Failure Probability Analysis of an Automatic Landing System for a General Aviation Aircraft Using "subset Simulation"

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This paper is about investigating stochastic safety properties of an automatic landing system for a general aviation aircraft (CS23) which has been developed at the Institute of Flight System Dynamic at Technical University of Munich. According to European Aviation Safety Agency (EASA) automatic landing systems have to satisfy certain failure probability limits for certain error events, such as abnormal runway contact or exceeding vertical loads of the landing gear, with acceptable failure probability limits of 10e-6 or even below. The system is represented in terms of a closed loop high fidelity simulation model in MATLAB/SIMULINK. In order to significantly reduce the required number of samples for a numeric stochastic analysis Subset Simulation is applied.

I. Introduction

Take-off and landing are one of the most critical or dangerous flight phases, as the aircraft is very slow and close to ground, which means that available time to react and recover the aircraft from undesired flight state is very short. Over decades landing an aircraft has become much more safe due to the development and integration of automatic landing systems which utilize for example ILS to guide the aircraft down to the runway. In civil applications [1] different categories for automatic landing are defined, where only the third category, CAT III, refers to automatic landing in that sense that guidance to runway, flare, decrab, touchdown and derotation are performed automatically. All other categories only cover the automatic guidance of the aircraft until a certain, so-called decision height. After passing this height a manual landing is performed by the pilot. For the purpose of this paper the term "automatic landing" covers the full procedure, including guidance to the runway, flare, decrab and derotation.

In order to be able to certify an automatic landing system, beside the general formal certification process, the system has to satisfy certain failure probability limits for hazards, which would lead to injury or death of passengers and damage or total loss of the aircraft. Unfortunately the total system including aircraft, flight control system, actuators and environmental conditions is highly non-linear and complex, so that a closed analytical solution for the failure probability of the system cannot be given. Therefore numerical methods have to be applied, which in general suffer from the high number of evaluations that are required [5].

In order to overcome this problem, this paper applies an advanced Monte Carlo simulation method, which significantly reduces the required number of samples, but at the same time does not require any apriori knowledge of the system. The method is called Subset Simulation and was originally invented for civil engineering failure probability problems. Several application have shown good results and performance of this method [2–4].

II. Nomenclature

The following symbols are applied in this paper

Symbol	Distribution
s_{LH}	Left hand landing gear compression
s_{RH}	Right hand landing gear compression

Symbol	Distribution
s_N	Nose gear compression
x_{TD}	Longitudinal touchdown point position
Δy	Lateral deviation from runway center line
Θ_{TD}	Pitch angle at touch down
Φ_{TD}	Bank angle at touch down
$\dot{h}_{TD}$	Vertical speed at touch down
$\Delta\Psi_{TD}$	Crab angle at touch down

The following nomenclature is applied for distributions:

Symbol	Distribution
$\mathcal{N}(\mu,\sigma)$	Gaussian Normal Distribution with mean μ and standard deviation σ.
$\mathcal{U}(a,b)$	Uniform distribution over the interval $[a,b]$

A general stochastic parameter vector is denoted as $\boldsymbol{\Theta}$**.** The k-th realization of a stochastic parameter is denoted as $\boldsymbol{\Theta}_k$**.**

IV. Simulation Model and Environment

In order to investigate the behavior of the automatic landing system a complex test simulation model has been build up in MATLAB/SIMULINK environment. The test harness covers a high fidelity flight dynamics model running an 1kHz including non-linear aerodynamic data. The flight control system including the automatic landing system is running at 100Hz as it will do on the real Flight Control Computer (FFC) later on in the real aircraft. The actuators are represented by a high fidelity non-linear simulation model, where the controller, and the gear of the actuators are modeled separately.

Figure 1 shows the general structure of the test harness. The Flight dynamics Model is subjected to parameter uncertainties $\boldsymbol{p}_{FDM}$ and the sensor model is subjected to sensor uncertainties $\boldsymbol{p}_{Sensor}$. The actuators are assumed being known to a reasonable level of accuracy hence there are no parameter uncertainties assumed and of course there is no parameter uncertainty in the flight controller software.

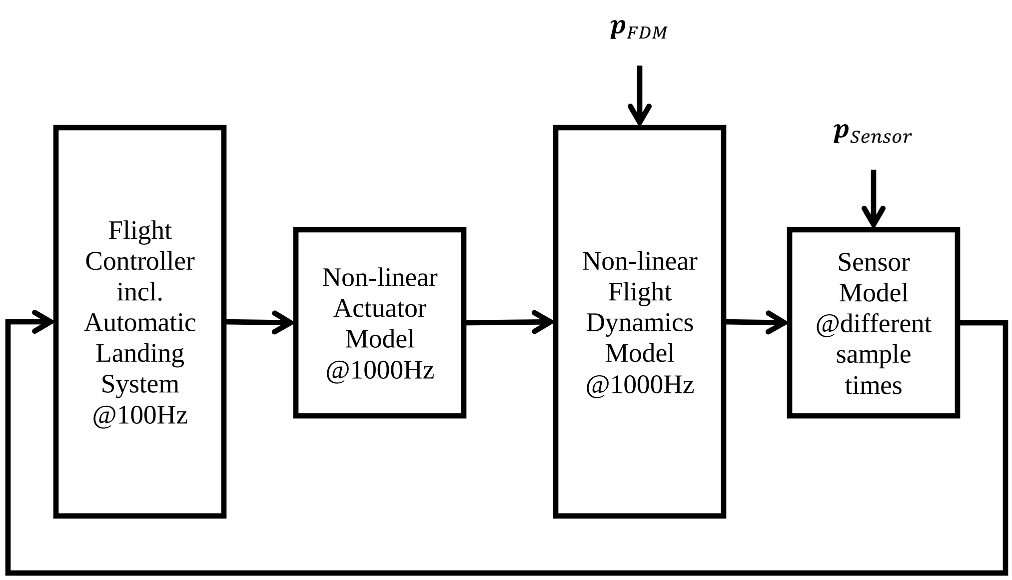


Figure 1 General structure of the applied test harness simulation model

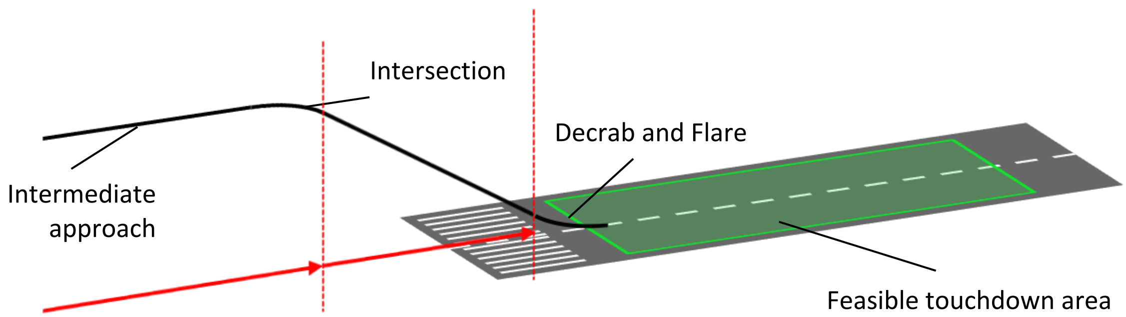
For automatic landing the aerodynamic model and the environment model uncertainties play the major roll for evaluation as the dynamic system responds characteristics of the aircraft and the environmental conditions, such as wind and turbulence effects mainly contribute to hazardous flight states during landing. Currently only environmental disturbances are considered

In order to investigate the effect of wind and turbulence conditions on the aircraft's automatic landing performance, the wind model according to CS-AWO [1] is applied. It consists of a wind shear part, which means that the mean wind velocity changes in magnitude and direction over height above ground and a wind turbulence part. The stochastic distribution for the mean wind magnitude and direction can be directly inherited from CS-AWO and is assumed not to change during one approach.

For turbulence a Dryden-Turbulence model is applied which generates a stochastic wind component over time applying a linear filter to a white noise input $\boldsymbol{\eta}(t)$ [7].

V. Automatic landing system Architecture

The automatic landing procedure is separated into different flight phases and is designed according to a standard 3° ILS approach, with a horizontal, an intercept part and a final approach part at a 3° glideslope. Close to ground a flare maneuver with vertical speed command is performed. Simultaneously the crab angle is reduced and the aircraft is aligned with the runway center line just before touchdown.



Beside this nominal functionality there are several additional functions implemented for safety reasons. First of all, if certain limits are violated during approach an automatic go around is triggered, which brings the aircraft back to air. This is a common fall back strategy and may be triggered at any time, even close to ground, if anything goes wrong during approach. Among others, the indicated airspeed, vertical speed, lateral and vertical deviation from desired track are monitored. A limit may be exceeded due to heavy wind and turbulence effects or other reasons.

Additionally an active Pitch angle protections is implemented in order to prevent the aircraft from abnormal runway contact.

III. Numeric Probability Estimation

III.A. Monte Carlo Method

The basic and most popular numeric probability estimation method is the "Monte Carlo" Method. Monte Carlo can be seen as a virtual experiment which is performed a large number of times with parameters being subjected to certain probability distributions [5]. Based on a large sample size N the sample mean $\bar{z}$ of an uncertain parameter z is given by

$$\bar{z} = \frac{1}{N} \cdot \sum_{i=1}^{N} z(\boldsymbol{\Theta}_i) \qquad \textbf{(1)}$$

If N is very large the law of large numbers states, that the sample mean approaches the Expectation value $\mathbb{E}(z)$ for N going to infinity with:

$$\lim_{N\to\infty} \bar{z} = \mathbb{E}(z) \tag{2}$$

In order to calculate the probability of a certain event, hence $P(\boldsymbol{x} \in F)$, the function z can be choosen as an indicator function $I_F(\boldsymbol{x})$ which is 1, if $\boldsymbol{x}$ lies in F and 0 otherwiese.

$$z(\boldsymbol{x}) = I_F(\boldsymbol{\Theta}) = \begin{cases} 1, & \forall \boldsymbol{x} \in F \\ 0, & \forall \boldsymbol{x} \notin F \end{cases} \tag{3}$$

Consequently one simply counts the number of trials that lie in the failure domain F and divides this number by the total sample size N.

Unfortunately this method becomes inefficient if $P(F)$ is very small, as it is the case for failure probabilities in aerospace applications. Therefore a different approach is applied, which is “Subset Simulation” that originally has been developed for small failure probability problems in civil engineering.

III.B. Subset Simulation

The method of “Subset” Simulation was first introduced by [6] for failure probability estimation problems in civil engineering. The basic idea is to split up the original problem with a very small failure probability into a sequence of intermediate failure events F_i with higher probability, where each F_i is a subset of the preceding F_{i+1}. The advantage is, that the total probability $P(F)$ can be expressed as a product of conditional probabilities $P(F_{i+1}|F_i)$ giving

$$P(F) = P(F_1) \cdot \prod_{i=1}^{m-1} P(F_{i+1}|F_i) \tag{4}$$

Hence instead of evaluating $P(F)$ directly, first the failure probability $P(F_1)$ for a significantly larger F_1 is estimated. Subsequently the conditional failure probability $P(F_{i+1}|F_i)$ which is the probability of F_{i+1} under the condition F_i is evaluated. Due to the properties of equation (4) each probability $P(F_1)$ and $P(F_{i+1}|F_i)$ has a significantly higher probability than the original problem $P(F)$ and are therefore much more efficient to estimate.

The probability of the first intermediate event $P(F_1)$ is evaluated from standard Monte Carlo Methods with sample number N. According to chapter III.A the following estimation holds:

$$P(F_1) \approx \bar{P}(F_1) = \frac{1}{N}\sum_{k=1}^{N} I_{F_1}(\boldsymbol{\Theta}_k) \tag{5}$$

Where $\boldsymbol{\Theta}_{\mathbf{k}}$ is drawn from the corresponding probability density function q. In similar way the probability for any subsequent intermediate event is given by

$$P(F_{i+1}|F_i) \approx \bar{P}(F_{i+1}|F_i) = \frac{1}{N}\sum_{k=1}^{N} I_{F_{i+1}}(\boldsymbol{\Theta}_k) \tag{6}$$

In order to evaluate the conditionally probability $P(F_{i+1}|F_i)$ $\boldsymbol{\Theta}_k$ may not be drawn from its original density function q, but from the conditional probability density function $q(\vec{\boldsymbol{\Theta}}|\, F_i)$. Hence only those parts of the original density function are regarded that lead to a realization of Θ which is located in the failure domain F_i.
There are common methods for drawing parameters from arbitrary distributions, such as the conditional distribution $q(\vec{\boldsymbol{\Theta}}|\, F_i)$, e.g. the Metropolis algorithm. Nevertheless, [6] proposed a modified Metropolis Algorithm with better performance, which is also applied in this paper. Details on this algorithm can be found in [6].

V. Assessment and Analysis Criteria

In order to evaluate, if an automatic landing is successful, different parameters of the aircraft are investigated. All constraints are inherited from CS-AWO [1], but sometimes modified in their applicable values in order to match for landing on smaller runways.

First of all it has to be detected, if the aircraft has ground contact, which is simply given by the fact, that at least one landing gear is deflected.

$$I_{GroundContact} = \big((s_{LH} > 0)\ OR\ (s_{RH} > 0)\ OR\ (s_N > 0)\big) \tag{7}$$

One condition for a safe landing is that the longitudinal touchdown point must be located at least 60m behind runway threshold. Hence one failure domain $F^{(1)}$ is defined by

$$F^{(1)} = \{x_{TD} < 60m\} \tag{8}$$

In similar way the longitudinal touchdown point must not be located too close to the runway end in order to prevent a runway overrun. Usually the maximum position is the middle of the runway. Assuming an runway with 800m the corr4esponding constraint is given by

$$F^{(2)} = \{x_{TD} > 400m\} \tag{9}$$

In order to evaluate this equation, x_{TD} is defined as that longitudinal position at which the aircraft first has ground contact.

In similar way the aircraft shall keep rolling on the runway, hence the lateral deviation $|\Delta y|$ must be smaller than the half of the runway width plus a certain amount of safety overhead.

For the purpose of this paper the following limits apply for a runway with a width of $b = 24m$

$$F^{(3)} = \{|\Delta y| > 8m\} \tag{10}$$

In order to prevent the aircraft from tail strike, the pitch angle Θ of the aircraft must not exceed certain limits. For the purpose of this paper the following values are applied

$$F^{(4)} = \{\Theta_{\mathrm{TD}} > 10°\} \tag{11}$$

In contrast to the longitudinal touchdown position, the pitch angle limits may not be exceeded at any time where the aircraft is on ground, hence for the upper limit, Θ_{TD} is defined by the maximum value of Θ where the aircraft has contact to ground.

A lower limit for Θ is defined by the fact, that the aircraft shall first contact the ground with the main landing gear, not the nose gear. But instead of evaluating $\Theta < 0°$ it is better to directly investigate if the main landing gear touches the ground before the nose gear does.

During landing the wings of the aircraft may also not touch the ground. Therefore the bank angle Φ is limited according to

$$F^{(5)} = \{|\Phi_{\mathrm{TD}}| > 5°\} \tag{12}$$

Of course $|\Phi_{\mathrm{TD}}|$ is once again given by the maximum value where the aircraft has ground contact.

In order to not overload the landing gear of the aircraft, the vertical speed of the aircraft must not exceed a certain value with

$$F^{(6)} = \left\{\dot{h}_{TD} < -2\frac{m}{s}\right\} \tag{13}$$

In lateral direction the maximum carb angle, which is the deviation between aircraft azimuth angle and runway centerline direction must not exceed the following limit:

$$F^{(7)} = \{|\Delta\Psi_{TD}| > 5°\} \tag{14}$$

As the target aircraft in this paper is a CS23 aircraft the acceptable failure probability limits for each constraint in this paper are set to $P_F < 10^{-6}$.

V. Simulation Results

For evaluation different Subset Simulations have been performed. The proposal distribution for the modified Metrpolis Hasting algorithm is chosen to a uniform distribution within the interval $[-0.25; 0.25]$:

$$q = \mathcal{U}(-0.25, 0.25) \tag{15}$$

This proposal distribution resulted in an average acceptance ratio of about 0.5 through out the simulations.

The following parameter were assumed to be stochastic with corresponding distribution:

Symbol	Description	Distribution
$\left(u_{W_{20}}\right)_N^E$	Mean wind at 20ft above runway in runway direction [m/s]	$\mathcal{N}(1.27, 6.67)$
$\left(v_{W_{20}}\right)_N^E$	Mean cross wind at 20ft above runway [m/s]	$\mathcal{N}(0, 6.85)$
w_1	First White noise for Turbulence model	$\mathcal{N}(0,1)$
w_2	Second White noise for Turbulence model	$\mathcal{N}(0,1)$
w_3	Third White noise for Turbulence model	$\mathcal{N}(0,1)$
w_4	Fourth White noise for Turbulence model	$\mathcal{N}(0,1)$
k_{CL}	Lift uncertainty factor for ground effect	$\mathcal{U}(1,1.2)$
$\delta h_{AGL_{meas}}$	Radar Altimeter Bias [ft]	$U(-1,1)$
$\left(x_{v_W}\right)_N$	Mean position in respect to runway threshold of lateral gust [m]	$\mathcal{U}(-500, 500)$
σ_{v_W}	Width of lateral gust [m]	$\mathcal{U}(1, 250)$
$\left(I_{v_w}\right)_N$	Maximum amplitude of lateral gust [m/s]	$\mathcal{U}(-2, 2)$
$\left(x_{w_W}\right)_N$	Mean position in respect to runway threshold of vertical gust [m]	$\mathcal{U}(-500, 500)$
σ_{w_W}	Width of vertical gust [m]	$\mathcal{U}(1, 250)$
$\left(I_{w_w}\right)_N$	Maximum amplitude of vertical gust [m/s]	$\mathcal{U}(-2, 2)$

The distribution for the mean wind at 20ft above runway are taken and derived from CS-AWO and cutted off at $\pm 5kts$ for the current evaluation. The white noise inputs w_1 to w_4 are normally distributed values over time. As the radar altimeter triggers the flare maneuver of the aircraft, a bias error in the measurement is assumed. This error is assumed being uniform distributed.

For automatic landing changes in wind close to ground are very dangerous. Therefore the effect of sudden gusts close to ground and located near threshold are taken into account. The maximum assumed amplitude is $2\frac{m}{s}$ in any direction.

The assessment criteria are defined according to section **V**. Usually one separate Sub Simulation run has to be performed for each single constraint in order to state on the failure probability of each single requirement. Nevertheless any approach is only successful if all constraints are satisfied. Therefore each simulation run is checked against all constraints and in case of violation the corresponding data is stored.

In the following the results for the crab angle constraint $F^{(7)}$ and the bank angle constraint $F^{(5)}$are shown exemplarily. Both Subset Simulations are run with a sample size of $N = 500$. And the subset Simulation is stopped after 6 iterations. The target probability for intermediate events is set to $P_{des}(F_i) = 0.1$ which makes the algorithm to stop if the probability of 10^{-7} has been reached. The resulting coefficient of variation (COV), for the probability estimation of 10^{-6} is approximately $COV = 0.35$.

V.A Crab angle constraint

Figure 2 shows the subset simulation results for the crab angle constraint. The crab angle describes the deviation of the aircraft's orientation with the runway center line or the direction of flight respectively. In order to not damage the landing gear during touch down, this angle may not be too large. The maximum crab angle depends on the corresponding landing gear. Airbus for example allows a maximum crab angle of 5° for the A320 at touchdown [8].

For the aircraft considered in this paper the maximum crab angle is correspondingly set to 5° as well. Figure 2 shows the subset simulation results for the crab angle constraint during touch down. It can be seen, that the failure probability limit $P(F) = 10^{-6}$ is located at $\Delta\Psi = 2.5°$ which is only the half of the acceptable limit. Hence also regrading the COV of 0.35 the approach is safe in respect to the crab angle limit.

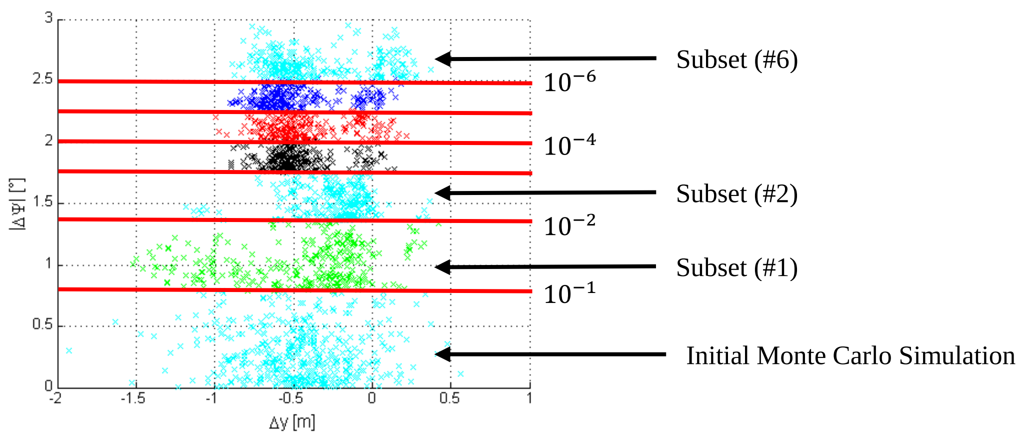


Figure 2 Subset Simulation result for crab angle limit

From Figure 2 the general principal of subset simulation can be seen quite well. During each subset set the algorithm places points closer and closer to the final target. In this example the algorithm does reach the final target failure probability of 10^{-6} before it reaches the maximum limit and it consequently stops.

V.B Bank angle constraint

Figure 3 shows the subset simulation results for the bank angle limit. It reveals that the probability for the bank angle $|\Phi|$ exceeding 3.6° is $P = 10^{-6}$. Even regarding the coefficient of variation with $COV = 0.35$ it reveals that $\mathrm{P}(F^{(5)}) < 10^{-6}$ and hence the system is safe in respect to this constraint.

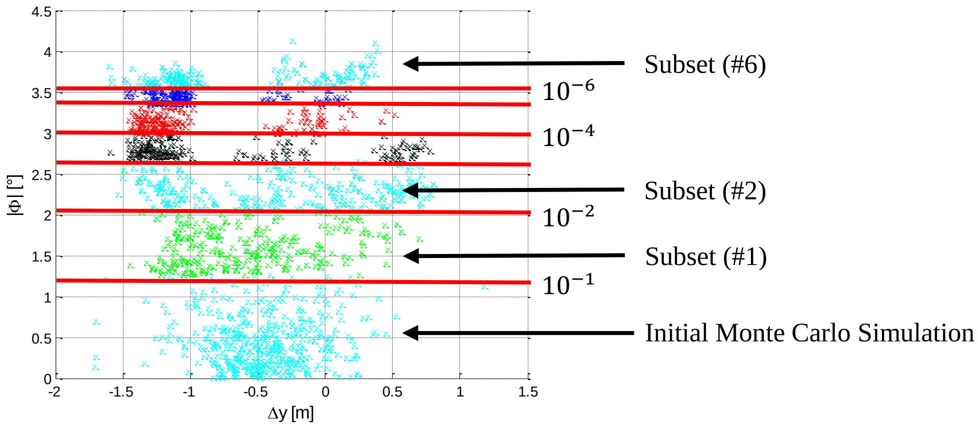


Figure 3 Subset Simulation result for bank angle limit

VI. Conclusions

In this paper a developed automatic landing system is investigated if it satisfies safety requirements specified in terms of very low failure probability limits. For that purpose "Subset Simulation" is applied. The investigation criteria are specified and the Subset Simulation results for the crab angle and bank angle constraint are displayed. The investigation currently only covers environmental disturbances, ground effect uncertainties and an offset in the radar altimeter measurement. Additional investigations on the effect of aerodynamic parameter uncertainties, additional sensor errors and higher wind conditions have to be performed and analyzed.

The subset Simulation is suitable to evaluate the small failure probabilities for automatic landing conditions.

ACKNOWLEDGMENTS

This research is funded by the German Federal Ministry of Economics and Technology. The authors wish to thank the active support and engagement of the project sponsors from the DLR Space Administration.

Supported by Federal Ministry of Economics and Technology on the basis of a decision by the German Bundestag

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