A NON-PROBABILISTIC APPROACH FOR LEVEL-2 UNCERTAINTY PROPAGATION BASED ON UNCERTAINTY THEORY

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Level-2 uncertainty propagation separates aleatory uncertainty and epistemic uncertainty and therefore receives significant interests in risk assessment. In this paper, a novel approach is developed for level-2 uncertainty propagation based on uncertainty theory. In the developed approach, aleatory uncertainty is characterized by probability distributions and epistemic uncertainty affecting the model parameters is described by uncertain variables. Therefore, the probability of interest becomes an uncertain measure in uncertainty theory. The uncertain distribution of the probability of interest is, then, calculated based on the operation law of uncertain variables or uncertain simulation method. Several metrics, e.g., the average probability, the quantile probability, etc., are defined and calculated to consider both types of uncertainty and assess the risk. The developed approaches are implemented in a benchmark case study of flood risk assessment. The results are compared to those that are obtained from some commonly-used level-2 uncertainty propagation methods, e.g., probability-based method, evidence theory, etc., in order to highlight the strength and weakness of the developed approach.

I. INTRODUCTION

Risk has two main dimensions, consequences and uncertainty, and a risk description is obtained by specifying the consequences and using a description of the uncertainty [1]. Thus, uncertainty analysis is one of the most important part of risk analysis. Broadly speaking, uncertainty can be categorized as aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty refers to the uncertainty inherently exist in the physical behavior of a system, and epistemic uncertainty refers to the uncertainty caused by our lack of knowledge [2]. In literatures, the separation of aleatory and epistemic uncertainty is encouraged [3, 4].

Uncertainty analysis include two main steps: uncertainty modeling and uncertainty propagation. Uncertainty modeling means we need to choose a proper mathematical representation, which may be probabilistic or non-probabilistic, to describe the uncertainty of input parameters. Commonly, we use probability theory to model aleatory uncertainty, and epistemic uncertainty is described by Bayesian probability, evidence theory, fuzzy theory, etc [5]. Once the uncertainty has been modeled, it must be propagated through the models used in the risk assessment. Depending on the type of uncertainty affecting the model input quantities, methods for uncertainty propagation can be classified into level-1 and level-2 methods [6]. For a level-1 uncertainty propagation setting, the input parameters are divided into two groups, i.e., one group is subject to aleatory uncertainty and the other is subject only to epistemic uncertainty. A level-2 uncertainty propagation setting will be used when the input parameters are subject to aleatory uncertainty described by probability theory with distribution parameters subject to epistemic uncertainty.

Level-2 uncertainty propagation in risk assessment has drawn a lot of interests these years. Aven et al. [1] introduced level-2 uncertainty propagation method in detail. They gave the algorithms for level-2 pure probabilistic uncertainty propagation and level-2 probabilistic-evidence theory uncertainty propagation, and both of the algorithms performed a two-level Monte Carlo simulation. Limbourg and Rocquigny [6] applied evidence theory to level-1 and level-2 uncertainty propagation, and give some difficulties and challenges when using level-2 uncertainty settings. A new benchmark risk assessment problem was also proposed to illustrate the evidence-theory-based method. Limbourg et al. developed an accelerate level-2 uncertainty propagation method, where the two-level Monte Carlo simulation is simplified by Monotonous Reliability Method (MRM), considering the monotonous properties of the model. Baraldi et al. [7] introduced hybrid Monte Carlo-evidence theory uncertainty propagation method in maintenance policy performance assessment.

As pointed out in Ref. 8, uncertainty theory founded by Liu [9] can be used to describe the belief degree of events affected by epistemic uncertainty. Therefore, in this paper, we use uncertainty theory to model the level-2 uncertainty parameters, and a novel level-2 uncertainty propagation method based on uncertainty theory is developed. We will also define some metrics, such as average probability and quantile probability, to further assess the risk.

The remainder of this paper is structured as follows: Section II introduced some basic concepts about uncertainty theory. In Section III, level-1 and level-2 uncertainty models are first reviewed, and level-2 uncertainty propagation method for monotone risk index is then developed based on operation laws of uncertainty theory. Then the uncertain simulation method for level-2 uncertainty propagation is proposed. In Section IV, the developed approaches are implemented in a benchmark case study of flood risk assessment, and the results are compared to propagation methods based on evidence theory.

II. PRELIMINARY ON UNCERTAINTY THEORY

Uncertainty theory is founded by Liu [9] in 2007 and refined by Liu [10] in 2010 as a branch of mathematics for modeling uncertainties. A new measure, called uncertain measure, is defined in uncertainty theory based on the following four axioms[9]:

Axiom 1. (Normality Axiom) $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ.

Axiom 2. (Duality Axiom) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ.

Axiom 3. (Subadditivity Axiom) For every countable sequence of events $\Lambda_1,\Lambda_2,\ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Axiom 4. (Product Axiom) Let $(\Gamma_k,L_k,\mathcal{M}_k)$ be uncertainty spaces for $k=1,2,\ldots$. The product uncertainty measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\}=\bigwedge_{k=1}^{\infty}\mathcal{M}\{\Lambda_k\},$$

where L_k are σ-algebras over Γ_k, and Λ_k are arbitrarily chosen events from L_k for $k=1,2,\ldots$, respectively.

As an extension of Axiom 4, Liu gives the uncertain measure of any events in the uncertainty space $(\Gamma,L,\mathcal{M})$, based on maximum uncertainty principle, i.e., if there are multiple reasonable values that an uncertain measure may take, then the value as close to 0.5 as possible is assigned to the event [11]. Therefore, for each event $\Lambda\in L$, we have

$$\mathcal{M}\{\Lambda\}=\begin{cases}\sup\limits_{\Lambda_1\times\Lambda_2\times\ldots\subset\Lambda}\min\limits_{1\le k<\infty}\mathcal{M}_k\{\Lambda_k\}, & if\ \sup\limits_{\Lambda_1\times\Lambda_2\times\ldots\subset\Lambda}\min\limits_{1\le k<\infty}\mathcal{M}_k\{\Lambda_k\}>0.5\\ 1-\sup\limits_{\Lambda_1\times\Lambda_2\times\ldots\subset\Lambda^c}\min\limits_{1\le k<\infty}\mathcal{M}_k\{\Lambda_k\}, & if\ \sup\limits_{\Lambda_1\times\Lambda_2\times\ldots\subset\Lambda^c}\min\limits_{1\le k<\infty}\mathcal{M}_k\{\Lambda_k\}>0.5\\ 0.5, & otherwise\end{cases}\tag{1}$$

According to Liu [11], a measurable function ξ from an uncertainty space $(\Gamma,L,\mathcal{M})$ to the set of real numbers is called an uncertain variable. Thus, the uncertainty distribution can be defined by $\Phi(x)=\mathcal{M}\{\xi\le x\}$ for any real number x. For example, a linear uncertain variable $\xi\sim\mathcal{L}(a,b)$ has an uncertainty distribution

$$\Phi_1(x)=\begin{cases}0, & if\ x<a\\ (x-a)/(b-a), & if\ a\le x\le b,\\ 1, & if\ x>b\end{cases}\tag{2}$$

and a normal uncertain variable $\xi\sim\mathcal{N}(e,\sigma)$ has an uncertainty distribution

$$\Phi_2(x)=\left(1+\exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1},\ x\in\Re.\tag{3}$$

An uncertainty distribution Φ is said to be regular if it is a continuous and strictly increasing with respect to x at which $0<\Phi(x)<1$, additionally $\lim\limits_{x\to-\infty}\Phi(x)=0$ and $\lim\limits_{x\to+\infty}\Phi(x)=1$. A regular uncertainty distribution has an inverse uncertainty distribution, denoted as $\Phi^{-1}(\alpha)$, for $\alpha\in(0,1)$. It is clear that linear uncertain variables and normal uncertain variables are regular, and their inverse uncertainty distributions are (4) and (5), respectively.

$$\Phi_1^{-1}(\alpha)=(1-\alpha)a+\alpha b, \tag{4}$$

$$\Phi_2^{-1}(\alpha)=e+\frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} \tag{5}$$

To calculate the uncertain distribution of function of uncertain variables, Liu gives an operational law (Theorem 1) for strictly monotone functions.

Theorem 1. [11] Let $\xi_1,\xi_2,...,\xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1,\Phi_2,...,\Phi_n$, respectively. If $f(\xi_1,\xi_2,...,\xi_n)$ is strictly increasing with respect to $\xi_1,\xi_2,...,\xi_m$ and strictly decreasing with respect to $\xi_{m+1},\xi_{m+2},...,\xi_n$, then $\xi=f(\xi_1,\xi_2,...,\xi_n)$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha)=f\left(\Phi_1^{-1}(\alpha),...,\Phi_m^{-1}(\alpha),\Phi_m^{-1}(1-\alpha),...,\Phi_n^{-1}(1-\alpha)\right).$$

III. LEVEL-2 UNCERTAINTY PROPAGATION BASED ON UNCERTAINTY THEORY

In this section, we first introduce level-1 and level-2 uncertainty models in risk analysis in subsection III.A. In III.B, we develop a level-2 uncertainty propagation method for monotone interested index, and a simple example is given to illustrate the method. Considering the non-monotone situations, a level-2 uncertainty propagation method based on uncertain simulation is proposed in subsection III.C.

III.A. Level-1 and level-2 uncertainty models in risk analysis

In probabilistic risk analysis, the system output can usually be modeled by

$$z=g(\mathbf{x}), \tag{6}$$

where g is the system model and $\mathbf{x}=(x_1,x_2,...,x_n)$ denotes the vector of input parameters [1]. We are interested in the probability that z exceeds its threshold z_{th}, which is denoted by $p=\Pr\{z>z_{th}\}$.

In level-1 uncertainty models, the input vector can be separated into $\mathbf{x}=(\mathbf{a},\mathbf{e})$. Variables $\mathbf{a}=(x_1,x_2,...,x_m)$ represent parameters affected only by aleatory uncertainty, and are usually modeled by probability theory, i.e., the probability density function (pdf) $f(x_i,\theta_i), i=1,2,...,m$, where θ_i is deterministic parameter of the pdf. Other variables $\mathbf{e}=(x_{m+1},x_{m+2},...,x_n)$ represent the parameters that are not subject to any kind of random variation, and are only affected by epistemic uncertainty due to lack of knowledge [6]. Different paradigms can be used to describe $\mathbf{e}$, such as evidence theory and fuzzy theory. For example, if evidence theory is preferred, then BPAs can be obtained to represent epistemic uncertainty, and if we choose fuzzy theory, the membership function will be used. Under level-1 settings, Monte Carlo simulation methods or fuzzy extension theorem is usually used to propagate uncertainty. Through the propagation method, the interval of p can be calculated.

Level-2 uncertainty models are derived based on level-1 uncertainty models. In these models, parameters which determine the aleatory uncertainty model are subject to epistemic uncertainty. Thus, θ_i s in pdf $f(x_i,\theta_i)$ are no longer deterministic due to our lack of knowledge. In literatures, evidence theory is preferred to describe this kind of parameters, and two-levels Monte Carlo simulation is used for level-2 uncertainty propagation [6].

III.B. Level-2 uncertainty propagation method for monotone interested index

In this paper, we use uncertainty theory to describe the level-2 epistemically uncertain parameters. Let Φ_i be uncertainty distributions of $\theta_i, i=1,2,...,n$, and assume there is no level-1 epistemic uncertainty, and the interested probability can be explicitly expressed as

$$p=h(\Theta), \tag{7}$$

where $\Theta=(\theta_1,\theta_2,...,\theta_n)$ denotes the vector of level-2 uncertain parameters. If h is strictly monotone with respect to Θ, the uncertainty distribution of p can be obtained through Theorem 1 as follows:

$$\Psi_p^{-1}(\alpha)=h\left(\Phi_1^{-1}(\alpha),...,\Phi_m^{-1}(\alpha),\Phi_m^{-1}(1-\alpha),...,\Phi_n^{-1}(1-\alpha)\right). \tag{8}$$

Take a simple fault tree (shown in Fig. 1) for example. We are interested in the probability of occurrence of top event A at time t_0. Assume that the failure time of B_1 and B_2 follow exponential distributions with parameters λ_1 and λ_2, respectively. Therefore, the probability can be calculated as:

$$p=1-e^{-(\lambda_1+\lambda_2)t_0} \tag{9}$$

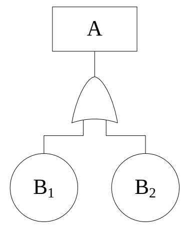

Fig. 1. A simple fault tree for risk analysis

Due to the effect of epistemic uncertainty, we cannot get precise values of λ_1 and λ_2. Thus we assume the two parameters are all linear uncertain parameters with uncertainty distribution given in (2), that is, $\lambda_1 \sim \mathcal{L}(a_1,b_1)$ and $\lambda_2 \sim \mathcal{L}(a_2,b_2)$. According to Theorem 1, there is

$$\Psi_p^{-1}(\alpha)=1-\exp\left[-(1-\alpha)(a_1+a_2)t_0+\alpha(b_1+b_2)t_0\right],$$

which indicates that the uncertainty distribution of p is

$$\Psi(p)=\frac{-\frac{1}{t_0}\ln(1-p)-(a_1+a_2)}{(b_1+b_2)-(a_1+a_2)}. \tag{10}$$

III.C. Uncertain simulation method for level-2 uncertainty propagation

In many situations, we cannot get a strictly monotone interested index with respect to the level-2 uncertain parameters. Therefore, an uncertain simulation method is introduced in this subsection to propagate level-2 uncertainty.

Uncertain simulation method is proposed in Ref. 12 by Zhu, to calculate an appropriate uncertain measure of non-monotone functions of uncertain variables. This method requires the corresponding uncertain vector $\boldsymbol{\xi}$ is common. The definition of common is given in Definition 1.

Definition 1 [12]. An uncertain variable ξ is common if it is from the uncertain space $(\Re,\mathcal{B},\mathcal{M})$ to $\Re$ defined by $\xi(\gamma)=\gamma$, where $\mathcal{B}$ is the Borel algebra over $\Re$. An uncertain vector $\boldsymbol{\xi}=(\xi_1,\xi_2,...,\xi_n)$ is common if all the elements of $\boldsymbol{\xi}$ are common.

Based on (1), Zhu gives a theorem as a basis for uncertain simulation method.

Theorem 2. [12] Let $f:\Re^n\to\Re$ be a Borel function, and $\boldsymbol{\xi}=(\xi_1,\xi_2,...,\xi_n)$ be a common uncertain vector. Then the uncertainty distribution of f is:

$$\begin{aligned}\Psi(x)&=\mathcal{M}\{f(\xi_1,\xi_2,...,\xi_n)\le x\}\\&=\begin{cases}\sup\limits_{\Lambda_1\times\Lambda_2\times...\times\Lambda_n\subset\Lambda}\min\limits_{1\le k\le n}\mathcal{M}_k\{\Lambda_k\}, & if \sup\limits_{\Lambda_1\times\Lambda_2\times...\times\Lambda_n\subset\Lambda}\min\limits_{1\le k\le n}\mathcal{M}_k\{\Lambda_k\}>0.5\\ 1-\sup\limits_{\Lambda_1\times\Lambda_2\times...\times\Lambda_n\subset\Lambda^c}\min\limits_{1\le k\le n}\mathcal{M}_k\{\Lambda_k\}, & if \sup\limits_{\Lambda_1\times\Lambda_2\times...\times\Lambda_n\subset\Lambda^c}\min\limits_{1\le k\le n}\mathcal{M}_k\{\Lambda_k\}>0.5.\\ 0.5, & otherwise\end{cases}\end{aligned} \tag{11}$$

In (11) $\Lambda=f^{-1}(-\infty,x)$, $\{A_i\}$ denotes a collection of all intervals of the form $(-\infty,a]$, $[b,+\infty)$, $\varnothing$ and $\Re$, and each $\mathcal{M}_k\{\Lambda_k\}$ is derived by (12):

$$\mathcal{M}\{B\}=\begin{cases}\inf\limits_{B\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}, & \inf\limits_{B\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}<0.5\\ 1-\inf\limits_{B^c\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}, & \inf\limits_{B^c\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}<0.5\\ 0.5, & otherwise\end{cases} \tag{12}$$

where $B\in\mathcal{B}$, and $B\subset\bigcup\limits_{i=1}^{\infty}A_i$.

According to this theorem, an numerical algorithm is given in Ref. 12. Through the algorithm, we can theoretically calculate the uncertain measure $\mathcal{M}\{p=c\}$, where p is the interested probability and c is a constant. When c ranges from the minimum of p to the maximum of p, the uncertainty distribution $\Psi(p)$ can be obtained. However, since the uncertain measure given in this theorem satisfies the maximum uncertainty principle, the uncertain measures of p at many possible values may be assigned to 0.5 due to the sever uncertainty in level-2 settings. Thus, the bounded uncertainty distribution may be more proper to describe the uncertainty of p.

Actually, (12) gives the bound of each $\mathcal{M}_k\{\Lambda_k\}$ in (11). Let $m=\inf\limits_{B\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}$, $n=\inf\limits_{B^c\subset\cup A_i}\sum\limits_{i=1}^{\infty}\mathcal{M}\{A_i\}$. It's clear that m and $1-n$ are both reasonable value that $\mathcal{M}\{B\}$ may take. Noting that $m>1-n$ for any $B\in\mathcal{B}$, we can use m and $1-n$ as the upper and lower bounds of $\mathcal{M}_k\{\Lambda_k\}$, respectively. Therefore, a numerical algorithm is developed to calculate upper bound L_U and lower bound L_L of $\mathcal{M}\{p=c\}$.

Algorithm 1.

Step 1. Set $m_1(i)=0$ and $m_2(i)=0$, $i=1,2,...,n$.

Step 2. Randomly generate $u_k=\left(\gamma_k^{(1)},\gamma_k^{(2)},...,\gamma_k^{(n)}\right)$ with $0<\Phi_i\left(\gamma_k^{(i)}\right)<1$, $i=1,2,...,n$, $k=1,2,...,N$.

Step 3. From $k=1$ to $k=N$, if $f(u_k)\le c$, $m_1(i)=m_1(i)+1$, denote $x_{m_1(i)}^{(i)}=\gamma_k^{(i)}$;
otherwise, $m_2(i)=m_2(i)+1$, denote $y_{m_2(i)}^{(i)}=\gamma_k^{(i)}$, $i=1,2,...,n$.

Step 4. Rank $x_{m_1}^{(i)}$ and $y_{m_2}^{(i)}$ from small to large, respectively.

Step 5. Set $a^{(i)}=\Phi\left(x_{m_1(i)}^{(i)}\right)\wedge\left(1-\Phi\left(x_1^{(i)}\right)\right)\wedge\left(\Phi\left(x_1^{(i)}\right)+1-\Phi\left(x_2^{(i)}\right)\right)\wedge...\wedge\left(\Phi\left(x_{m_1(i)-1}^{(i)}\right)+1-\Phi\left(x_{m_1(i)}^{(i)}\right)\right)$;

$$b^{(i)}=\Phi\left(y_{m_1(i)}^{(i)}\right)\wedge\left(1-\Phi\left(y_1^{(i)}\right)\right)\wedge\left(\Phi\left(y_1^{(i)}\right)+1-\Phi\left(y_2^{(i)}\right)\right)\wedge...\wedge\left(\Phi\left(y_{m_1(i)-1}^{(i)}\right)+1-\Phi\left(y_{m_1(i)}^{(i)}\right)\right).$$

Step 6. $L_{1U}^{(i)}=a^{(i)},L_{1L}^{(i)}=1-b^{(i)},L_{2U}^{(i)}=b^{(i)},L_{2L}^{(i)}=1-a^{(i)}$.

Step 7. If $a_U=L_{1U}^{(1)}\wedge L_{1U}^{(2)}\wedge...\wedge L_{1U}^{(n)}>0.5$, $L_U=a_U$; if $b_U=L_{2L}^{(1)}\wedge L_{2L}^{(2)}\wedge...\wedge L_{2L}^{(n)}>0.5$, $L_U=1-b_U$;
otherwise, $L_U=0.5$.
If $a_L=L_{1L}^{(1)}\wedge L_{1L}^{(2)}\wedge...\wedge L_{1L}^{(n)}>0.5$, $L_L=a_L$; if $b_L=L_{2U}^{(1)}\wedge L_{2U}^{(2)}\wedge...\wedge L_{2U}^{(n)}>0.5$, $L_L=1-b_L$;
otherwise, $L_L=0.5$.

Through Algorithm 1, we have the bounded uncertainty distribution of interested probability, denoted by $\left[\Psi_L(p),\Psi_U(p)\right]$. To analyze the risk of an event, we define average probability based on the expected value of uncertain variable:

$$\overline{p}=\int_0^{+\infty}\left[1-\frac{\Psi_L(p)+\Psi_U(p)}{2}\right]dp. \tag{13}$$

In addition, the quantile probability, such as 0.9 quantile, can be also calculated to reflect the uncertainty. In this method, the quantile probability is no longer a precise value, but an interval, denoted by $\left[Q_{0.9}(p_L),Q_{0.9}(p_U)\right]$.

IV. A NUMERICAL EXAMPLE

In this section, the developed approach in III.C is implemented in a benchmark case study of flood risk assessment. Subsection IV.A will briefly introduce the benchmark problem. In subsection IV.B, the results are compared to those that are obtained from evidence-theory-based level-2 uncertainty propagation method, in order to highlight the strength and weakness of the developed approach.

IV.A. The benchmark problem for uncertainty propagation

The benchmark problem investigates flood risk of a residential area which is closely located to a river. In this area, a dike is constructed for prevention. Thus, we are interested in the probability that the annual maximum water level exceeds the dike height.

The system is visualized in Fig. 2, and the maximal water level is calculated by

$$Z_c = g(Q, Ks, Zm, Zv, l, b) = Zv + \left(\frac{Q}{Ks \cdot b \cdot \sqrt{(Zm - Zv)/l}} \right)^{3/5}, \tag{14}$$

where Zm denotes the riverbed level at the upstream part of the river, Zv denotes the riverbed level at the downstream part of the river, Ks denotes the friction coefficient of the riverbed, Q denotes the yearly maximal water flow, l denotes the length of river, and b denotes the width of river [13].

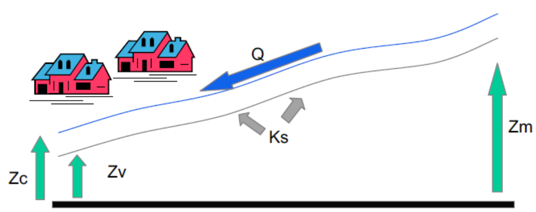

Fig. 2. [13] Visualization of the system

IV.B. Uncertainty propagation in a level-2 setting using different methods

IV.B.1. Uncertainty theory

According to Ref. 6, the input parameters are subject to two-level uncertainty. In this paper, we use probability to model level-1 uncertainty, and uncertainty theory to model level-2 uncertainty. For example, Q follows Gumbel distribution $Gum(\alpha, \beta)$, and α is an normal uncertain variable whose uncertainty distribution $\mathcal{N}_\alpha(e_\alpha, \sigma_\alpha)$ is given in (3). All the models or values of parameters are tabulated in TABLE I as follow.

TABLE I. Models and values of Level-1 and Level-2 parameters

Parameter	Level-1	Level-2	
Q	$Gum(\alpha, \beta)$	α	$\mathcal{N}_\alpha(1013, 48)$
		β	$\mathcal{N}_\beta(558, 36)$
Ks	$N(\mu_{Ks}, \sigma_{Ks}^2)$	μ_{Ks}	$\mathcal{L}(22.3, 33.3)$
		σ_{Ks}	$\mathcal{L}(2.5, 3.5)$
Zm	$N(\mu_{Zm}, \sigma_{Zm}^2)$	μ_{Zm}	$\mathcal{L}(54.87, 55.19)$
		σ_{Zm}	$\mathcal{L}(0.33, 0.57)$
Zv	$N(\mu_{Zv}, \sigma_{Zv}^2)$	μ_{Zv}	$\mathcal{L}(50.05, 50.33)$

	σ_{Zv}	$\mathcal{L}(0.28, 0.48)$
l	5000 (constant)	
b	30 (constant)	

Here, we let the threshold of Zc, i.e. the dike height to be $55.5m$. Through Algorithm 1, we have the bounds of uncertainty distribution of $p = \Pr\{Zc > Zc,th\}$, shown in Fig. 3.

IV.B.2. Evidence theory

Level-2 uncertainty propagation of the benchmark problem is also conducted using evidence theory in this paper. In this process, discretization method is used to obtain the basic probability assignment of input parameters, and a two-level Monte Carlo simulation is used to propagate the uncertainty. The belief function and plausibility function of interested probability is shown in Fig. 3.

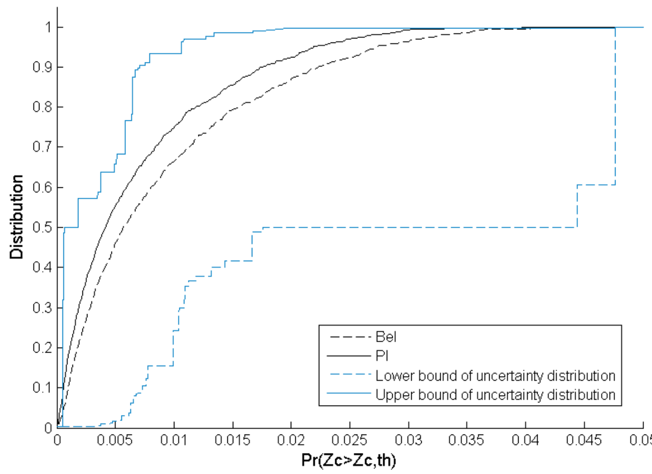

Fig. 3. Level-2 uncertainty propagation results based on uncertainty theory and evidence theory

IV.B.3. Discussion

Fig. **3** shows the results of level-2 uncertainty propagation based on uncertainty theory and evidence theory. It is clear that most belief function (Bel) and plausibility function (Pl) are covered by the uncertainty distribution bounds. This means that the results derived from uncertainty theory is more conservative, and the level-2 uncertainty affects uncertainty distribution bounds severer than Bel and Pl.

We can also calculate average probability of the two distribution bounds. Through (13), the average probability is calculated to be $\overline{p}_{UT} = 0.0161$. In evidence theory, Bel and Pl are regarded to be lower and upper bounds of probability, respectively. In [14], the probability can be estimated by the average value of Bel and Pl. Thus, the average probability is

$$\overline{p}_{ET} = \int_0^\infty \left(1 - \frac{Bel(p) + Pl(p)}{2}\right) dp = 0.0080.$$

The results mean that we believe the average probability of the occurrence of flood is 0.0161 in uncertainty theory, and be 0.008 in evidence theory. In other words, if we want to control the risk to some fixed level, the dike should be higher according to uncertainty theory. Thus, the average probability also shows that the result of uncertainty theory is more conservative. As for 90% quantile probability, the result of uncertainty theory and evidence theory is $[0.0073, 0.0476]$ and $[0.0175, 0.0223]$, respectively. In the perspective of decision maker, the first interval may be too large to make a decision, and this may be a shortcoming of the proposed method.

Therefore, when choosing the method in risk analysis from the proposed method and classical method, we need to consider the attitude of decision maker. For a conservative decision maker, the developed method may be better. In addition, the result of our method reflects that the uncertainty is much sever in level-2 uncertainty settings, and the uncertainty need to

II. CONCLUSIONS

In this paper, uncertainty theory is introduced to risk analysis and a new level-2 uncertainty propagation method is developed. For monotone interested risk indexes, we give the propagation method based on the operation laws of uncertainty theory. For the more common situations, an uncertain simulation method is developed to calculate bounds of interested probability. Average probability and quantile probability are defined to assess the risk. The developed approaches are implemented in a benchmark case study of flood risk assessment, and compared to the method based on evidence theory. The results show that the developed method is more conservative than the evidence-theory-based method. Propagating uncertainty in a level-2 setting is an interesting but challenging future research topic. In addition, the uncertainty controlling method should be considered in the future studies.

ACKNOWLEDGMENTS

This work has been performed within the initiative of the Center for Resilience and Safety of Critical Infrastructures (CRESCI, http://cresci.buaa.edu.cn). This work is supported by Natural Science Foundation of China (NSFC) under contract No. 61573043.

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