 2: Perform the initial pre-sampling to estimate the weights w_o in (9)
- Step 3: Update IS density model using CE-AIS-GM with respect to (9) (CE-CAIS-GM)
- Step 4: Repeat Step 3 until converged
- Step 5: Final sampling using the near-optimal IS density found in Step 4

Table I. Number of samples required to achieve target c.o.v 0.01

	Brute-force MCS	CE-AIS-GM	CE-CAIS-GM
Pre-sampling	-	61,000	7,000
Final-sampling	5,781,842	136,780	40,896
Total	5,781,842	197,780	47,896

In Step 1, a total of $3^8 \times 5^2$=164,025 combinations of damage states of 10 bridges result in only 11 possible network flow capacity values, i.e. O = 11 in (7). In Steps 2-4, a total of 7,000 pre-samples were enough to find a near-optimal IS density for overall convergence of the probabilities of 11 states – 1,000 samples in Step 2, and 1,000 samples for each of 5 updates in Step 3 and 4. Setting our target c.o.v as 0.01, Table I shows that convergence was achieved using only 47,896 samples while brute-force MCS demanded 5,781,842 samples. As one can check in Table II, this superb improvement in efficiency does not hamper the accuracy of the simulation. Since the original CE-AIS approach does not provide any rule of concurrent sampling, each 11 state was investigated separately to test its efficiency (See Table III).

TABLE II. Estimated probabilities of reduced network traffic capacity

Max flow (veh/hr)	12,600	12,300	12,150	12,000	11,850	11,700	11,400	11,100
Brute-force MCS	0.148	2.668×10^{-5}	2.465×10^{-4}	1.132×10^{-5}	0.615×10^{-5}	1.819×10^{-4}	0.659×10^{-5}	1.630×10^{-5}
CE-AIS-GM	0.151	2.717×10^{-5}	2.508×10^{-4}	1.141×10^{-5}	0.612×10^{-5}	1.845×10^{-4}	0.665×10^{-5}	1.647×10^{-5}
CE-CAIS-GM	0.148	2.647×10^{-5}	2.403×10^{-4}	1.130×10^{-5}	0.612×10^{-5}	1.802×10^{-4}	0.658×10^{-5}	1.628×10^{-5}
Max flow (veh/hr)	10,800	10,500	10,200					
Brute-force MCS	5.717×10^{-5}	0.284×10^{-5}	1.734×10^{-5}					
CE-AIS-GM	5.872×10^{-5}	0.289×10^{-5}	1.732×10^{-5}					
CE-CAIS-GM	5.717×10^{-5}	0.284×10^{-5}	1.727×10^{-5}					

Table III. Number of samples to implement the original CE-AIS for each systemic event with the same target c.o.v 0.01

Max flow (veh/hr)	12,600	12,300	12,150	12,000	11,850	11,700	11,400	11,100	10,800	10,500	10,200	Total
Pre-sampling	1,000	6,000	5,000	8,000	5,000	5,000	6,000	7,000	6,000	7,000	6,000	61,000
Final-sampling	-	12,355	5,463	10,299	19,134	7,036	10,680	13,374	8,335	16,063	34,041	136,780
Total	1,000	18,355	10,463	18,299	24,134	12,036	16,680	20,374	14,335	23,063	40,041	197,780

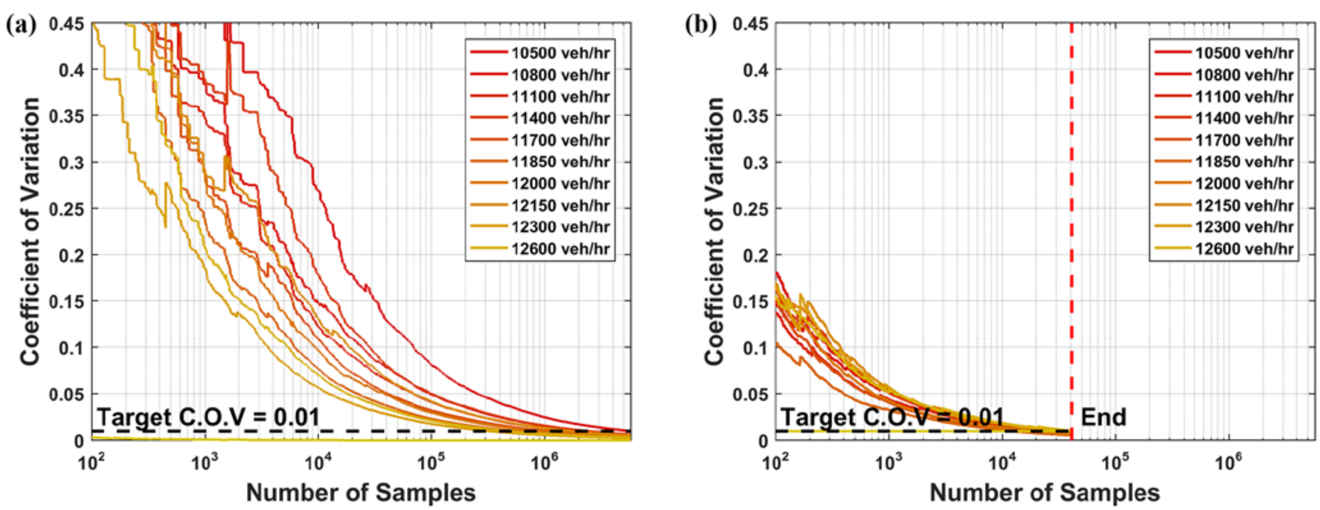

Fig. 3. (a) Coefficient of variation (c.o.v) with the number of samples when using brute-force MCS; and (b) c.o.v with the number of samples during the final sampling in CE-based concurrent AIS (CE-CAIS); when using CE-CAIS, the number of samples required to achieve the same target c.o.v 0.01 is much less than that of MCS (47,896 vs the 5,781,842). In addition, c.o.v values by CE-CAIS descend concurrently when compared with those with MCS.

As shown in the Table I, CE-CAIS achieves the same target c.o.v. with only 0.8% of the simulations by brute-force MCS and 24% of CE-AIS. Table II also confirms that the estimates converge to the same probabilities. Fig. 3(a) shows the convergence histories of the brute-force MCS while Fig. 3(b) provides those of the proposed CE-CAIS. The plots confirm that CE-CAIS enables rapid and collective convergence among the multiple states although the levels of probabilities show great variability. This example demonstrates that the CE-CAIS approach provide powerful means for PSRA of lifeline network, and in particular, facilitates application of IS methodology to network problems having multiple post-disaster scenarios.

VII. CONCLUSIONS

A new simulation-based approach is proposed for probabilistic seismic risk assessment (PSRA) of post-hazard flow capacity of traffic network. The new approach, termed as cross-entropy-based concurrent adaptive importance sampling (CE-CAIS), improves the efficiency of the sampling method when the reliability of multiple states of the system has to be investigated. Originated from CE-AIS, the proposed approach does not require any subjective judgement, assumption or additional reliability analysis to identify a near-optimal density to perform efficient sampling. The great advantage of using CE-CAIS is that this method provides a near optimal sampling density which would give fast convergence for all possible states in the system while the original CE-AIS needs pre-sampling to identify a near-optimal sampling density for each network state. This feature makes the proposed approach superior to the original CE-AIS approach for network-level PSRA because numerous outcomes of the network performance often exist under seismic hazards. Numerical examples showed superb efficiency of CE-CAIS compared to brute-force MCS and CE-AIS. Using only a small number of pre-samples, CE-CAIS found a near-optimal IS density which gives superb performance for PSRA of lifeline network while not relying on any subjective knowledge to identify IS density model. It is expected that the proposed approach will provide a scalable PSRA framework especially for complex infrastructure systems which may exhibit many possible post-disaster states.

ACKNOWLEDGMENTS

This research was supported by Integrated Research Institute of Construction and Environmental Engineering at Seoul National University, and the National Research Foundation of Korea (NRF) Grant (No.2015R1A5A7037372), funded by the Korean Government (MSIP).

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