A COMPUTATIONAL SCHEME FOR TORNADO MISSILE STRIKE PROBABILITY USING STOCHASTIC CORRELATION BETWEEN LOCAL WIND SPEED AND FLIGHT DISTANCE

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An efficient evaluation method for probability of tornado missile strike without employing Monte Carlo method is proposed in the paper, and software is developed to numerically compute such probability especially for unconstrained objects on the ground around a cylindrical structure. The probability evaluation is based upon the numerical results computed by TONBOS, which enables us to evaluate lift-off and flight behaviors of unconstrained objects on the ground driven by a tornado. In TONBOS, wind field of a tornado is modeled by Fujita's DBT-77, while motion of objects in flight is modeled with three dimensional translational equations where aerodynamic drag force, lift force due to ground effect and gravity are taken into account. Using TONBOS, we have obtained stochastic correlation between local wind speed, V, and flight distance, L, of each object, and such stochastic correlation was used to evaluate probability of missile strike. The evaluation method was applied to a sample problem to qualitatively confirm the validity, and quantitatively verify the results for two extreme cases where an object is located just in the vicinity of and far away from the structure.

I. INTRODUCTION

Tornado missile impact on structure, system and component (SSC) of a nuclear power plant can lead to physical and functional damages with possible deviation from normal plant conditions. To understand such risk induced by tornado missiles, we need to evaluate not only the degree of damages incurred but also the probability of the occurrence. In the United States, probabilistic evaluation of tornado missile hazard and fragility was intensively and extensively studied around 1980. Especially, Electric Power Research Institute (EPRI) developed a probabilistic evaluation code, TORMIS to compute SSC damage probability and its uncertainty range [1], [2a,2b]. In recent years, TMSC (Tornado Missile Strike Calculator) has been developed by Westinghouse Electric Company [3] and was shown to yield results comparable to those of TORMIS. In these tools, probability of missile strike is basically computed by a kind of Monte Carlo method with running an enormous number of simulation cases.

The objectives of this study are to formulate an efficient evaluation method for probability of tornado missile strike without employing Monte Carlo method, and to develop software to numerically compute such probability, especially for unconstrained objects (possible missiles) sitting on the ground. For the purpose, we have employed a computational code, named TONBOS [4], [5], which is able to evaluate lift-off and flight behaviors of unconstrained objects on the ground driven by a tornado. In TONBOS, tornado wind field is modeled by Fujita's DBT-77 model [6] as well as the Rankine vortex model [7], [8], while motion of objects in flight is modeled with three degrees-of-freedom translational equations where aerodynamic drag force and gravity are taken into account [7], [8]. The unique feature of TONBOS is that objects are assumed to be subject to lift force near the ground which is generated by asymmetric air flow around objects due to ground effect. Though TONBOS was mainly used in deterministic design of tornado missile speed, it has recently been applied to evaluate probabilistic characteristics by initially placing enough number of objects around and within a tornado in dense and computing the stochastic features [9], [10]. One of the conclusions derived from the stochastic results is that the design speed of automobile indicated in NUREG 1.76 [11] (computed for an object released from a specific point at 40m elevation with Rankine vortex model) is close to the median value of the horizontal speeds of many automobiles initially placed at 40m elevation around a tornado. It was also seen that the design speeds computed by the method of NUREG 1.76 are comparable to the maximum horizontal speeds computed for car and truck on the ground using the wind field of the Fujita model.

Furthermore, the authors have extended the probabilistic application of TONBOS to missile strike computation [12]. In the present paper, we will fully explain the evaluation scheme for annual missile strike probability with stochastic correlation between local wind speed, *V*, and flight distance, *L.* The method will be applied to automobiles on the ground, demonstrating that such an annual strike probability can be efficiently computed without employing Monte Carlo method.

II. THEORY

II.A. Overall Framework

First, basic equations to describe the lift-off and flight of a missile are explained in II.B as well as the tornado wind filed model in II.C, which are employed in TONBOS. In section II.D, it is explained how we can obtain stochastic correlation between local wind speed, V, and flight distance, L, of each object initially sitting at a specific local point relative to the tornado position and moving direction. These results are used to compute conditional probability density function, $S_V(L)$, with respect to flight distance under a specific local wind speed condition. Furthermore, strike probability, $q(r, L)$, of an object at position, r, whose flight distance is L, will be defined by geometrical condition, which will be explained in section II.E. Strike probability, $Q_V(r)$, under a specific local wind speed condition can be obtained *via* convolutional integration of product of $S_V(L)$ and $q(r, L)$ over L, as shown in the left hand side of Fig.1. On the other hand, annual exceedance probability of local wind speed, $H(V)$, can be computed by a tornado hazard analysis code, *e.g.*, TOWLA (Tornado Wind Speed Hazard Model for Limited Area) [13], and the probability density function, $p(V)$, with respect to local wind speed is obtained by the partial differentiation of $H(V)$ with respect to V. Finally, we can get annual probability of tornado missile strike on a structure with the convolutional integration of product of $Q_V(r)$ and $p(V)$ over V, which will be explained in section II.F. As an example of this evaluation method, probability that a car on the ground strikes on a tall circular cylinder standing vertically was computed in section III. We will confirm not only qualitative validity of the result, but also quantitative consistency for two special cases where an object is located in vicinity of and far away from the structure.

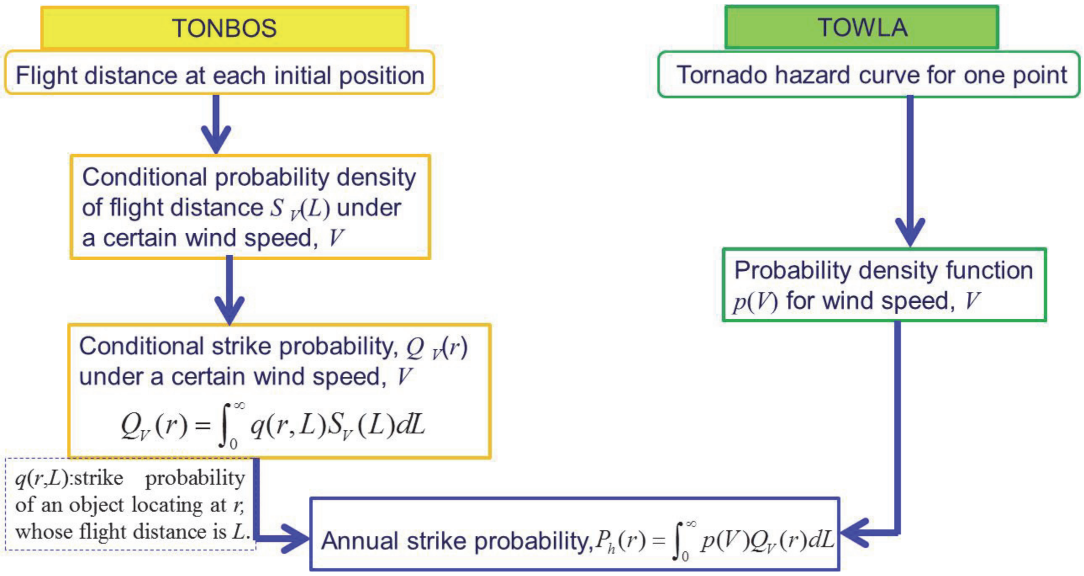


Fig.1 Computational flow of annual strike probability

II.B. Missile Lift-off and Flight Model

II.B.1. Lift-off Model

Objects on and near the ground are assumed to be lifted up by lift force generated by asymmetric air flow around the objects due to ground effect as shown in Fig.2. Such a lift force, F_L, is expressed by a lift coefficient, $C_L(z)$ as follows, with horizontal relative wind speed, $|\mathbf{V}_w\text{-}\mathbf{V}_M|$ and projection area, a, where $\mathbf{V}_w$ and $\mathbf{V}_M$ are wind velocity vector and missile velocity vector, respectively.

$$F_L = \frac{1}{2}\rho C_L(z_c)a\left|\mathbf{V}_w - \mathbf{V}_M\right|_{x,y}^{2} \qquad (1)$$

In eq.(1), ρ is air density and suffix *x, y* attached to | | denotes horizontal component magnitude. To maintain conservatism and to increase practicability, the authors proposed the following model instead of eq. (1) in TONBOS.

$$F_L = \frac{1}{2}\rho C_D A\, f(Z/d)\left|\mathbf{V}_w - \mathbf{V}_M\right|_{x,y}^{\;2} \tag{2}$$

where C_DA and $f(Z/d)$ are define as follows.

$$C_D A = \frac{C_{D_x}A_x + C_{D_y}A_y + C_{D_z}A_z}{3} \quad (3), \qquad f(Z/d) = \begin{cases} \dfrac{1-(Z/3d)}{1+(Z/d)} & (0 \le Z \le 3d) \\ 0 & (3d < Z) \end{cases} \quad (4)$$

In the above, C_{Di} and A_i are respectively a drag coefficient and a projection area for *i*-direction flow, while Z is a gap between an object and the ground, *i.e*, $Z=z_c-d/2$, where z_c is elevation of object center from the ground level, and d is object height. Conservatism of eq.(2) can be justified with various experimental data described in relevant literatures [14]-[19].

II.B.2.Flight Model

In TONBOS code, the three DoF (degree-of-freedom) dynamic equation is employed for missile flight motion, as originally proposed by Simiu and Cordes [7] as well as Simiu and Scanlan [8]. The schematic image of the dynamic motion of a missile is depicted in Fig.3, while the basic equation can be written in the following form, with taking the ground effect into account.

$$\frac{d\mathbf{V}_M}{dt} = \frac{1}{2}\rho\frac{C_DA}{m}\left|\mathbf{V}_w - \mathbf{V}_M\right|(\mathbf{V}_w - \mathbf{V}_M) - (g-L)\mathbf{k} \tag{5}$$

where C_DA/m is the flight parameter, g is gravitational acceleration and $\mathbf{k}$ is unit vector toward z direction. The lift acceleration due to the ground effect, L, is defined as follows.

$$L = \frac{1}{2}\rho\frac{C_DA}{m} f(Z/d)\left|\mathbf{V}_w - \mathbf{V}_M\right|_{x,y}^{\;2} \tag{6}$$

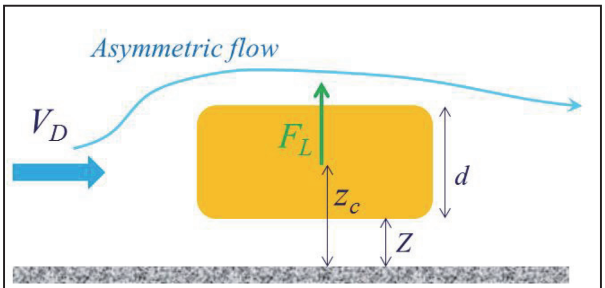
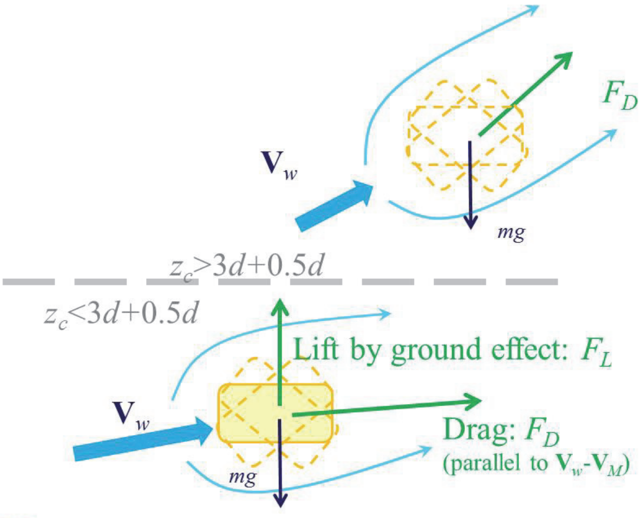


Fig.2 Flow pattern around object on/near the ground



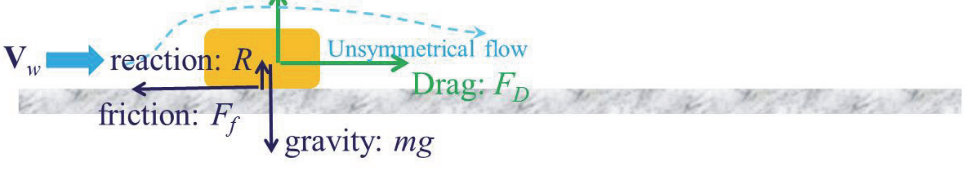


Fig. 3. Schematic image of the dynamic motion of a missile modeled in TONBOS

II.C. Tornado Wind Field Model

In the Fujita model[6], tornado core is divided into inner core and outer core as shown in Fig.4, whose maximum radii are denoted by R_i and R_m, respectively, and both are related with the following formula proposed by Fujita.

$$R_i = \nu R_m \quad (7), \qquad \nu = 0.9 - 0.7\exp(-0.005 R_m) \quad (8)$$

where unit of R_m is meter. The Fujita model is also composed of a boundary layer near the ground, named inflow layer, whose height is denoted by H_i and is related with the following empirical formula.

$$H_i = \eta R_m \quad (9), \qquad \eta = 0.55(1-\nu^2) \quad (10)$$

In the following, all the velocity components of the Fujita model are written as function of non-dimensional radius $\underline{r}$ ($=r/R_m$) and non-dimensional axial co-ordinate $\underline{z}$ ($=z/H_i$). The tangential wind velocity V_θ is expressed as follows.

$$V_\theta(r,z) = \min(\underline{r}, \underline{r}^{-1}) \times \min(\underline{z}^{k_0}, \exp\{-k(\underline{z}-1)\}) \times V_m \quad (11)$$

where V_m is the maximum horizontal velocity, which is equal to the maximum tangential velocity in the Fujita model.

For constants k_0 and k, we set k_0=1/6 and k=0.03 as recommended. The radial component, V_r, which is positive for outward direction, is derived from continuity of fluid as follow.

$$V_r(r,z) = \begin{cases} 0 & (\underline{r} \le \nu) \\ \dfrac{V_\theta \tan\alpha_0}{1-\nu^2}\left(1 - \dfrac{\nu^2}{\underline{r}^2}\right) & (\nu < \underline{r} < 1) \\ V_\theta \tan\alpha_0 & (\underline{r} \ge 1) \end{cases} \quad (12), \qquad \tan\alpha_0 = \begin{cases} -A(1-\underline{z}^{1.5}) & (\underline{z} < 1) \\ B\{1-\exp(-k(\underline{z}-1))\} & (\underline{z} \ge 1) \end{cases} \quad (13)$$

The upward wind velocity V_z, is defined as follows, which exists only in the outer core region and is independent on radius in the region.

$$V_z(z) = \begin{cases} \dfrac{3}{28}\dfrac{\eta V_m}{1-\nu^2} A(16\underline{z}^{\frac{7}{6}} - 7\underline{z}^{\frac{8}{3}}) & (\underline{z} < 1) \\ \dfrac{\eta V_m B \exp(-k(\underline{z}-1))}{k(1-\nu^2)}\{2 - \exp(-k(\underline{z}-1))\} & (\underline{z} \ge 1) \end{cases} \quad (14)$$

where A and B are constants, and we employ a recommended value A=0.75, while constant B is given by $B=3kA/(k_0+1)(k_0+2.5)$ due to continuity constraint.

Typical velocity vectors on tornado center cross-section are shown in Fig.4. Wind velocity at (x, y, z) at $t=T$ is given by assuming that the center of tornado is located at origin initially (t=0), and that the tornado translationally moves toward x-direction at constant speed of V_{tr} as shown in Fig.5.

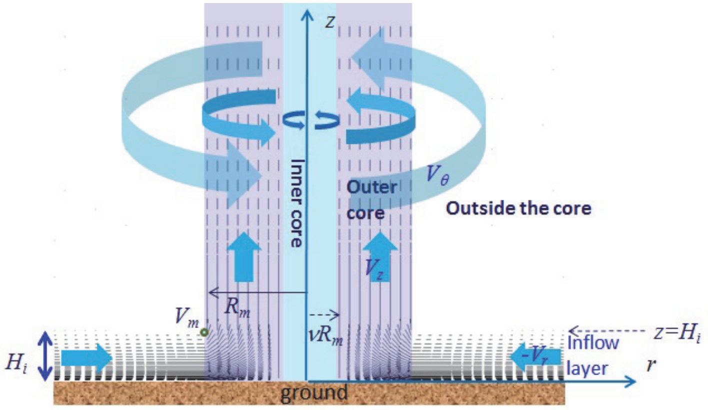


Fig. 4 Cross-sectional view of wind field of Fujita model.

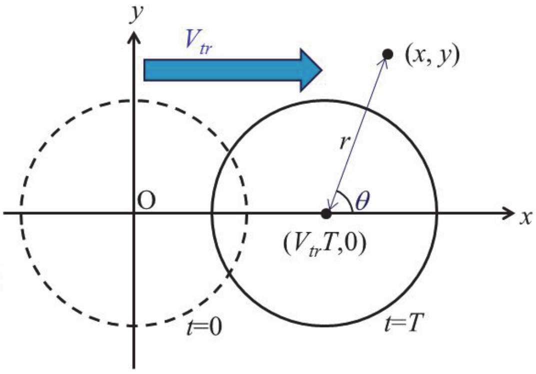


Fig. 5 Translational motion of tornado.

II.D. Stochastic Correlation between Wind Speed and Flight Distance

Assuming the wind field of a translating tornado defined by the Fujita model, maximum local wind speed, *V*, experienced at the 10m above the ground level (AGL), can be calculated at arbitrary points in (*x,y*) plane. On the other hand, flight distance, *L*, of each object initially placing at arbitrary points in (*x,y*) plane as shown in Fig. 6 can be computed by TONBOS. The flight distance, *L*, is dependent on the location of the local point relative to the tornado position and moving direction. Especially, objects around the origin tend to fly for large distance, because the tornado is assumed to initially appear at the origin, imposing high wind on the objects around origin. On the other hand, objects in downwind side (+*x* region) tend to fly for short distance, because low wind triggers the lift-off of the objects and hits the ground shortly. To be conservative, such a flight distance was replaced by maximum value at upwind side in this study. Figures 7 (a) and (b) show the spatial distributions of maximum local wind speed, *V*, and that of flight distance, *L*, respectively, computed for automobiles (flight parameter:0.0097m^2/kg, height:1.5m) for a design F3 tornado (maximum wind speed:92m/s, radius:30m) with TONBOS. Then, the correlation between *V* and *L* is obtained as shown in Fig.8.

Based on these numerical results, we are able to compute conditional probability density function, $S_V(L)$, with respect to flight distance under a specific local wind speed condition, so that the probability density function, $S_V(L)$, should satisfy the following mathematical restriction, $\int_0^\infty s_V(L)dL = 1$.

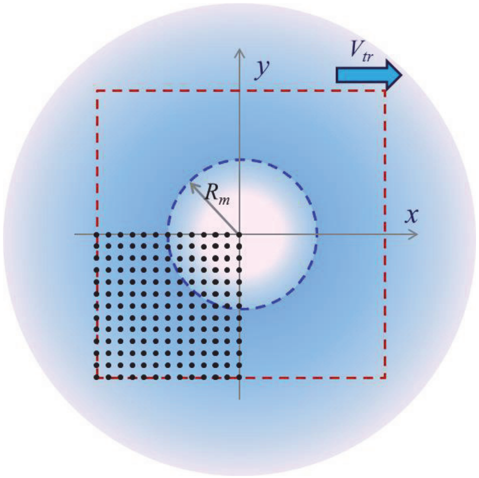


Fig. 6. Arrayed arrangement of objects around a tornado

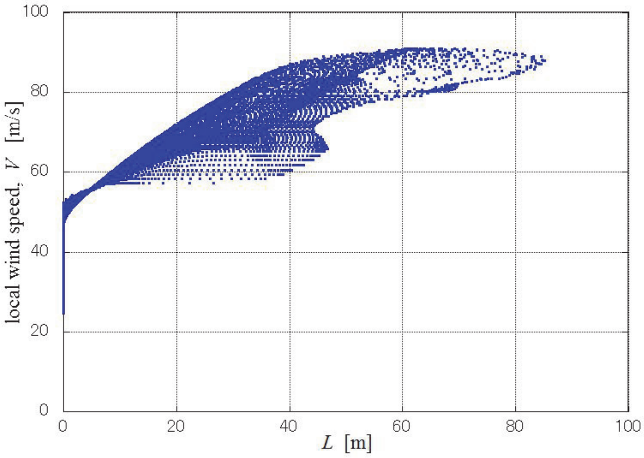


Fig.8 Plot on local wind speed, *V*, and flight distance, *L* plane

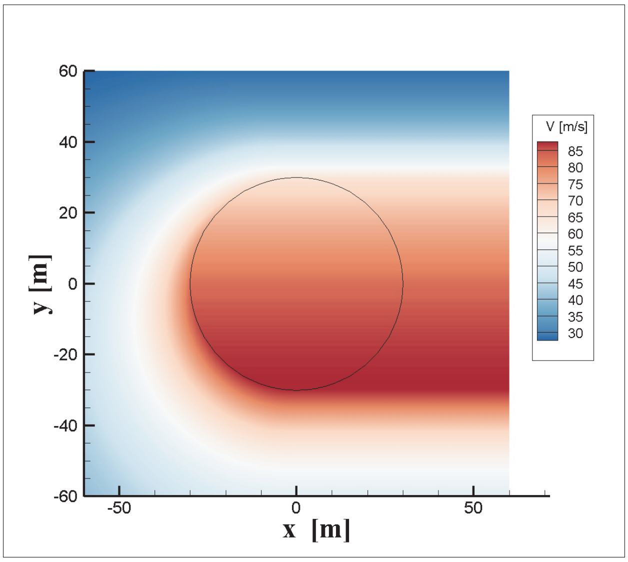


Fig. 7 (a) Distribution on maximum local wind speed, *V*

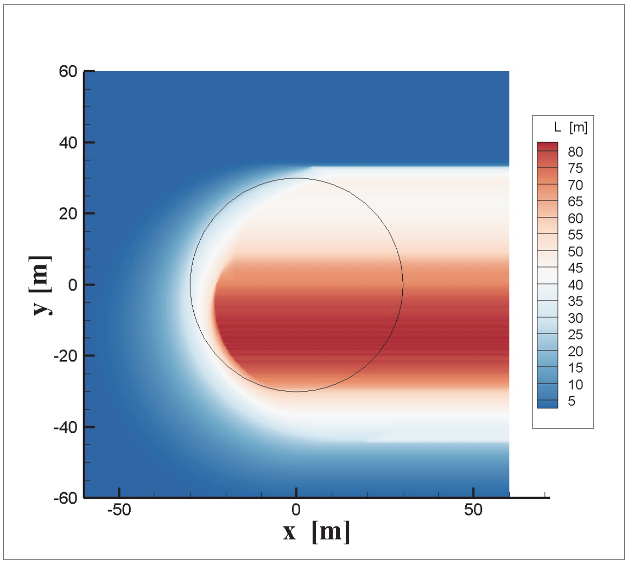


Fig. 7 (b) Distribution of flight distance of automobile, *L*

An example of the probability density function, $S_V(L)$, corresponding to Fig.8, is shown in Fig.9. High values of $S_V(L)$ are seen near the right-hand side end (boundary between zero and non-zero distribution) in this figure. This is because a flight distance in the downwind region was replaced by maximum value at upwind side to assure conservatism. It should be also noted that the probability density function, $S_V(L)$, can be expressed as a delta function $\delta(L)$ for $V<V_c$, where V_c is the critical local wind velocity below which the object will not be lifted-off and not be transported by tornado at all (*i.e.*, L=0).

II.E. Conditional Strike Probability

Assuming that the target structure is a tall cylinder or pillar whose radius is R_t, as shown in Fig.10, the strike probability, $q(r, L)$, of a missile object placed at radius, r, whose flight distance is L, can be conservatively calculated *via* eq.(15) derived from the geometrical condition.

$$q(r,L)=0 \quad (if \quad 0\le L\le r-R_t), \quad q(r,L)=\frac{1}{\pi}\sin^{-1}\left(\frac{R_t}{r}\right) \quad (if \quad r-R_t<L) \tag{15}$$

Then, conditional strike probability under a specific local wind speed condition, $Q_V(r)$, can be obtained by the integration of product of $S_V(L)$ and $q(r, L)$ over L, yielding eq. (16) .

$$Q_V(r)=\int_0^{\infty} q(r,L)S_V(L)dL=\frac{1}{\pi}\sin^{-1}\left(\frac{R_t}{r}\right)\int_{r-R_t}^{L_m} S_V(L)dL \tag{16}$$

II.F. Annual Strike Probability

Annual exceedance probability of local wind speed, $H(V)$, can be computed by a tornado hazard analysis code, *e.g.*, TOWLA (Tornado Wind Speed Hazard Model for Limited Area)[13]. The probability density function, $p(V)$, with respect to maximum local wind speed, V, is obtained by the partial differentiation of $H(V)$ with respect to V, *i.e.*, $p(V)=-dH(V)/dV$.

Then, the annual probability of tornado missile strike on a structure, $P_h(r)$, is computed with the convolutional integration of product of $Q_V(r)$ and $p(V)$ over V, as below.

$$P_h(r)=\int_0^{V_m} p(V)Q_V(r)dV=\int_{V_c}^{V_m} p(V)Q_V(r)dV=\frac{1}{\pi}\sin^{-1}\left(\frac{R_t}{r}\right)\int_{V_c}^{V_m} p(V)\left\{\int_{r-R_t}^{L_m} S_V(L)dL\right\}dV \tag{17}$$

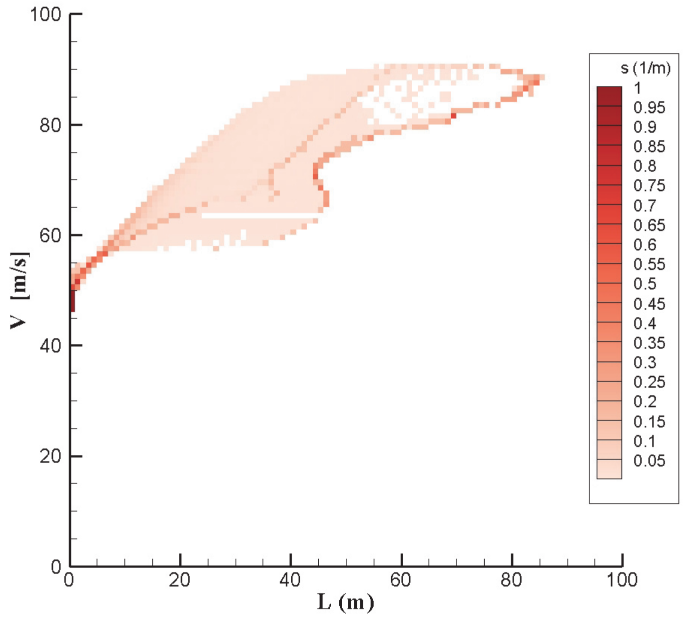


Fig.9 Example of the probability density function, $S_V(L)$

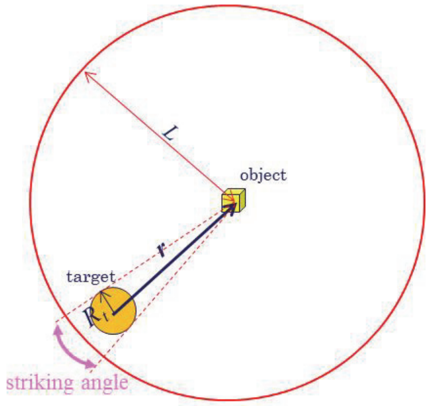


Fig.10 Geometrical condition of missile and target

III. NUMERICAL RESULTS

The assumptions of this numerical example is that an automobile whose flight parameter is 0.0097m^2/kg, and height is 1.5m, is located at radius r around a tall cylindrical structure of radius of 20m (R_t=20m). As for the tornado condition, maximum wind speed of 92m/s, translational speed of 14m/s, and core radius of 30m are assumed. Spatial distributions of maximum local wind speed, V, and that of flight distance, L, are computed with TONBOS, as shown in Figures 7 (a) and (b), respectively. Then, the probability density function, $S_V(L)$ is obtained as shown in Fig.9. On the other hand, annual exceedance probability of local wind speed, $H(V)$, is assumed such that $H(V)=10^{-6}$ year^{-1} at V=70m/s, while $H(V)=10^{-7}$year^{-1} at V=92m/s, which implies $H(V)=0.00152\exp(-0.10466V)$. With these assumed data, the annual strike probability of an automobile on the structure, $P_h(r)$, is computed via eq.(17), yielding the result shown in Fig.11.

The above result can be partially verified as follows. Figure 7 or 9 indicates that the minimum local wind velocity at 10m AGL required for lift-off, V_c, is about 45 m/s, which is a little larger than the critical wind speed at the level of an automobile on the ground, $U_c=(g/0.5\rho(C_DA/m))^{0.5}$=40.6m/s. Then, annual exceedance probability of local wind speed, $H(V)$, at the minimum local wind velocity, 45m/s, is calculated as 1.37x10^{-5}year^{-1} *via* $H(V)=0.00152\exp(-0.10466V)$, which is the probability that an automobile detaches from the ground and fly for any distance. If the automobile is located just in the vicinity of the structure, and if $V>V_c$, the possibility of strike should be an half depending on the flight direction. Therefore, the probability of an automobile strike on the structure is the half of 1.37x10^{-5}, *i.e.* 6.85x10^{-6}year^{-1}. The P_h value at $r=R_t$ shown in Fig.11 is very close to this value, indicating the consistency of the result. Furthermore, the P_h value beyond $r=80+R_t$ drastically decreases as seen in Fig.11, which is consistent with the maximum flight distance of automobile, *i.e.* about 83m.

IV. CONCLUSIONS

The authors have formulated an efficient evaluation method for probability of tornado missile strike without employing Monte Carlo method, and developed software to numerically compute such probability, especially for unconstrained objects (possible missiles) sitting on the ground. The stochastic correlation between local wind speed, V, and flight distance, L, of each object initially placing around a tornado was computed by a computational code, named TONBOS, which enables us to evaluate lift-off and flight behaviors of unconstrained objects on the ground driven by a tornado. Such stochastic correlation was used to evaluate annual probability of missile strike. The evaluation method was applied to the evaluation of strike probability of an automobile around a tall cylindrical structure, demonstrating the qualitative validity, and quantitative consistency for special cases where an object is located just near and away from the structure.

As future works, authors will extend the method to more general tool so that variations of tornado characteristics such as intensity and scale can be incorporated into the strike probability evaluation. The authors also believe that the method should be improved so that various geometries of targets can be dealt with for the purpose of more practical applications.

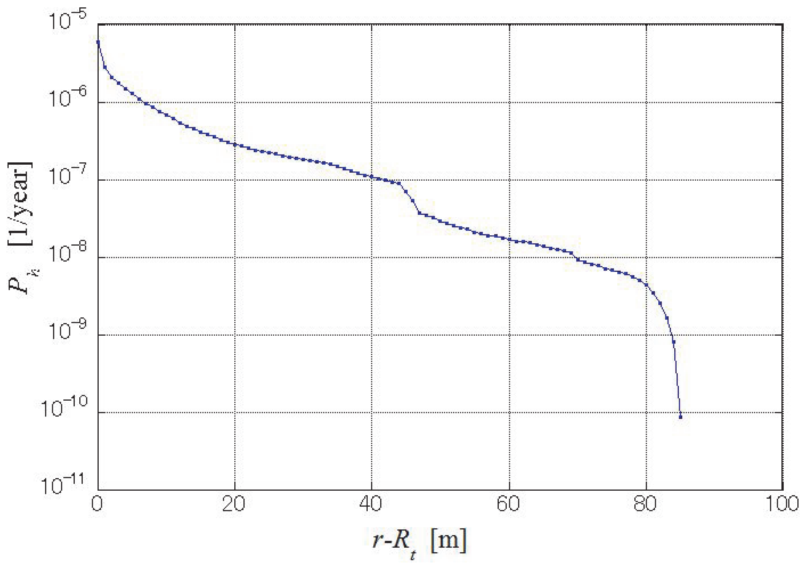


Fig.11 Annual strike probability of an automobile on the structure, $P_h(r-R_t)$

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