Exploring Relations between Graph Metrics and Importance Measures in PRA Sequences

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Abstract

In this paper, we explore the relations between Probabilistic Safety Assessment models and graph theory (complex networks). From an event sequence diagram, representing the scenarios following the occurrence of an initiator, we built a corresponding network which represents the different relations between the network nodes and the different consequences of the event sequence diagram. The undesired consequence is then represented as an s-t network which is analyzed using different metrics.

In this study we show that for some metrics, for instance, the mean centrality metric, there is a strong relation between a high mean centrality and the frequency of occurrence in the cutset list of the corresponding consequence. Some similarities between he way PSA models are solved and the construction of the networks were highlighted and other experiments using simulation are considered.

The experimentation was done using different tools:

- A knowledge based system expert KB3 for representing system structures from which deduced graphs are driven to represent system missions.
- Gephi a graph visualization and manipulation software.

1 Introduction

In a serie of experimentations around complex network theory, we started with [9] and [1] to use network algorithms to search for minimal cutsets that lead to an undesired event following an initiator. The different consequences were modelled in the form of a network for which network algorithms were used to find cutsets. In [1], we showed that it was possible to compute the sequence frequencies using direct computation over networks built over system networks and a standard representation of the scenarios in the form of event sequence diagrams.

In this paper, we explore the relations between *Probabilistic Safety Assessment* metrics and *Graph Theory* metrics (complex networks). From an event sequence diagram, representing the scenarios following the occurrence of an initiator, we built a corresponding network which represents the different relations between the network nodes and the different consequences of the event sequence diagram. The undesired consequence is then represented as an s-t network which is analyzed using different metrics.

In this study we show that for some metrics, for instance, the betweenness centrality metric, there is a strong relation between a high betweenness centrality and the frequency of occurrence in the cutset list of the corresponding consequence. Some similarities between he way PSA models are solved and the construction of the networks were highlighted and other experiments using simulation are considered.

2 Centrality measures

As soon as we face a complex network, it becomes valuable to identify its main components. The first attempts to describe the importance of a graph's element were conducted together with the study of social networks. So, the idea of centrality first appeared in A. Bavelas and H. Leavitt works (cf. [10]). It was then related to the idea of influence. Afterwards, the concept went through a mathematical description and one can find a handful number of measurements to evaluate centrality, depending on how the importance of the network elements is defined. Altough, as L.C. Freeman early stated it in 1979 in his fundamental paper [8]:

" *there is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is very little agreement on the proper procedure for its measurement* ".

Freeman himself strengthened the theoretical basis regarding the definition of centrality, but his statement still echoes today. This paper will present the mostly used and relevant centrality measurements, while generally proposing application examples for each of them.

Several ways of approaching centrality will first be presented, leading to a possible classification of the measurements. The tedious task of describing the various measurements will follow.

As foretold, centrality now claims to be able to evaluate the importance of a graph's nodes regarding a multitude of criteria. Our aim in this paper is to take a look of the properties of the components of important centralities. Let us consider the four following classes:

- A vertex is said to be central if he has a privileged access to the network/graph other vertices, or if it is easy to reach that vertex from the other vertices.
- A vertex is central if the neighbouring vertices are central themselves.
- A central vertex is one through which a lot of information can pass.
- The more a graph "works" correctly with a given node, the more this node is considered important. Vitality measurements are based on other measurements by evaluating the impact of removing a vertex.

This section is dedicated to the introduction of the access centrality measurements, which are the most simple and intuitive ones. Most of them were already described by Freeman in [8]. They were built based on the following considerations: in a *star graph*, the node at the centre of the star is the most central one for three specific reasons: it has the highest *degree* possible, it is the *closest* to the highest number of other vertices, and is a mandatory *intermediary* in the highest number of paths. The "highest" referring to the possible values for a graph with the same number of vertices and edges.

Based on these three points, Freeman proposed the firsts centrality measurements.

2.1 Degree centrality

Degree centrality is probably the most intuitive way to define centrality. Analyzing a social network, it would be natural to consider important someone who interferes with a lot of other people. Thus a person's importance is given by the number of its neighbours. For a general graph, the importance is hence given by the number of adjacent edges, named its *degree*.

The degree can be defined using the adjacency matrix. For an undirected graph, it can be written as:

$$C_{degree}(v) = \frac{1}{n-1} \sum_{j=1}^{n} A_{vj}$$

we can also omit the summation to only use a matrix formalism:

$$C_{degree} = \frac{A \cdot 1_n}{n-1}$$

where $1_n = (1, 1, ..., 1)^T$.

For directed graphs, two degree centralities can be defined depending on which edges are considered: the incoming ones or the outgoing ones.

$$C_{degree}^{out}(v) = \frac{1}{n-1} \sum_{j=1}^{n}$$

$$C_{degree}^{in} = \frac{1}{n-1} \sum_{j=1}^{n} A_{jv}$$

Which is re-written solely using matrices:

$$C_{degree}^{out} = \frac{A \cdot 1_n}{n-1} \text{ and } C_{degree}^{in} = \frac{A^T \cdot 1_n}{n-1}$$

meanwhile, for directed graphs, the "classical" degree is now defined as the summation of both in and out centralities:

$$C_{degree} = \frac{\left(A + A^T\right) \cdot 1_n}{n-1}$$

Note that degree centrality is a local measurement of a node's importance since it only depends on the direct neighbourhood of each vertex. As such, it does not give any valuable information regarding the importance of a vertex in regards of the whole graph's structure.

2.2 Closeness centrality

Closeness centrality evaluates the centrality of a vertex in terms of distance to the other vertices. The underlying idea is that the position of a node is interesting when this node can rapidly access any point of the network. Once again, applying this measurement to social networks, closeness centrality tells how fast an individual can spread information to the whole network.

In practice, closeness centrality is expressed as the inverse of the average distance of a vertex to all other vertices. This time again we start defining it for undirected graphs.

[Closeness centrality, [8]]

$$C_{closeness}(v) = \frac{n-1}{\sum_{t \in V} d_G(v,t)}, \qquad \forall\, v \in V$$

Note that if the graph is not connected, closeness centrality is null for every vertex $v \in V$ since infinite distances do exist. It might then be more informative to instead define the measurement for every connected component. This is done by only summing over reachable vertices and modifying the normalizing factor by counting the number of such reachable vertices.

$$C_{closeness}(v) = \frac{|\{t \in V, v \rightsquigarrow t\}| - 1}{\sum_{\substack{t \in V \\ v \rightsquigarrow t}} d_G(v,t)}$$

Considering directed graphs, we now distinguish the paths originating from a vertex v and the ones ending at it. Thus defining two measurements. This time again the summation will only be done over existing paths in order to avoid infinite values.

$$C^{out}_{closeness}(v) = \frac{|\{t \in V, v \rightsquigarrow t\}| - 1}{\sum_{\substack{t \in V \\ v \rightsquigarrow t}} d_G(v,t)}, \qquad v \in V$$

$$C^{in}_{closeness}(v) = \frac{|\{t \in V, t \rightsquigarrow v\}| - 1}{\sum_{\substack{t \in V \\ t \rightsquigarrow v}} d_G(t,v)}, \qquad v \in V$$

2.3 Betweenness centrality

Betweenness centrality gives importance to a node if it is a mandatory passageway when travelling between two other nodes. If a node is an individual which has to transmit information, one can easily understand that a node through which a lot of information transits has a lot of power in the network: apart from being responsible of the maintain of the communication, he can decide to alter or retain it to influence the rest of the network. This concern led to the notion of betweenness, which was, once again, firstly introduced in a study of social relationships.

To express the betweenness centrality of a vertex $v \in V$, let us consider that it lies on a path between every other vertices $(s,t) \in V$. Only shortest paths will be considered here. When several geodesic exist between s and t with some of them not passing by v, the probability that information will indeed use the path containing v has to be taken into account. Such a probability is given by $\frac{\sigma_{st}(v)}{\sigma_{st}}$, where the notations are the ones introduced earlier.

[Betweenness centrality, Freeman, 1979]

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}, \qquad \forall\, v \in V$$

The measurement should now be normalized to make it independent of the network size in order to allow comparison of betweenness values between graphs of different sizes. To achieve that, we have to express the maximum achievable value of C_B. This is done by answering the following question: to how many shortest paths can a node v belong in a graph with n vertices ? Since only the path in which v is not an extremity are considered, choosing such paths is equivalent to choose its extremities s and t in the set $V \backslash \{v\}$. Thus leaving the following number of choices:

$$\frac{(n-1)(n-2)}{2}$$

Now considering the central node v_{centre} of a star graph with n vertices, one can easily check that it satisfies $C_B(v_{centre}) = \frac{(n-1)(n-2)}{2}$. This is hence the actual maximum value for C_B, allowing us to correctly define the normalised betweenness centrality:

[Normalised betweenness centrality, Freeman, 1979]

$$C_{betweenness}(v) = \frac{2}{(n-1)(n-2)} \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}, \qquad \forall\, v \in V$$

2.4 Eccentricity and graph centre

Another intuitive way to evaluate the centrality of a node is to characterize it using the distance to the *borders* of a graph. Rather than considering the average distance between a vertex and all other vertices as it is done with closeness centrality, we will here consider the upper bound of such distances. This is done using the notions of *radius*, also called *eccentricity*, and *center* of a graph.

[Eccentricity] The eccentricity of a vertex is the maximal distance to all other vertices

$$e(v) = \max_{t \in V} d_G(v,t), \qquad \forall v \in V$$

The centre of the graph is then defined as the set of vertices with the smallest eccentricity. It is the set of nodes which distance to the remotest vertices is the smallest.

[Graph centre, Hage and Harary, 1995]

$$C_{centre}(v) = \frac{1}{\max_{t \in V} d_G(v,t)}, \qquad \forall\, v \in V$$

As for closeness centrality, the problem of connectivity has to be addressed. If a graph is not connected, the centre value will always be null. Centre centrality thus becoming an indicator of connectivity, which one might still find useful. Nonetheless, directed graphs being expectedly rarely connected, the graph centre will be defined for both incoming and outgoing paths:

$$C^{out}_{centre}(v) = \frac{1}{\max_{\substack{t \in V \\ v \rightsquigarrow t}} d_G(v,t)}, \qquad \forall\, v \in V$$

$$C^{in}_{centre}(v) = \frac{1}{\max_{\substack{t \in V \\ t \rightsquigarrow v}} d_G(t,v)}, \qquad \forall\, v \in V$$

2.5 Eigenvector centrality

Eigenvector centrality, which denomination will appear justified afterwards, defines the centrality of a vertex such as it is directly proportional to the centrality of its neighbours. To link this measurement to social network studies, we can state that an individual is not necessary someone with a lot of contacts, but rather someone whose relations are influential. This idea has been presented and developed by P. Bonacich in [5] and [4] and is frequently used.

Starting from the proportionality idea, let c_v denote the centrality of the vertex $v \in V$ and λ^{-1} be the proportionality coefficient. The eigenvector centrality hence satisfies:

[Eigenvector centrality, [5]]

$$c_v = \sum_{\substack{s \\ (v,s) \in E}} \lambda^{-1} c_s = \lambda^{-1} \sum_{s=1}^{n} A_{vs} c_s$$

from which the following form directly results:

$$Ac = \lambda c$$

where c is the vector composed of the $c_v,\ v \in V$.

c is hence an eigenvector of the adjacency matrix A, whence the name *eigenvector centrality*. Moreover, if we force the centrality measurements to always be non-negative, then, according to Perron-Frobenius theorem c is the eigenvector associated with the largest eigenvalue of A. This results in the following property:

The eigenvector centrality of a graph's vertices is given by the eigenvector associated with the largest eigenvalue of the adjacency matrix of the graph.

For directed graphs, one can distinguish two kinds of eigenvector centralities: one describing the influence (a vertex that leads to many other vertices), and the other one reflecting the popularity (a vertex which has many predecessors). These measurements satisfy:

$$Ac^{out} = \lambda^{out} c^{out} \text{ and } A^T c^{in} = \lambda^{in} c^{in}$$

But since A is not symmetrical for directed graphs, eigenvectors might be complex, which prevents the centrality measurement from providing a global ordering of the vertices. This can be overcome by simply considering the modulus of the eigenvectors.

2.6 PageRank

At least one other area managed to raise interest for centrality: the search of documents on the web. An efficient search engine has to be able to identify the most "relevant" documents in a whole complex network. To provide an acceptable answer, the engine thus has to identify the most *central* document among those that answer the request specifications. The most commonly used algorithm, such as Google's, are based on feedback centrality: a document is important if the ones pointing to it are important themselves.
We will here describe a simplified version of the PageRank algorithm that was created by Brin and Page and presented in 1998 in [7] and is the algorithm with which they built Google. The vocabulary originally associated with the algorithm will be used, such that we will rather speak of a *page* rather than a vertex or a node. Due to the naturally directed structure of the web, the PageRank is defined for directed graphs.

Let $p(v)$ be the PageRank of page $v \in V$ and d a damping factor between 0 and 1.

$$p(v) = (1-d)\frac{1}{n} + \frac{d}{n} \sum_{t \in \Gamma^-(v)} \frac{p(t)}{k_t^{out}}$$

To keep the presentation consistent with what will follow, we will already slightly modify the definition of p. The reasons behind such a choice will become clear later on. Let us thus define the set Θ as follows:

$$\Theta = \{t \in V, k_t^{out} = 0\}$$

and now write p as:

$$p(v) = (1-d)\frac{1}{n} + d\left(\sum_{t \in \Gamma^-(v)} \frac{p(t)}{k_t^{out}} + \sum_{t \in \Theta} \frac{p(t)}{n}\right)$$

We can then re-write the PageRank introducing the normalized adjacency matrix, which will be denoted $\mathbf{P}$.

$$\mathbf{P}_{ij} = \begin{cases} \frac{A_{ji}}{\sum_{k=1}^{n} A_{jk}} = \frac{A_{ji}}{k_j^{out}} & \text{if } k_j^{out} \neq 0 \\ \frac{1}{n} & \text{otherwise} \end{cases}$$

Let p denote the vector indexed by the pages. We have that

$$p = \frac{1-d}{n}\mathbf{1}_n + d\mathbf{P}p$$

leading to

$$p = \frac{1-d}{n}(I - d\mathbf{P})^{-1}\mathbf{1}_n$$

for the values of d that make $(I - d\mathbf{P})$ invertible.
Notice that the PageRank is in fact a probability distribution on all the web pages (thanks to the normalization we proceeded to introduce). The sum of all PageRanks over the network is hence equal to 1. This leads to another interpretation of the PageRank, which was the one proposed by Brin and Page.

3 From event sequence diagrams to networks via system topology

To develop event trees and fault trees used in PSA models we are dealing with a process of generating event trees and fault trees (cf. figure 1). The first from the event sequence diagrams that represent all the scenarios from the initiator, in a given configuration, to the different outcomes, and the second represent the system missions which are generated from the system topology using an expert system software (KB3 cf. [11]).

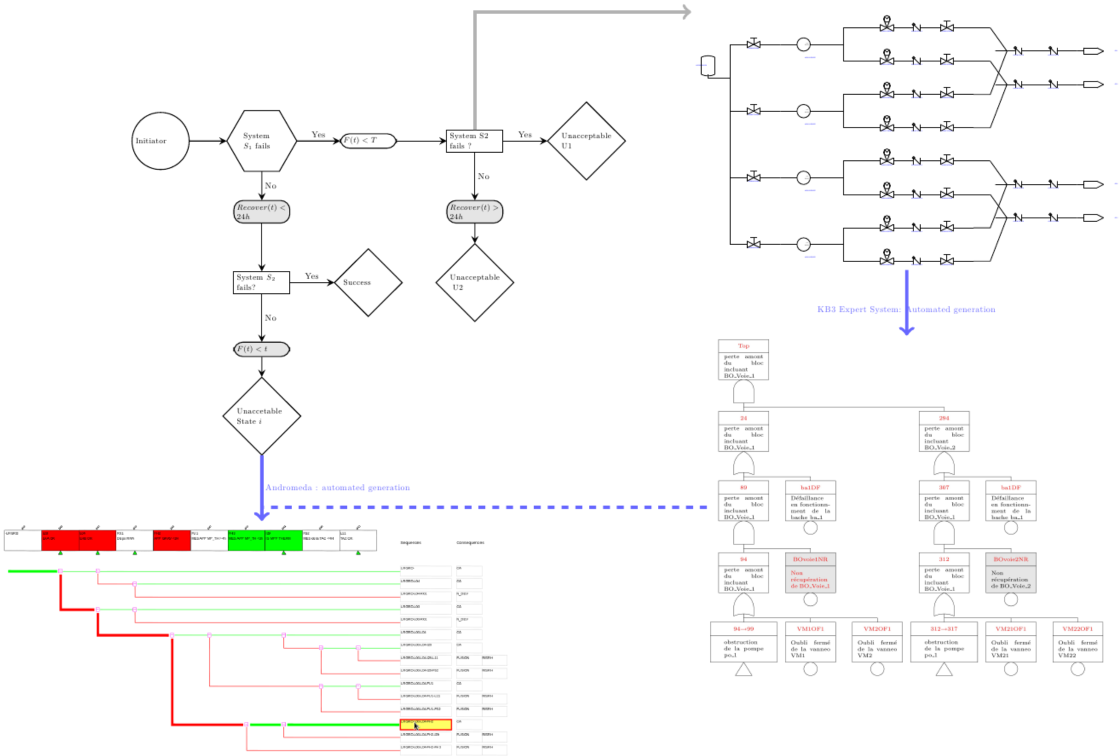


Figure 1: General scheme of model construction

Given an event sequence diagram representing the different scenarios starting from an initiator. One can obtain an equivalent s-t network which is then submitted to network metric analysis (cf. [9], [1]). In [1] it was shown that using network algorithms, one can find the different cutsets leading to an undesired events exploring the notion of networks cut-sets ([3], [12]) instead of solving Boolean master fault trees.

In this paper, another approach is proposed to have some insight on the problem characteristics by computing network metrics which are of less computational complexity and could give deep insight on the structure of our models.

We use directed graphs whose nodes symbolize physical components of a system (e.g. valves, pumps etc) and when appropriate a functional group of components and whose edges are physical wires between these components (pipes for hydraulic components, wires for electrical or instrumentation and control systems). These networks represent relations between the physical system components.

The first step of the procedure consists of the event sequence graph construction. This is a graph representing the different scenario but at a level of system, I&C and electrical missions or operator actions. This graph may be considered as a "macro graph". The second step consists of the extraction of the graphs corresponding to each system mission regarding the context of the mission. This means the configuration of the plant (state of the reactor, the system alignments and maintenance assumptions and success criteria requirements . . .). For this task we use the *KB3 expert system* [11] [1] which allows to model the systems in question with all the needed attributes and configurations. The KB3 archive files store all this information. Therefore, it is possible to get the topology of the graphs that represent the systems modeled in the event sequence diagram.

The procedure we used is based on the following scheme:

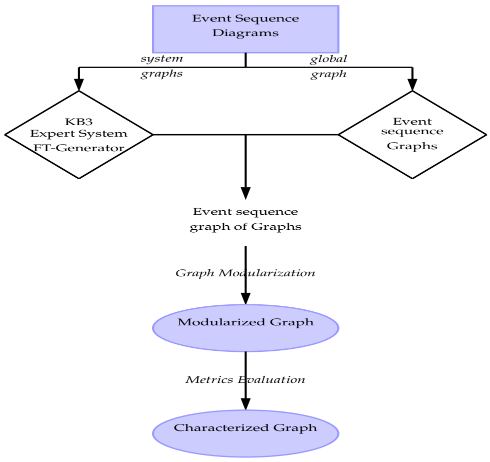


Figure 2: Graph Characterization

This work consists in three main steps:

- Representation of safety systems based on *Event Sequence Diagrams* (ESD) as a network
- Visualization of the resulting graphs
- Complex network metrics analysis

First, the mission diagrams expressing ESD will be converted to a network that should give a faithful representation of the safety systems and procedures, and where there is only one instance of every element (node). To achieve such a work data sources of ESD are imported from a software called KB3, and a new module must be added to another application Andromeda developed for modular PSA analysis to transform this data to graphs.

Once the graph constructed, it should be visualized. Many tools allow the visualization or spacialization of graphs. An algorithm based on attractive forces is used to get a pleasant view and topology. And then the complex network analysis metrics could be applied.

3.1 Principle of representation

Depending on the nature of links and nodes, a network can have one of many representations:

- Simplex graph
- Oriented graphs
- Weighted graphs
- Multiplex graphs, contain various type of links (group of layers of graphs of the same nodes)
- Multigpahs

[1] KB3 is based on the concept of knowledge bases that capitalizes, for a specific domain, a set of information on the system and component characteristics and behavior. This is done through a number of generic descriptions and rules expressed via a declarative programming language called [6]. In the PSA domain KB3 serves for instance to generate fault trees from the topology of the systems regarding their profiles and the different success criteria [11].

Many metrics were developed for the simplex type, and in some works other forms are simplified, and a in a few metrics were adapted to be applied for the last form. In this study we focus on centrality metrics for complex graphs. This experimentation is ongoing with different scopes. In particular, with multiplex modeling of external hazards.

ESDs are based on functional requirements diagrams, where every bloc stands for a system mission. Every mission is made of a group of elements or components that do gather an information (a fluid for example) from some sinks or sources to a group of terminal points.

The failure of the mission means that none or not enough terminal points received the information, and is modeled by a fault tree where the leaves are activation tests of terminals in the process system diagram.

The first step of graph construction is then, to resolve a minimal Boolean expression of the undesired events tree, which will give us a logical relationship between the different system's tested elements. For every one of these tested systems we have one elementary mission gathering information from one or many sources into that terminal point. There are three Boolean laws to link the basic events: `AND`, `OR` and `k/n`, and then rules to connect elementary graphs depending on the Boolean relation that links them should be developed.

The Boolean logic however does not give a view of the time evolution factor, which means the elementary graphs order in one mission. Let's take the OR gate example, if tow base elements; and let them be A and B, are linked by OR law in the causes tree, meaning that the fail of one results in the failing of the mission, and then are represented by linking the two respective graphs (GA, GB) in serial, though should we put first GA, or GB, and what this means, and what it changes. Making a generalization of such an example to all the Boolean expression, gives an overview of the explosion in term of possibilities.

3.2 k/n Case

- In the fault tree: `k/n` gate isn't passing only if at least k between n of its inputs are activated. This gate will be replaced by a test node that carries the same function
- In the systems functions: k/n nodes type are elements that are activated only when k between n of their inputs receives information fluid.

We can think as a solution of the OR gate problem about using a k/n gate with k=n, but this issue wasn't used here.

Once sub-graphs of missions are built, they must be linked to construct a sequence leading to a consequence as defined in the functional request diagram.

3.3 Representation problem

The aim of the analysis being the calculation of the structural indices of the network, and particularly robustness and resilience to defaults of one or many components, thus, care must be taken to defaults propagation, and for that we can have two kinds of links:

- Fluid links: ordinary links where the graphical representation and behavior is obvious.
- Functional links: gives a dependency between an element and its antecedent, if one of the antecedents of an element is down then the component become non active.

A good representation of the system (as a complex system) must take in account the electrical system, the automation system, and human operator layers. And thus the best model to take is the multiplex graph model.

As a starting of study, we'll try to make some simplifications to work with a simplex graph:

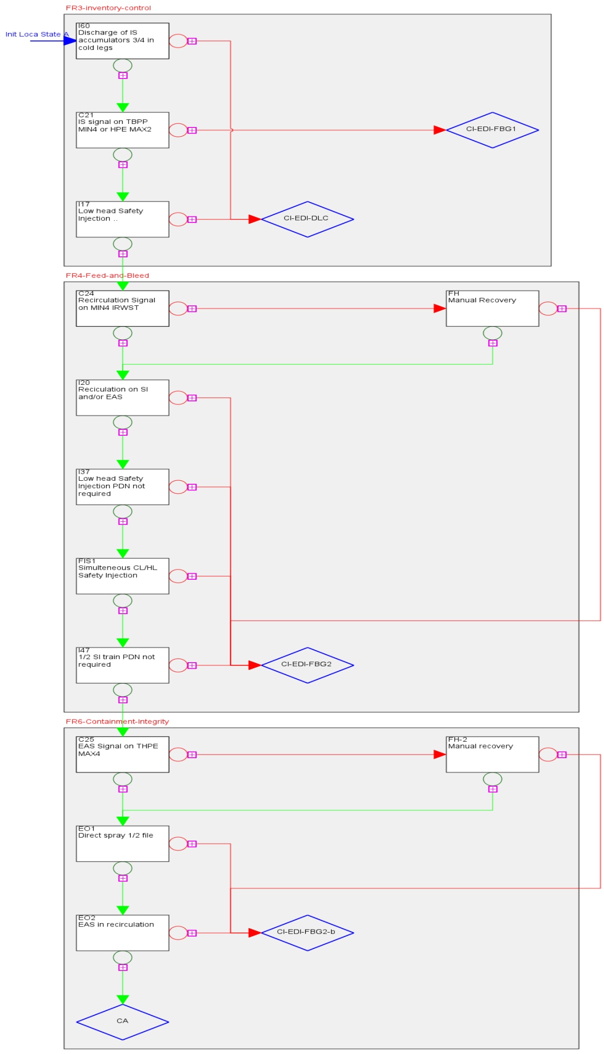
- In the graph, only fluid links are represented.
- Functional links, are kept as meta information of the nodes, and are used in the default propagation when calculating robustness factors.

- Only the hydrau-pneumatic components layer is modeled, and the other layers will not be represented. If a mission concerns a human factor or a control command system, this one is performed by a simple node, and its relative network (power supply and communication) ignored.
- `k/n` node is kept as a node, and the information is preserved as meta data.

The final result will be an oriented simplex graph of hydraulic systems.

4 A LOCA example

The example we are dealing with in this study is a Loss of Coolant accident in power state A with a break of diameter greater than 12". The following diagrams shows a representation of the different scenarios.



- Event I60: Discharge of IS accumulators 3/4 in cold legs
- Event C21: IS signal on TBPP MIN4 or HPE MAX2
- Event I17: Low head Safety Injection ..
- Event C24: Recirculation Signal on MIN4 IRWST
- Event FH: Manual Recovery
- Event I20: Reciculation on SI and/or EAS

- Event I37: Low head Safety Injection PDN not required
- Event FIS1: Simulteneous CL/HL Safety Injection
- Event I47: 1/2 SI train PDN not required
- Event C25: EAS Signal on THPE MAX4
- Event FH-2: Manual recovery
- Event EO1: Direct spray 1/2 file
- Event EO2: EAS in recirculation

The system missions are represented in the form of graphs. For instance the EFWS missions are represented through the graph of figure 3 and the electrical supply is represented in the graph of figure 4.

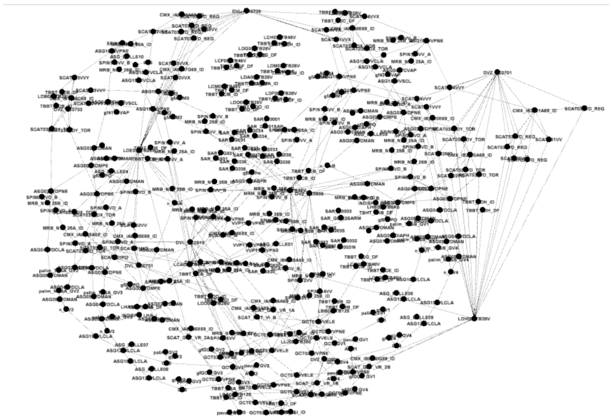


Figure 3: EFWS network

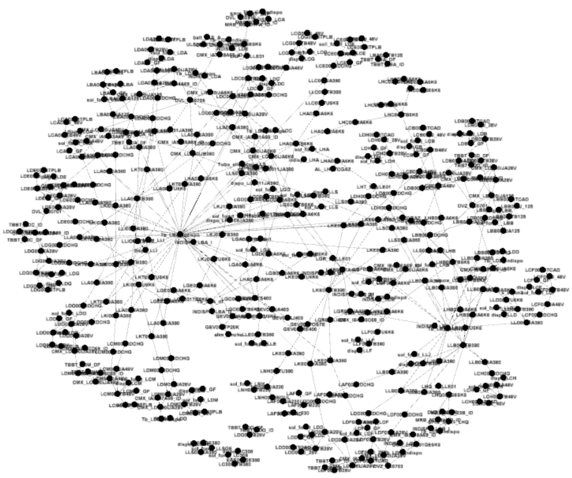


Figure 4: Electrical supply network

The whole graph representing the main ESD is shown in figure 6 where the node size correspond to the betweenness centrality measure.

In figure 5, we see a zoom on one of the most important components in terms of betweenness centrality measure.

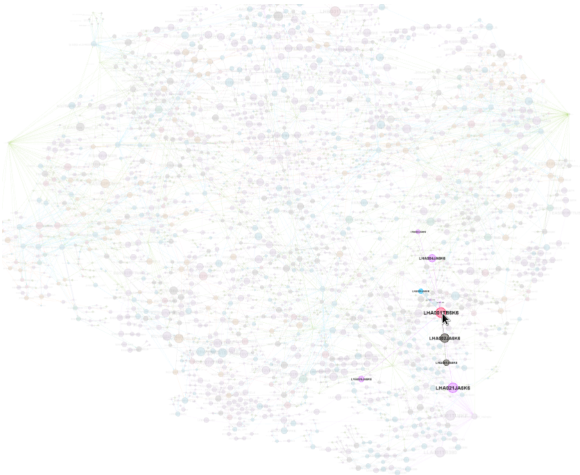


Figure 5: A zoom of most mean central components

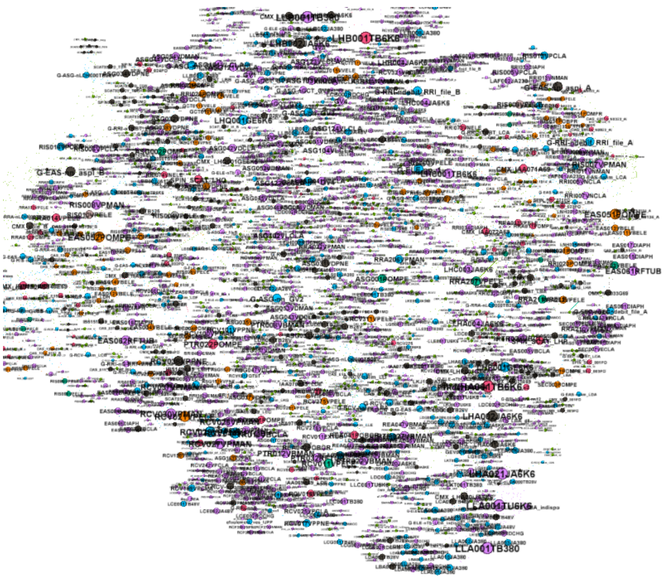
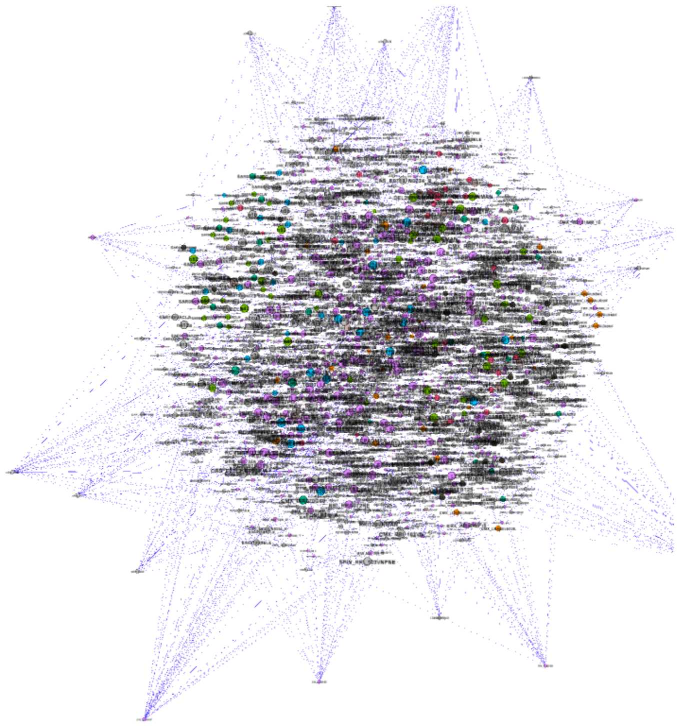


Figure 6: Betweenness Centrality Network Based Metrics

The network structure can help understand the PSA model in many ways. Indeed, for a global perspective one can highlight the different network metrics (using colors or node size or combining the two) and for more focus perspective, one can use different spacialization algorithms to highlight components of interest. For instance, we showed in figure 6 a mean centrality perspective over network. We can also highlight eccentric components using a different spacialization technique see figure 4. These eccentric elements correspond to either hydraulic sources of water and some main support systems.

The network measure of this study showed many characteristics. In the following, we show the different measures and their distribution over the objects in our model.

If we consider the most important betweenness central components we have a link to the frequency of their occurrence in the cutset list. A reduction of the main graph to those components of betweenness centrality on the interval [6.62E^{-5},4.61 E^{-4}] gives the following components. The occurrence of their failures in the cutset list is given in the table below.



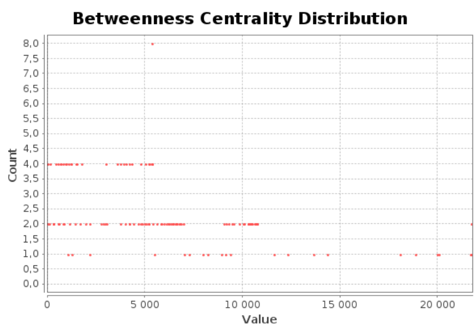
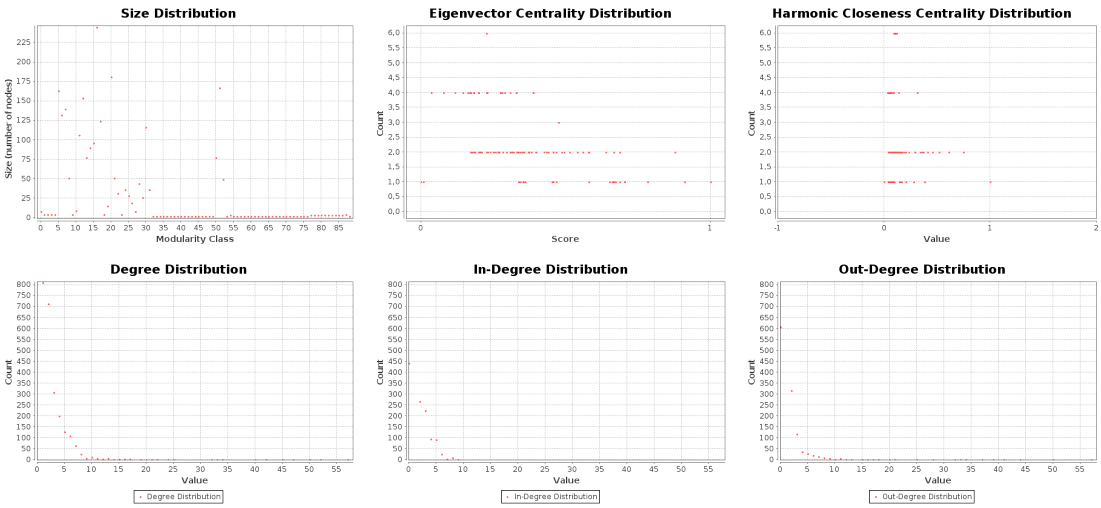


Figure 7: Betweenness Centrality

Table 1: Frequency of ocurrences in the cutset list of main betweenness central

component	number of matches	notable CCFs
EAS051POMPE	2370 matches	80
EAS052POMPE	1720 matches	80
RIS009VPELE	1506 matches	
LHP001GE6K6	**1330** matches	302
LHQ001GE6K6	**1323** matches	302
RIS010VPELE	1238 matches	
RIS007VPMAN	749 matches	
RIS008VPMAN	559 matches	
LHA001TB6K6	**539** matches	
ASG001POMPE	453 matches	
ASG002POMPE	453 matches	
LHB001TB6K6	**450** matches	22
CMXIAA071A69	194 matches	
EAS061RFTUB	96 matches	
EAS062RFTUB	78 matches	
LHB002JA6K6	26 matches	
LLB001TB380	19 matches	22
LHA002JA6K6	16 matches	
RRA014VPELE	5 matches	
LHB021JA6K6	4 matches	
LLB001TU6K6	4 matches	



5 Conclusion and perspectives

In this paper, we proposed an experimentation of graph theory metrics to analyze probabilistic safety assessment models for nuclear power plants. We showed that it may be quite simple to identify some central components using very low complexity algorithms which may help in decision making for instance to prepare sensitivity analysis in PSA models. This study showed many other properties related to some of the main support functions and the role they may play in different system missions.

This approach is now a simple experimentation with a relaxation of some constraints. This means that we are not necessarily considering all the assumptions in the model but an attempt to model different relationships between components of the model in one side and the relations between the system missions, the support systems and the way mitigation actions are performed against the effect of an initiator.

In this study we show that for some metrics, for instance, the betweenness centrality metric, there is a strong relation between a high betweenness centrality and the frequency of occurrence in the cutset list of the corresponding consequence. Some similarities between he way PSA models are solved and the construction of the networks were highlighted and other experiments using simulation are considered.

This experimentation was done using different tools:

- A knowledge based system expert KB3 for representing system structures from which deduced graphs are driven to represent system missions.
- Gephi a graph visualization and manipulation software (cf. [2] [2]).

Other studies are ongoing to consider external hazard modeling using multiplexed networks. Many approaches are explored in this directions: modeling external hazard impact as nodes attributes or as new layer of the multiplexed network.

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